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PUBLISHER'S NOTE

The articles in this volume, as in all others in the Collected Studies Series, have not been given a new, continuous pagination. In order to avoid confusion, and to facilitate their use where these same studies have been referred to elsewhere, the original pagination has been maintained wherever possible.

Each article has been given a Roman number in order of appearance, as listed in the Contents. This number is repeated on each page and quoted in the index entries.

Where the author has indicated amendments to the text, as listed under 'Corrections', the amendment has been marked by an asterix in the margin for ease of reference.

PREFACE

The articles assembled in this volume span a period of some thirty years, during which my teaching and research interests focused almost entirely on Arabic science and philosophy. While being a selection of studies concerned with various aspects of the intellectual endeavour of medieval Islam, the articles intersect in so many points as to justify their being brought together between two covers—a fact which I was able to convince myself of only when I sat down to prepare the Index.

The reader should know that some of these articles are complemented by other published studies which it was not possible to include in this volume for technical or other reasons. Thus the piece on "The appropriation and subsequent naturalization of Greek science in medieval Islam" is complemented by the chapter on "The scientific enterprise" which I contributed to the volume edited by Bernard Lewis, *The World of Islam*, published by Thames and Hudson, London, and Alfred Knopf, New York, 1976 (1st edn), and by two pieces on "Islamic optics" and "Science" in the *Dictionary of the Middle Ages*, ed. J.R.S. Strayer, Charles Scribner's Sons: New York, vols IX & XI (1987, 1988). The reader should also know that Ibn al-Haytham's text on "The appearance of the stars", referred to but not reproduced *in toto* in Chapter IX, is now published, together with English translation, in *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften*, 7 (1991/92), pp. 31-72. Edition of another text discussed in the same chapter, Ibn al-Haytham's "Solutions" to problems raised by one of his contemporaries concerning some optical statements in the *Almagest*, is forthcoming in the same journal.

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A. I. Sabra

I

THE APPROPRIATION AND SUBSEQUENT NATURALIZATION OF GREEK SCIENCE IN MEDIEVAL ISLAM: A PRELIMINARY STATEMENT

I. APPROPRIATION VERSUS RECEPTION

A merely kinematic account of the transmission of scientific knowledge from one culture to another (where culture is understood mainly as a spatio-temporal region) would be a description of movements of scientific products (texts, concepts, theories, techniques, etc.), in abstraction from the forces underlying these movements. The principal aim of such a description would be to state when and where the transport of these products took place, and thus to determine the path of their movement. Since transmission is often accompanied by some kind of change, a kinematic approach is bound to take account of certain facts of transformation, such as the fact that the occurrence of a text at a later time differs linguistically from its occurrence at an earlier time (translation); or that the later occurrence is an otherwise different form (summary, revision, development, etc.) of the earlier one. When possible, names of persons are attached to such events: they serve as shorthand symbols that designate certain points or, rather, small regions on the space-time continuum — namely, the regions occupied by the named persons.

As a partial description of transmission events, the kinematic picture is of course indispensable, and it will always constitute part of the story that the historian will want to tell. In some cases, it is all that the historian may believe he can provide with confidence. However, when joined, as it often is, with statements of interpretation, it tends to invite intrusion of one or two well-known extremes: reductionism and precursorism. This is not a necessary consequence of the kinematic approach, but frequently seems to be called upon to fill a vacuum from which this approach is felt to suffer. As far as Islamic science is concerned, reductionism is the view that the achievements of Islamic scientists were merely a reflection, sometimes faded, sometimes bright or more or less altered, of earlier (mostly Greek) examples. Precursorism (which has a notorious tendency to degenerate into a disease known as

'precursitis') is equally familiar: it reads the future into the past, with a sense of elation.

These two extreme views have given rise — if only half-consciously — to a counter research program which strives to see things as they really are and, accordingly, seeks to emphasize the differences between the products of two cultures rather than their similarities. It is in the application of this program that the word 'context' inevitably appears. We must see things in their proper context, it is asserted with appropriate urgency. But what this recommendation amounts to in practice is a question which historians of Islamic science have yet to determine in a rigorous manner.

The transmission of Greek science to medieval Islam was an important event in the history of civilization; it had far-reaching consequences for the history of the classical heritage, for the development of Islamic thought and culture, and for the European renaissance of the twelfth and later centuries. The student of Islamic civilization is concerned with the meaning of this event for this civilization itself and, if he is an historian of science, with what it can reveal about the origins and the specific character of the scientific tradition it initiated. His approach is not, therefore, that of the classicist who might, as such, be interested in Arabic scientific texts solely for the purpose of 'recovering' lost Greek works or 'reconstructing' obscure episodes in Greek thought. Nor is it the approach of the Latin medievalist who will seek knowledge of Islamic science and philosophy only in so far as they influenced later developments in Europe. Both of these other approaches are of course quite legitimate from their own perspectives, and there are many examples to show that Arabic material selected from their points of view can be put to good historical use. The Islamist may himself be inclined to use his knowledge and expertise in his own field to contribute to their successful implementation. But the results of this research, whether it is done by an Islamist or a non-Islamist, cannot be regarded as contributions to the study of Islamic science for its own sake, except, perhaps, in a partial and indirect way. The reason is that the problems posed in such research, and the terms in which they are formulated, belong to areas other than those with which the Islamist, *qua* Islamist, is chiefly concerned.

Put simply and briefly, the Islamist's approach is that which looks at science in Islam as a phenomenon of Islamic civilization — a phenomenon which must be understood and explained in terms peculiar to that civilization. Before going into a discussion of some of the implications of this point of view, it might be well to point out here that, far from being totally indifferent to the perspectives of the classicist and the Latin medievalist, it promises to be of value to both groups of scholars, as well as to the not-so-specialized historian who is generally concerned with the cross-cultural migration of scientific ideas. Three points may be made briefly. (1) What we call the Greco-Arabic transmission of science and philosophy was a complex process in which translation was often much influenced by interpretations imparted through a prior scholastic tradition and, sometimes, in terms already in technical use in the newly formed disciplines concerned with the Arabic language and with Islamic religion. As a consequence, the task of 'reconstructing' a scientific or philosophical Greek source cannot always be expected to take the form of a straightforward translation back into a supposed Greek original, but must take into account activities which took place outside the extant text and which therefore have to be reconstructed independently of the text. (2) Islamic 'extensions', 'developments', and 'reworkings' of Greek ideas from al-Kindī to Averroes and al-Biṭrūjī (to choose the period of interest to the medievalist) cannot be fully appreciated without reference to cultural situations that conditioned the direction and character of these works.[1] As for cross-cultural transmission, it is clear (3) that its presentation in isolation from cultural factors would remain an incomplete description, one which cannot by itself explain large transformations that frequently occur when cultural boundaries are crossed.

The fact that Islamic science, or rather a segment of it, happened to perform an intermediary role between the Greek and Latin medieval traditions has had certain untoward consequences, other than attracting the attention of scholars serving the interests of research in the two adjacent fields. Despite the profound effects of these consequences there is little evidence to show that historians of Islamic science have been critically aware of them. Apparently because of the importance of that role in world intellectual history many scholars have been led to look at the medieval Islamic period as a period of reception, preservation and transmission, and this in turn has affected the way in which they have viewed not only individual achievements of that period but the whole of its profile. To illustrate this last point let us consider the very word 'reception' which is commonly applied to the initial event in the history of Islamic science — I mean the transfer of the corpus of Greek science to Islam in the eighth and ninth centuries. 'Reception' can of course be used, and is sometimes used, as a value-free word referring to the bare movement of translation; and as a kinematic term it may be quite harmless. But does it adequately describe the event in question? 'Reception' might connote a passive receiving of something being pressed upon the receiver, and this might reinforce the image of Islamic civilization as a receptacle or repository of Greek learning. This, however, was not quite what happened; the transmission of ancient science to Islam would be better characterized as an act of appropriation performed by the so-called receiver. Greek science was not thrust upon Muslim society any more than it was later upon Renaissance Europe. What the Muslims of the eighth and ninth centuries did was to seek out, take hold of and finally make their own a legacy which appeared to them laden with a

variety of practical *and* spiritual benefits. And in so doing they succeeded in initiating a new scientific tradition in a new language which was to dominate the intellectual culture of a large part of the world for a long period of time. 'Reception' is, at best, a pale description of that enormously creative act.

It is true that the agents of this transmission (the Ḥunayns and Thābits) were non-Muslims who had espoused a Hellenistic outlook which had been developed in pre-Islamic times and continued to be cultivated in non-Muslim institutions after the Arab conquests. But their activity as translators was positively and generously supported (not just tolerated) by the Muslim ruling establishment — the caliphal court itself and individuals closely associated with it. The extent of their spectacular achievement would not have been possible in the absence of attitudes such as those which manifested themselves, for example, in the energetic and extensive efforts to acquire Greek manuscripts and in the founding of Bayt al-Ḥikma, the great library and institute for the promotion of the philosophical sciences, which the early ᶜAbbāsids established at Baghdad.

A particularly significant representative of such attitudes in the ninth century was the philosopher Abū Yūsuf Yaᶜqūb ibn Isḥāq al-Kindī, the son of a former governor of Kūfa. Al-Kindī was not only eager and able to absorb and expound an astounding amount of Greek science and philosophy, but also wrote openly and forcefully against religious bigotry and intellectual insularity.[2] Being himself an Arab and a Muslim and an intimate associate of the ᶜAbbāsid court, al-Kindī was destined to serve the cause of Hellenism more effectively than was possible for its non-Muslim advocates to whom he owed his knowledge of the contents of Greek thought.

To look at Islamic civilization as a transmitter of knowledge is to view it from a point outside it. It is an extraneous view that suggests externally inspired and externally selected research programs. An historian concerned with the Arabo-Latin transmission of science may not, for example, be compelled to learn about al-Bīrūnī, one of the greatest of Islamic scientists, because al-Bīrūnī was not known to the Latin West. I have even heard some medievalists say that they need not deal with the Arabic originals of Latin translations because it was only these translations that were read and interpreted by European thinkers. While a similar attitude should certainly *not* be recommended to Arabists in regard to the Greek sources, the medievalist may be trying to make a valid point. To take an example that comes quickly to mind: the Latin Averroes, namely Averroes as interpreted and understood by Christian philosophers, may indeed turn out to be quite different from the Averroes who was responding to a specific situation at a specific time and place in Islamic history. But herein, precisely, is the danger that lurks for the Islamist who allows his own choice of research problems to be unduly determined by facts of transmission, however important these may be. An illustration of what I have in mind is provided by recently published research in Islamic planetary theories. It will be true to say that interest in the so-called "non-Ptolemaic" models imagined by Islamic astronomers from al-Ṭūsī to Ibn al-Shāṭir, was initially spurred by the observation of certain close similarities between these models and their counterparts in Copernicus.[3] And the hypothesis seemed plausible that Copernicus might have been influenced by his Islamic predecessors. The hypothesis certainly was, and still is, worth pursuing. If future research produces a definitely positive answer, we will gain in our understanding of Copernicus's work and may perhaps discover a hitherto unknown channel of transmission of Arabic science to Europe.

But what about the significance of the new models for the history of Islamic astronomy itself? What do we learn from them about the scientific outlook and motivation of Islamic astronomers? What do these models reveal concerning their author's interpretation of Ptolemaic astronomy or their understanding of the nature and aim of astronomical investigations? To what extent do they help us in evaluating other types of astronomical research by the same authors or by others in the same period? Is it true that they demonstrate a certain 'poverty' or lack of inventiveness on the part of Islamic mathematicians, as Carra de Vaux thought on the basis of al-Ṭūsī's example;[4] or that the pervasive concern for physical reality (which undoubtedly motivated the new models) betrayed an essentially non-Ptolemaic predilection for sensualism, as Duhem believed?[5] These are obvious and interesting questions that concern Islamic astronomy *per se*, and therefore one would have expected them to receive a fair share of the attention given to these models. But though many excellent articles have been devoted to their analysis since 1957, they show no determined effort to deal with these questions in detail.[6] It is reasonable to assume that preoccupation with the question of influence has been responsible for this neglect.

I hope I shall not be understood as advocating a parochial point of view. Historians of *Islamic science* are also historians of *science*, and, as such, they have a serious interest in scientific developments wherever and whenever they have occurred. It is also natural for historians of Islamic science to be interested in external developments that connect in some way or another with events in the period of their primary concern. But that should not be allowed to weaken their search for a perspective of their own or interfere with their attempt to define the scope and methods of their own discipline.

I have suggested that we describe the transmission of ancient science to Islam as an act of 'appropriation' rather than a mere 'reception'. This is not to propose replacing one term by another, but to indicate something of the nature of that event. Both terms imply the prior existence and subsequent transfer of what was to be appropriated or received. But they carry different interpretations, and an act of appropriation may demand explanations which

an act of reception may not call for, at least not with equal emphasis. In the case under consideration 'prior existence' refers primarily to the existence (though not necessarily the immediate availability) of the texts that later were translated; but also involved is the prior existence of a translation activity from Greek into Syriac and the existence of individuals capable of rendering scientific and philosophical works from these (and other) languages into Arabic. As is well known these initial conditions had obtained for a long time in the Near East. Thanks to the continued existence of Hellenized, mainly Christian, communities and centres of learning, and soon after Islamic rule had settled over that area, it appears that some Arabic translations of scientific and pseudo-scientific literature began to be made in sporadic, unregulated fashion. This was a natural, spontaneous and perhaps inevitable, development, in keeping with a centuries-old practice in the Near East. But it was not until after the ʿAbbāsid revolution and the establishment of the ʿAbbāsid empire with Baghdad as its capital, and especially during the reigns of al-Rashīd and al-Maʾmūn (786-809, 813-833), that we witness what can only be described as a cultural explosion of which the translation of ancient science and philosophy was a major feature. That occurred roughly a century and a half after the Arab conquest of Egypt and a century and a quarter after the Umayyads came to power in Damascus.

The translation movement in the early ʿAbbāsid period was not a sideline affair conducted by a few individuals working in the dark under threat of being found out and thwarted. It was a massive movement which took place in broad daylight under the protection and active patronage of the ʿAbbāsid rulers. Indeed, in terms of intensity, scope, concentration and concertedness, it had had no precedent in the history of the Middle East or of the world. Large libraries for books on "the philosophical sciences" (*ḥikma*, or *al-ʿulūm al-ḥikmiyya*) were created, embassies were sent out in search of Greek manuscripts, and scholars (Christians and Sabians) were employed to perform the task of translation, all of this at the instigation and with the financial and moral support of the ʿAbbāsid caliphs.[7]

What, then, were the forces that combined to produce this great cultural movement? What was it that urged the ʿAbbāsid authorities to embark on this enterprise? Why the emphasis on "the philosophical sciences", not just useful medicine and astrology and alchemy? What were the relations of this enterprise to other trends — cultural, religious and political — in the Islamic world? And what is the exact meaning of the frequently asserted connection between this early interest in the secular sciences and an inherently Muslim concept of knowledge? In short, how do we explain that forceful and, in a sense, unexpected act of appropriation?[8] Again, these are obvious questions and, again, it is noticeable that they have been neglected by historians of Islamic science. Other perspectives have turned their attention to other tasks.

It is true that here and there in the literature we encounter references to a possible connection between al-Maʾmūn's enthusiasm for the philosophical sciences and his support for Muʿtazilism against traditionalist theology, or to the part played by the Persian family of the Barmakids in the promotion of the new learning, or to the role of the rising middle class of merchants and high officials as supporters of and participants in intellectual activities. But, barring a few exceptions,[9] we shall look in vain for monographs that develop these hints into full fledged theses.

II. THE MARGINALITY THESIS

There is a certain view of Islamic science and philosophy which appears to have given encouragement and validity to their treatment as a foreign object in the body of Islamic civilization. The view is widely held, though not always fully or explicitly stated, and seems to be accepted by some historians of these disciplines, at least as an implicit premise. For the purpose of the present discussion it may be expressed as the thesis consisting of the following statements: That scientific and philosophical activity in medieval Islam had no significant impact on the social, economic, educational and religious institutions; that this activity remained itself unaffected by these institutions, except when it was finally crushed by their antagonism or indifference; and that those who kept the Greek legacy alive in Islamic lands constituted a small group of scholars who had little to do with the spiritual life of the majority of Muslims, who made no important contributions to the main currents of Islamic intellectual life, and whose work and interests were marginal to the central concerns of Islamic society. In what follows I shall refer to this view as "the marginality thesis".

If the marginality thesis were true, then it might indeed seem possible or legitimate to study the history of Islamic science and philosophy in isolation from other aspects of Islamic civilization. In fact, however, the thesis is obviously paradoxical and, in my view, downright false. It is paradoxical because one would not normally expect a marginal activity of a minor group of individuals to rise to the high level of achievement reached by Islamic scientists and philosophers, nor would one expect such an activity to maintain a great deal of its initial vigour and determination, not fifty or a hundred but more than six hundred years after it had been launched. After all, as George Sarton has remarked, Arabic science lasted longer than Greek or Latin medieval or, for that matter, modern, science. As for the falsity of the marginality thesis, this can best be demonstrated by offering a description of an alternative picture — one which shows the connections with cultural factors and forces, thereby explaining (or proposing to explain) not only the external

career of science and philosophy in Islam, but at least some of their inherent characteristics, possibilities and limitations. Since I am not able to present such a picture here, not even in outline, I shall have to limit myself to a few general remarks. While these may not suffice as a convincing argument in favour of the opposite thesis, they will at least give some indication of the questions involved in my present approach.

A number of considerations may be cited in support of the marginality thesis. Chief among these are the following three general observations which we find scattered in the literature: (1) Right from the time of their importation into Islamic society and throughout their career, the philosophical or rational sciences met with opposition from two powerful camps: religious 'orthodoxy' and the champions of an indigenous Arabic culture. (2) The rational sciences, called "sciences of the ancients" (*ʿulūm al-awā'il*) by medieval scholars to distinguish them from the newly developed disciplines concerned with Islamic religion and the Arabic language, were excluded from the curricula of the advanced institutions of learning known as the *madrasas*, and thus did not form part of the mental equipment of the majority of educated Muslims. (3) Because of their predominantly theoretical character the ancient sciences were not in a position to render tangible services to Muslim society and, accordingly, were forced to lead a bookish existence away from interaction with the demands of society.

First I shall comment on each of these three observations one by one.

(1) In medieval Islam, and especially in the early period of Islamic history, the intellectual arena was occupied by various groups who competed vigorously for the minds and hearts of the Muslim community. *Aṣḥāb al-ḥadīth* (who espoused an ideology based on the paradigmatic authority of the sayings and actions of the prophet Muḥammad) contended with the *mutakallimūn* (or "dialectical theologians"), the latter with the *ṣūfīs* (or mystics), and the *ṣūfīs* with the *fuqahā'* (or jurisconsults). The champions of the ancient sciences were no exception and they inevitably suffered attacks from various quarters. But instead of being always intimidated by these attacks they frequently responded in a self-assured, even aggressive manner. Philosophers, for example, scoffed at the arguments of *kalām* which they regarded as muddleheaded, and they derided the *ṣūfīs* for abiding by standards of knowledge (intuition and immediate experience) below those of human reason. And mathematicians boasted about the certainty of their proofs and the superiority of their brand of knowledge to all others.

In 1916 the great Hungarian Islamist Ignaz Goldziher published his important study on the attitude of what he called "the old Islamic Orthodoxy" toward the ancient sciences.[10] By "die alte islamische Orthodoxie" he obviously meant the party of *ahl al-sunna*, those Muslim religious scholars

(traditionists and jurisconsults) who claimed to walk in the path of the prophet and his companions, uncontaminated by adventitious and pernicious influences. Contrary to a possible misconception for which the author would certainly not be responsible, but which might be induced by the word 'Orthodoxy', it was not Goldziher's intention to describe an attitude of "Islam in general" (whatever that is) or of Islamic society; such a generalization is nowhere suggested in his article. Rather, the position he set out to illustrate on the basis of a wide variety of sources drawn from practically all periods of Islamic history was that of a specific and fairly well defined, albeit a very important and influential, group of scholars. The resulting picture shows a decidedly negative attitude — which does not mean that consequences can be drawn from it in a simple or straightforward manner. That 'Orthodox' scholars, or a large number of them, were suspicious or antagonistic towards the philosophical sciences can be easily documented from the historical reports of the time and from their own writings. But what this vocal antagonism implies, in every case, with regard to the actual practice of these sciences cannot be so readily inferred. Here are a few observations (marked (a), (b) and (c)) suggested by the evidence compiled in Goldziher's study.

(a) It is important to bear in mind that the *faqīh* is simply a scholar who has reached a certain understanding (*fiqh*) of matters relating primarily to *sharīʿa* or Islamic law — an understanding which, in the eyes of his fellow scholars, allows him to exercise legal judgments. The *faqīh*, unlike the judge, was not appointed to his role as legist or jurisconsult by a government office. His legal authority was a self-appointed authority that depended on his standing as a scholar, his reputation and his power of persuasion. To put his legal opinions into practice he, therefore, had to win the conviction of the ruler (or the judge appointed by the ruler) or the public. As a consequence, the effectiveness of opposition mounted by the *faqīhs* was invariably a function of their capacity to manipulate political power or to stir up popular sentiment — a capacity which, as one might expect, was not always available to them. We know, for example, that 'Orthodox' protestations were ignored, indeed suppressed, by the caliph al-Ma'mūn, who favoured the traditionists' adversaries, the rationally inclined Mu'tazila. And in twelfth century Andalusia, Averroes, while he remained a protégé of the Almohad rulers, went on writing his commentaries on Aristotle, untroubled by the condemnations of strict Mālikite jurists.

(b) Note must also be taken of the fact that the object of attack was not always the same or always clearly defined. Sometimes the term 'ancient sciences' designated everything that came to be known in Arabic through the translation effort. Frequently, however, it primarily referred to the occult branches of Hellenistic lore, such as magic, astrology, and witchcraft. Most often, certain other branches were explicitly spared the attack, as in the case of arithmetic and medicine. The former (together with algebra) even came to be

incorporated in an auxiliary branch of Islamic law, called *farā'iḍ* (a branch concerned with the division of legacies), and as such became part of the equipment of the *faqīh*. Astronomy was in an ambiguous position: on the one hand it was strongly associated with astrology, a usually suspect subject; on the other, it appeared innocuous as a neutral description of heavenly phenomena, and even a useful tool in the performance of Muslim ritual. The case of logic was perhaps the most complicated of all. It was rejected as the basis of a false metaphysics, but ended up being accepted, within a large section of the 'Orthodox' camp itself, as an indispensable instrument of reasoning, even in the religious sciences. It had an equally complicated relationship with the 'indigenous' science of Arabic grammar, whose practitioners by no means had a uniform attitude towards it. One distinguished grammarian in the tenth century, al-Sīrāfī, ridiculed the logicians' claim to be the qualified teachers of the rules of 'sound discourse'.[11] Another distinguished grammarian from the same century, al-Rummānī, was perceived by some of his contemporaries as having gone too far in mingling grammatical concepts with those of Aristotelian logic. Again, the views of the 'strict' jurisconsult Ibn Taymiyya (d. 1328), to whom Greek logic was anathema, contrasted greatly with those of the eleventh century 'literalist' Ibn Ḥazm, who readily employed it in his legal theorizing.

(c) Finally it is of some significance to note that the religious science of *kalām* was sometimes coupled with ancient science as objects of derision. In this case the implied antithesis was not that of religious versus secular, or of indigenous versus foreign, but rather of a traditional way of receiving and communicating knowledge (associated with the study of *ḥadīth* or prophetic traditions) versus *all* forms of independent exercise of reason.

Thus we see from these remarks, brief and inadequate though they are, that the question of religious opposition to Greek science and philosophy is far more complex than might appear from the general statements usually made about it, or from the reports and anecdotes encountered in medieval works of history and biography. Even as these reports and anecdotes reveal the persistence and vehemence of attacks in the name of 'Orthodox' belief (or what is taken by some to be such), they simultaneously furnish proof of the tenacity and penetration of those intellectual trends which they continually opposed. Whether the eventual decline of the rational sciences in medieval Islam was solely, or largely, due to religious opposition therefore remains an open question. The indications are, however, that this difficult question is not likely to receive an affirmative answer.

(2) As has been mentioned earlier the marginality thesis relies in part on the fact that the major Islamic institution of higher learning, the *madrasa*, formally ignored the rational or philosophical sciences. The consequence

drawn from this fact is that Muslim institutionalized education, having excluded these sciences from its purview, could not serve as a means for their promotion or propagation. Science was accordingly forced to lead a separate, private and precarious existence which, so the argument would go, it could not maintain indefinitely. The argument is compelling and may even contain a large portion of the truth. For my part, I am convinced that the character of the *madrasas*, and the circumstances and motivations that brought about their proliferation under the Saljūqs in the second half of the eleventh century, are important factors that must be considered in any attempt to understand the future career of Islamic science. What has yet to be made clear, however, is the precise nature of these factors and the precise way in which they affected the course of science.

The *madrasa*, as George Makdisi has repeatedly emphasized, is a *waqf* institution, or charitable foundation, and, as such, it belonged to a type of institutions which any Muslim could endow in his capacity as a private individual.[12] But this legal aspect of the *madrasa* need not of course eliminate the non-pietistic motivations that may lie behind the founder's action. Niẓām al-Mulk, the man directly responsible for initiating the system of *madrasas* which quickly spread over ʿIrāq and Khurāsān, may have *legally* acted as a private individual, but he also served the Saljūqs as their trusted and effective vizier or chief administrator at a time when subversive Ismāʿīlī propaganda was acutely felt. It may be debated whether the Niẓāmiyya *madrasas* were originally conceived as rivals or emulators of *dūr al-ʿilm*, the library-cum-teaching institutions which, like the original *Dār al-ʿIlm* in Fāṭimid Cairo, had made room for the philosophical sciences. It remains true in any case that they quickly replaced the *dūr al-ʿilm*, thus bringing to an end one of the few institutional homes in which the foreign sciences had been cultivated without inhibition. There are still other debatable questions of interpretation concerning the turn taken by higher education under the Saljūqs, and the significance of that event for the future of science in later centuries. But I am inclined to the view of van Berchem, Goldziher and others who detect a theologico-political motivation behind the establishment of the Niẓāmiyya system of *madrasas*.[13]

The exclusion of science from the *madrasa* could be viewed as an omission simply resulting from the fact that the *madrasa* was primarily intended for teaching the religious sciences with law at their centre. If, however, the *madrasa* was theologically and politically motivated, then there is reason for regarding the exclusion as a repudiation. The latter view would seem to be supported by the explicit educational philosophy of Abū Ḥāmid al-Ghazālī, a prestigious and influential early professor at the Niẓāmiyya *madrasa* in Baghdad. It is well-known that Ghazālī was wary of exposing the Muslim student to the foreign sciences, including those which (like arithmetic and astronomy) he considered benign. But whatever view is accepted, we know

that scientific activity did not cease to exist as a result of not being formally included in the teaching curriculum of the *madrasa*. (I say "not formally included" because it can be shown that, thanks to the informal character of all medieval Muslim education, the rational sciences, or some of them, were able to penetrate even the *madrasas*.) An important task of the historian of Islamic science is, therefore, to answer the question: How did a significant scientific tradition maintain itself for such a long time largely outside the only stable institution of higher learning in medieval Islam? It seems to me that this question is as important as inquiring into how the same situation was related to the eventual decline of science.

Anyone familiar with the course of Islamic science after the eleventh century will immediately recognize that there can be no one answer that is valid for all periods and all geographical areas. The circumstances that surrounded the flowering of science, philosophy and medicine in twelfth century Spain, for example, hardly resembled those in which the later flowering of mathematics and astronomy took place under the Mongols in Ādharbayjān; and the context in which Ibn al-Shāṭir was able to make his astronomical contributions in fourteenth century Damascus differed greatly from the conditions provided in the following century by Ulūgh Beg for the mathematicians gathered round him in Samarqand.

What we need, then, are monographic studies that deal with particular situations in particular places and times. One or several of such studies should be devoted to the *madrasas* themselves. For, as my parenthetical remark above has indicated, these institutions were not totally devoid of rational science, despite their pronounced emphasis on religious disciplines.[14] There were two main reasons for this. One is the well-known fact that some of the rational sciences (arithmetic and astronomy in particular) had become indispensable to the *faqīh* and the *muwaqqit*[15] respectively. The other, not so well appreciated, reason is that teaching activity in the *madrasa* often reflected the interests of the individual teacher rather than any rigid curriculum to which he is supposed to have confined himself. An example from the thirteenth century is that of Kamāl al-Dīn ibn Yūnus, a renowned scholar who had studied principles of jurisprudence at the Niẓāmiyya *madrasa* in Baghdad before he became a teacher of *fiqh* in the *madrasas* of his native Mawṣil, in northern ᶜIrāq. His biography relates that besides the usual religious sciences (remarkably enough, these were not restricted to Islam) his teaching activity included astronomy and mathematics, among others.[16] One notable scholar who studied the *Almagest* under him is Athīr al-Dīn al-Abharī (d. 1265). ᶜAlam al-Dīn Qayṣar (d. 1242) was another, a Ḥanafī jurist from Upper Egypt who went all the way to Mawṣil to study musical theory under Kamāl al-Dīn. One should not therefore be surprised to find it stated in many surviving manuscripts on the "foreign sciences" that they were copied inside one of the famous *madrasas*.

This is simply evidence of the fact that this branch of human knowledge was not barred from entering the libraries attached to these institutions. And one need only remember that the vast majority of the extant Arabic scientific *and* philosophical manuscripts have been preserved in mosque libraries to realize that their presence there was rather the rule, not the exception.

(3) As a court-sponsored enterprise the promotion of the foreign sciences under the early ᶜAbbāsids was certainly motivated in part by practical considerations. The patronized scholars were physicians, astrologers and engineers, all able and eager to render valued services in their respective spheres of competence. Physicians were in possession of a long and rich tradition of medical experience which had been incorporated and codified in the works of Galen. Astrologers claimed to be able to make predictions of important cosmic and earthly events, and this gave them authority as advisers on a whole range of public and private affairs that came under the influence of those events. To do their job properly they had to have the time and equipment necessary to pursue their multifarious mathematical and astronomical investigations. Hence the establishment of libraries and observatories from the very beginning. Books on *ḥisāb* (arithmetic) and algebra included sections on practical problems in land surveying, business transactions, and the division of legacies according to Muslim law. Mathematicians were employed as engineers who supervised town planning, the erection of buildings and bridges, and the digging of canals. Al-Ma'mūn seems to have valued science beyond its practical applications. But even after the reaction, during al-Mutawakkil's reign (232/847-247/861), against the rationalizing Muᶜtazila whom al-Ma'mūn had favoured, mathematicians and astronomers like the Sons of Mūsā ibn Shākir, al-Farghānī and Sanad ibn ᶜAlī still continued to be employed by the court.

Islamic scientists continued to perform these and similar functions in subsequent centuries. But a great deal of research has to be done before a full picture of the role of the scientist in medieval Islam can be drawn. Written documents exist on the construction of mechanical devices and on the application of geometry to art and architecture. But the problem belongs to art history and archaeology as well to the history of scientific texts, and it has yet to be fully investigated in all of its aspects.

One important development should be mentioned here, namely the emergence (probably in the thirteenth century) of the mathematician-astronomer as a *muwaqqit* attached to a mosque. The *muwaqqit* was in charge of producing tables for the determination of the times of the five daily prayers for his locality. (*'Muwaqqit'* derives from *'waqt'*, the word for definite time.) Since these times had come to be defined by reference to astronomical phenomena, such as the morning and evening twilights, sunrise and sunset, and the length

of gnomon shadows, the job could be done only by someone with a knowledge of astronomy, and, in a few instances, the position of *muwaqqit* was filled by distinguished mathematicians.

Whether as physician, astrologer, engineer, *muwaqqit* or *faraḍī*, the scientist or scientifically informed scholar was thus able to render a number of important services to the society in which he lived. Having failed to recognize the social and religious benefits perceived to be obtainable from science, the marginality thesis is quite unable to help us to understand the nature of the process through which scientific knowledge came to be absorbed and harnessed in medieval Islamic society.

III. THE PROCESS OF NATURALIZATION

In opposition to the marginality thesis I would suggest that what we see in the history of Islamic science is a process of assimilation ending in a complete naturalization of the imported sciences in Muslim soil. Anticipating the remarks in Part IV of this paper I would also suggest that the eventual decline of these sciences, rather than being considered merely a natural result of indifference or rejection, would be better understood against the background of this assimilative process. The broad picture I would propose, with some qualifications to be indicated later, is that of a three-stage development followed by a fourth stage of sharp decline. In the first stage we witness the acquisition of ancient, particularly Greek, science and philosophy through the effort of translation from Greek and Syriac into Arabic. As I have tried to argue earlier in the paper, Greek science entered the world of Islam, not as an invading force setting off from a powerful stronghold in Alexandria, Antioch or Ḥarrān, but rather as an invited guest. The individuals who brought him in kept their reserve and aloofness with regard to the important matter of religion. But the guest quickly proved to hold an attraction for his hosts far beyond the promise of his practical abilities. His power of persuasion can be seen in the unexpected but almost immediate and almost unreserved adoption of Hellenism by Muslim members of the household, like al-Kindī. But the real measure of his spectacular success is shown in the emergence, during the second stage, of a large number of powerful Muslim thinkers whose allegiance to a comprehensive Hellenistic view of the world of matter and thought and values can be described only as a thoroughgoing commitment. Those were the Fārābīs, the Avicennas, the Ibn al-Haythams, the Bīrūnīs, and the Averroeses. I describe them as Muslims because they thought of themselves as such, and because they were attentive to problems generated by the collision between their religious beliefs and Hellenistic doctrines.

The third stage is that in which *falsafa*, the type of thought and discourse found in the writings of philosophers like Fārābī and Avicenna, began to be practised in the context of *kalām*; and in which the philosopher-physician (represented by Rāzī) was replaced by the jurist-physician (represented by Ibn al-Nafīs), the mathematician (*taʿlīmī*) by the *faraḍī*, and the astronomer-astrologer by the *muwaqqit*.

Of crucial importance in the first stage were of course the agents of transmission, the Christians and Sabians who served their Muslim employers. They did not for the most part adopt the new faith; and while they wrote on scientific matters in Arabic for their patrons, they continued to write in Syriac on matters of religious concern to their co-religionists. As genuine believers in the values of the Hellenistic tradition which they propagated they cannot be merely considered as mercenaries, but they remained, in a sense, outsiders. Their heirs in the second stage were mostly Muslims who came from all parts of the Muslim world. They were fully acquainted with Muslim learning, and some of them (like Averroes) were steeped in it. But the general outlook which determined the direction of their thought and in terms of which they sought to interpret their own religion and expound their views on the place of religion and of rational thought in the organization of society was uncompromisingly Hellenistic.

A look at the later centuries, what I called the third stage, reveals a clearly noticeable change. The carriers of scientific and medical knowledge and techniques now largely consisted of men who were not only Muslim by birth and faith, but who were imbued with Muslim learning and tradition, and whose conceptual framework had been produced in the process of forging a consciously Muslim outlook. No longer was the scientific scholar committed to the presuppositions of the earlier philosophers. Sometimes a scholar of this later breed distinguished himself equally in the religious and the rational sciences — such as Kamāl al-Dīn ibn Yūnus of Mawṣil, and sometimes he held an office in a religious institution (like Ibn al-Shāṭir). In many cases he was an expert on *fiqh*, or grammar, or Qurʾānic science, or all of these. In almost every case he had undergone a thorough Muslim education. The question, therefore, is not whether scientific education and practice mingled with traditional learning in this third stage, but how the process of combination developed and with what consequences for the character and progress of scientific thought.

The names of the scholars mentioned will give some indication of the time-intervals I have in mind for the first three stages. But the question of periodization has to be approached with due attention to discontinuities, geographical differences, and ups and downs. Spain, for example, is said to have always lagged behind the Muslim East; and the scholars working under the Mongols in Marāgha and Tabrīz seem to have more in common with their predecessors of two centuries earlier than with those who came soon after them in Mamlūk Syria and Egypt. What one can say with some assurance is that the history of science in Islam presents us with a number of clearly

distinguishable types of scientific scholarship some of which were predominant in certain periods; and, further, that the three major types outlined above followed the order I have just described. A fully developed typology of scientific scholars in medieval Islam would make a fascinating study; and although the task has not yet started, it will be helped by general surveys of categories of scientific writing and of historico-geographical areas such as those that have begun to appear in recent years.[17]

IV. THE PROBLEM OF DECLINE

E. S. Kennedy once made the remark that there was no essential reason why Tycho Brahe could not have had a Turkish name. What he meant was that observations matching those of Brahe in precision and significance could have been made by Islamic astronomers using instruments of the types they had already developed in ᶜAbbāsid times. And yet the man who carried out for the first time the kind of observations that allowed Kepler to make his great advance in planetary theory did not have a Turkish, Persian or Arabic name, and the question is: Why didn't he? That is only a peculiar formulation of the why-not question that has been much debated in recent years, especially, but not exclusively, with reference to Chinese science. As far as Islamic science is concerned the question frequently takes the form of why it declined in Islamic lands after the initial spectacular flowering. But sometimes it is formulated as the question of why the seventeenth century breakthrough had escaped the Islamic scientists who had based their endeavour on the same foundations that later served their European successors.

There are those who consider the question meaningless, especially in its latter form, and accordingly refuse to speculate about it. Others would go so far as to maintain that the historian of Islamic science cannot address a more important question. It has become clear in any case that this is one of those questions that need to be subjected to critical analysis before one sets about finding an answer. That the question cannot be easily dismissed is shown in the not insignificant literature devoted to it, and even more in the fact that it almost always crops up in general discussions. The following remarks will be concerned with the problem of decline rather than the question of why Muslim scientists failed to produce "the scientific revolution".

But first let me deliver the disappointing confession that I do not possess a solution to the problem of decline. That the phenomenon in question did in fact occur seems clear to me from comparing levels of scientific productions in, say, the fifteenth and eleventh centuries. Difficulties arise when one tries to assign a date to the occurrence, in part because decline is a process that occupies a time-interval, and it is difficult to determine when the process

began, but also because we are dealing with a vast geographical area in which not all centres of scientific activity were always in the same phase of development at the same time. Add to this the consideration that decline in one branch of science may coincide with progress in another. Much specific research must be done before we can produce reliable general descriptions, let alone plausible explanations. Nevertheless, one general observation appears to be strongly suggested by the available evidence, and it is this observation that I shall now outline.

In a chapter of his *Muqaddima* devoted to a lengthy refutation of philosophy the fourteenth century historian Ibn Khaldūn wrote that "The problems of physics [he was referring to Aristotelian natural philosophy] are of no importance for us in our religious affairs or our livelihoods. Therefore, we must leave them alone."[18] He was echoing a sentiment already expressed by Ghazālī three hundred years earlier. Underlying this sentiment was a certain view of knowledge which Ghazālī set forth in the important First Book of his greatly influential *magnum opus*, *The revival of the religious sciences*.[19] Ghazālī first remarks that, knowledge being a source of pleasure in itself (*ladhīdh fī nafsih*), all knowledge qualifies as an object of pursuit for its own sake. In his own words, "Knowledge is a virtue in itself [*faḍīla fī dhātih*], absolutely and without relation [to something else]". Religious knowledge has the additional value of being a means for attaining eternal happiness in the world to come. "Man has been created only to know" (*lam yukhlaq illā li-l-ᶜilm*) is Ghazālī's bold assertion, but the knowledge man has been created to seek is that which brings him closer to his creator. For the religiously committed Ghazālī this means, not only that religious knowledge is higher in rank and more worthy of pursuit than all other forms of knowledge, but also that all other forms of knowledge must be subordinated to it. No occupation, no pursuit, however virtuous in itself, should be allowed to divert man from his ultimate goal. Thus, among the non-revealed forms of knowledge: medicine is necessary only for the preservation of health; arithmetic for the conduct of daily affairs and for the execution of wills and the division of legacies in accordance with the revealed law; astronomy, a science praiseworthy in itself but blameworthy in some of its implications, is useful in performing an operation legitimated by the holy Qur'ān, namely the calculating of celestial movements; and logic is just a tool for weighing arguments in religious as well as non-religious branches of inquiry. The doctrines of natural science are of two sorts: those that contradict religious belief should of course be rejected; as for those that are concerned with the general properties of material objects, they can be ignored without loss. There is only one principle that should be consulted whenever one has to decide whether or not a certain branch of learning is worthy of pursuit: it is the all-important consideration that "this world is a sowing ground for the next"; and Ghazālī quotes in this connection the

Prophetic Tradition: "May God protect us from useless knowledge." The final result of all this is an instrumentalist and religiously oriented view of all secular and permitted knowledge. This is the view that accompanied the limited admission of logic and mathematics and medicine into the *madrasa* and the conditional admission of the astronomer into the mosque.

The philosophical sciences, *al-ʿulūm al-ḥikmiyya*, had entered Islamic intellectual life under the banner of an articulated concept of *ḥikma* (or philosophical wisdom) which involved a doctrine of knowledge quite distinct from the view just outlined. According to this doctrine, the aim of theoretical investigation, whether mathematical, physical or metaphysical, was to ascertain the nature of all things as they are in themselves to the farthest extent possible, and the aim of the investigator was to gain knowledge of the truth for the sake of knowing the truth. The metaphysically inclined seeker after the truth, namely all the great philosophers of Islam and the majority of mathematicians in the earlier period, also believed that the ultimate value of all science was to perfect the human soul and prepare it for a state of eternal happiness — that is the very same state which Ghazālī hoped to secure through religious knowledge. But, for the philosopher and the philosophically committed scientist, this was not a state that transcended philosophical activity: it was to be achieved in fact only through philosophical activity and one was closer to it by as much as one progressed in the acquisition of philosophical knowledge. Man's perfection thus lay in the perfection of his philosophical or scientific knowledge and the way to his salvation was none other than the way of science. By contrast, a logical consequence of Ghazālī's view was, as we have seen, to put a curb on theoretical inquiry.

With the help of the foregoing expositions I am now able to formulate a thesis which I shall express in three parts: (1) that the two theories of knowledge described above, in so far as they embody two distinct sets of intellectual commitments, may serve to characterize two types of scientific scholar, namely the philosopher-scientist and the jurist-scientist, respectively; (2) that the 'naturalization' of the Greek sciences in Islam may be understood as the process in which the philosophers' view of knowledge was replaced by the instrumentalist view proposed by Ghazālī; and (3) that the decline of science occurred, not in the context of opposition (as is usually thought) but in the context of acceptance and assimilation as here understood — that is, it set in when the sciences came to be accepted and practised only to the extent that they were legitimated by the instrumentalist view.

I must repeat that part (3) of my thesis is not intended as an explanation of the phenomenon of decline if by 'explanation' we mean a set of factors the existence of which was sufficient for the production of a certain event.[20] It is merely meant as a relevant and possibly illuminating observation that might help in future research by directing our attention in a certain direction rather

than others. The observation is surprising (it surprised me when I first came upon it) and may even seem paradoxical. Would it not be strange, it might be asked, to correlate decline and acceptance? Wasn't it rather fortunate for science and scientifically informed scholars to be granted a niche, however small, in permanent Islamic institutions? Didn't 'naturalization' (if that is what happened) ensure the continuity of a certain amount of instruction in the sciences that might otherwise have been absent? As for those who limited the application of their scientific expertise to the service of religious practice, didn't they sometimes produce astronomically significant works on timekeeping and lunar visibility and *qibla* determination? Wasn't one of the most distinguished astronomers in medieval Islam, Ibn al-Shāṭir, employed as a *muwaqqit* in the Umayyad mosque? And would we be really justified in regarding an instrumentalist view of science as an impediment to scientific progress?

There will be no point in extending the list of questions any further since I cannot possibly deal with them. Instead, I shall, by way of comment, devote one or two concluding remarks to a relevant aspect of the important question of the scientist's role in Islam. In the earlier period, and at certain times and places afterwards, the scientist (mathematician or natural philosopher) was more often than not a practising astrologer and physician, and it was mostly for his expertise in the arts of astrology and medicine that he was valued by his employers. But he was also recognized and revered as a *ḥakīm*, a word which I would not hesitate to translate in this context as "scientist" if we mean by "scientist" one who investigates any aspect or aspects of the world for the sake of knowing the truth about the world. But the mosque was certainly not installing a *ḥakīm* within its walls when it employed the astronomically informed scholar as a *muwaqqit*. His function and expected contribution would be those defined by this latter term: someone in charge of regulating the times of prayer. It was only natural of course if curiosity or interest or whatever would attract the individual *muwaqqit* into areas of research and discovery that lay outside the requirements of his office, as did sometimes happen. But such individual and exceptional effort was likely to receive only limited appreciation from his contemporaries: it was not what they demanded and expected of him. It may be significant, for example, to note that, as David King has observed,[21] the important research of Ibn al-Shāṭir on planetary theory does not seem to have influenced later astronomical work by *muwaqqits* in Syria and Egypt where his *New zīj*, incorporating his new planetary models, achieved some popularity.

Finally, with regard to instrumentalism, it should be noted that what we have to do with here is not a general utilitarian interpretation of science, but a special view which confines scientific research to very narrow, and essentially unprogressive areas. We may rightly admire the ingenuity, inventiveness and

computational prowess in some of the works of the *muwaqqits* on timekeeping and the *qibla*; but we have to realize that breakthroughs, if such were at all possible, could only have occurred elsewhere — for example, in geometry and theoretical algebra, in observational and theoretical astronomy and in various branches of experimental science, all fields in which, it must always be remembered, the earlier *ḥakīms* had made significant and rather promising advances.

REFERENCES

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1. For an illustration of this point, see the author's "The Andalusian revolt against Ptolemaic astronomy: Averroes and al-Biṭrūjī", in Everett Mendelsohn (ed.), *Transmission and tradition in the sciences: Essays in honor of I. Bernard Cohen* (Cambridge, 1984), 133-53.
2. See, for example, the introductory chapter to his "On First Philosophy", in Alfred I. Ivry, *Al-Kindī's metaphysics* (Albany, N.Y., 1974), 55-60.
3. See the first article to be published in the more recent research on this problem: Victor Roberts, "The solar and lunar theory of Ibn al-Shāṭir: A pre-Copernican Copernican model", *Isis*, xlviii (1957), 428-32.
4. Carra de Vaux, "Les sphères célestes selon Nasîr-Eddîn Attûsî", in Paul Tannery, *Recherches sur l'histoire de l'astronomie ancienne* (Paris, 1893), Appendice VI, pp. 337-61.
5. Pierre Duhem, *To save the phenomena: An essay on the idea of physical theory from Plato to Galileo*, trans. by Edmund Dolan and Chaninah Maschler (Chicago and London, 1969), 25-35; see esp. p. 26. Also: *idem, Le système du monde*, ii (Paris, 1954), 117-79; esp. pp. 117-19, on "Le réalisme des arabes".
6. Many of these articles are reprinted in E. S. Kennedy and ʿImād Ghānim, *The life and work of Ibn al-Shāṭir* (Aleppo, 1976). For additional literature see Sabra, "The Andalusian revolt ..." (ref. 1), 146, n. 5.
7. *Cf.* Franz Rosenthal, *The classical heritage in Islam* (Berkeley and Los Angeles, 1965), esp. pp. 1-14.
8. Surely it is not to be taken for granted that an intellectual tradition to which the Romans paid little attention, and which was almost extinguished during the long struggle between Christianity and Paganism, should undergo a spectacular revival in a semitic, hitherto 'unscientific' language and a religiously oriented civilization.
9. A notable exception is Rudi Paret, *Der Islam und das griechische Bildungsgut* (Tübingen, 1950). See also the relevant chapters in S. D. Goitein, *Studies in Islamic history and institutions* (Leiden, 1966).
10. Goldziher's still unsurpassed study, first published as no. 18 (1915) of *Abhandlungen der königlisch Preussischen Akademie der Wissenschaften* (Philosophisch-historische Klasse) (Berlin, 1916), is now available in English translation in Merlin Swartz (trans. and ed.), *Studies on Islam* (New York and Oxford, 1981), 185-215.
11. On Sīrāfī's views, see D. S. Margoliuth, "The discussion between Abū Bishr Mattā and Abū Saʿīd al-Sīrāfī on the merits of logic and grammar", *Journal of the Royal Asiatic Society*, (1905), 79-129; M. Mahdi, "Language and logic in classical Islam", in G. E. von Grunebaum (ed.), *Logic in classical Islamic culture* (Wiesbaden, 1970), 50-83. Some interpretative remarks are in A. I. Sabra, "Avicenna on the subject matter of logic", *The journal of philosophy*, lxxvii (1980), 745-64.

12. See George Makdisi's most recent and most developed statement in *The rise of colleges: Institutions of learning in Islam and in the West* (Edinburgh, 1981). An earlier study by Makdisi is "Muslim institutions of learning in eleventh-century Baghdad", *Bulletin of the School of Oriental and African Studies*, xxiv (1961), 1-56. See comments and criticisms in A. L. Tibawi, "Origin and character of *al-madrasa*", *ibid.*, xxv (1962), 225-38.
13. Makdisi discusses van Berchem's and Goldziher's views at some length in *The rise of colleges*, 296-304.
14. Mention should be made of an early study by a pioneering student of Islamic scientific institutions: Aydin Sayili, *The institutions of science and learning in the Muslim world*, unpublished Ph.D. dissertation, Harvard University, 1941. Sayili's *The observatory in Islam* (Ankara, 1960; repr., New York, 1981) is still the only major monograph on a scientific institution in the broad context of Islamic culture.
15. The man in charge of regulating prayer times in a mosque.
16. Only a very few of Kamāl al-Dīn's mathematical works have survived. His more recently discovered writings (in Manisa, Turkey) deal with conic sections.
17. I have in mind the kind of research exemplified by E. S. Kennedy's *A survey of Islamic astronomical tables* (*Transactions of the American Philosophical Society*, ns. xlvi, pt. 2 (1956)), and David King's "The astronomy of the Mamluks", *Isis*, lxxiv (1983), 531-55.
18. Ibn Khaldûn, *The Muqaddimah*, trans. by Franz Rosenthal (3 vols, Princeton, 1967). See vol. iii, 246-58, esp. pp. 251-2.
19. *Iḥyāʾ ʿulūm al-dīn*, ed. by al-Bābī al-ḥalabī (Cairo, 1967), i, 13ff, esp. pp. 17, 22, 23, 31-32, 35-36, 45-48, 57-60.
20. I am not arguing the simplistic thesis that Ghazālī, or his influence, was 'the cause' of scientific decline. Nor would it be correct to claim that the two theories of knowledge outlined above were the only ones conceived in Islam; even within the restricted group of scientific scholars. Ibn al-Shāṭir, for example, subscribed to a view which was distinct from both theories. For a survey of a wide range of concepts of knowledge in Islam, see Franz Rosenthal, *Knowledge triumphant: The concept of knowledge in medieval Islam* (Leiden, 1970).
21. See King's article referred to above (ref. 17), 538.

IBN AL-HAYTHAM

IBN AL-HAYTHAM, ABŪ ʿALĪ AL-ḤASAN IBN AL-ḤASAN, called **al-Baṣrī** (*i.e.* Baṣra, Iraq), **al-Miṣrī** (of Egypt); also known as **Alhazen**, the Latinized form of his first name, **al-Ḥasan** (*b.* 965; *d.* Cairo, *ca.* 1040), *optics, astronomy, mathematics.*

About Ibn al-Haytham's life we have several, not always consistent, reports, most of which come from the thirteenth century. Ibn al-Qifṭī (*d.* 1248) gives a detailed account of how he went from Iraq to Fāṭimid Egypt during the reign of al-Ḥākim (996–1021), the caliph who patronized the great astronomer Ibn Yūnus (*d.* 1009) and who founded in Cairo a library, the Dār al-ʿIlm, whose fame almost equalled that of its precursor at Baghdad (the Bayt al-Ḥikma, which flourished under al-Maʾmūn [813–833]). Impressed by a claim of Ibn al-Haytham that he would be able to build a construction on the Nile which would regulate the flow of its waters, the caliph persuaded the already famous mathematician to come to Egypt and, to show his esteem, went out to meet him on his arrival at a village outside Cairo called al-Khandaq.

Ibn al-Haytham, according to Ibn al-Qifṭī, soon went at the head of an engineering mission to the southern border of Egypt where, he had assumed, the Nile entered the country from a high ground. But even before reaching his destination he began to lose heart about his project. The excellently designed and perfectly constructed ancient buildings which he saw on the banks of the river convinced him that if his plan had been at all possible it would have been already put into effect by the creators of those impressive structures. His misgivings were proved right when he found that the place called *al-Janādil* (the cataracts), south of Aswan, did not accord with what he had expected. Ashamed and dejected he admitted his failure to al-Ḥākim, who then put him in charge of some government office. Ibn al-Haytham at first accepted this post out of fear, but realizing his insecure position under the capricious and murderous al-Ḥākim he pretended to be mentally deranged and, as a result, was confined to his house until the caliph's death. Whereupon Ibn al-Haytham revealed his sanity, took up residence near the Azhar Mosque, and, having been given back his previously sequestered property, spent the rest of his life writing, copying scientific texts, and teaching.

To this account Ibn al-Qifṭī appends a report which he obtained from his friend Yūsuf al-Fāsī (*d.* 1227), a Jewish physician from North Africa who settled in Aleppo after a short stay in Cairo where he worked with Maimonides.[1] Yūsuf al-Fāsī had "heard" that in the latter part of his life Ibn al-Haytham earned his living from the proceeds (amounting to 150 Egyptian dinars) of copying annually the *Elements* of Euclid, the *Almagest*, and the *Mutawassiṭāt*,[2] and that he continued to do so until he died "in [*fī ḥudūd*][3] the year 430 [A.D. 1038–1039] or shortly thereafter [*aw baʿdahā bi-qalīl*]." These words are immediately followed by a statement, of which the author must be presumed to be Ibn al-Qifṭī, to the effect that he possessed a volume on geometry in Ibn al-Haytham's hand, written in 432 (A.D. 1040–1041).

An earlier account of Ibn al-Haytham's visit to Egypt is given by ʿAlī ibn Zayd al-Bayhaqī (*d.* 1169–1170).[4] According to him the mathematician had only a brief and unsuccessful meeting with al-Ḥākim outside an inn in Cairo. The caliph, sitting on a donkey with silver-plated harness, examined a treatise composed by Ibn al-Haytham on his Nile project, while the author, being short of stature, stood on a bench (*dukkān*) in front of him. The caliph condemned the project as impractical and expensive, ordered the bench to be demolished, and rode away. Afraid for his life, Ibn al-Haytham immediately fled the country under cover of darkness, going to Syria, where he later secured the patronage of a well-to-do governor. But this account, vivid though it is, must be discarded as being unsupported by other evidence. For example, we are told by Ṣāʿid al-Andalusī (*d.* 1070) that a contemporary of his, a judge named ʿAbd al-Raḥman ibn ʿĪsā, met Ibn al-Haytham in Egypt in 430 A.H., that is, a short time before the latter died.

Ibn Abī Uṣaybiʿa (*d.* 1270) gives the name of Ibn al-Haytham as Muḥammad (rather than al-Ḥasan) ibn al-Ḥasan; and he joins Ibn al-Qifṭī's story (which he quotes in full with the omission of the last statement about Ibn al-Haytham's autograph of 432) to a report which he heard from ʿAlam al-Dīn Qayṣar ibn Abi 'l-Qāsim ibn Musāfir, an Egyptian mathematician who resided in Syria and died at Damascus in 649 A.H./A.D. 1251.[5] According to this report, Ibn al-Haytham at first occupied the office of minister at Baṣra and its environs, but to satisfy his strong desire to devote himself entirely to science and learning he feigned madness until he was relieved from his duties. Only then did he go to Egypt, where he spent the rest of his life at the Azhar Mosque, living on what he earned from copying Euclid and the *Almagest* once every year. We may add that the title of one of his writings (no. II 13, see below) appears to imply that he was at Baghdad in 1027, six years after al-Ḥākim died.[6]

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It is unfortunate that the autobiography of Ibn al-Haytham, which Ibn Abī Uṣaybiʿa quotes from an autograph, throws no light on these different reports. Written at the end of 417 A.H./A.D. 1027, when the author was sixty-three lunar years old, and clearly modeled after Galen's *De libris propriis*,[7] it lists the works written by Ibn al-Haytham up to that date but speaks in only general terms about his intellectual development.

As cited by Ibn Abī Uṣaybiʿa, Ibn al-Haytham, reflecting in his youth on the conflicting but firmly held beliefs of the various religious sects, was led to put them all in doubt and became convinced that truth was one. When in later years he was ready to grasp intellectual matters, he decided to turn his back on the common people and devote himself to seeking knowledge of the truth as the worthiest possession that could be obtained in this world and the surest way to gain favor with God—a decision which, using Galen's expressions in *De methodo medendi*,[8] he attributed to his "good fortune, or a divine inspiration, or a kind of madness." Frustrated in his intensive inquiries into the religious sciences, he finally emerged with the conviction that truth was to be had only in "doctrines whose matter was sensible and whose form was rational." Such doctrines Ibn al-Haytham found exemplified in the writings of Aristotle (of which he here gave a conspectus) and in the philosophical sciences of mathematics, physics, and metaphysics. As evidence of his having stood by his decision, he provided a list of his writings to 10 February 1027, containing twenty-five titles on mathematical subjects (list *Ia*) and forty-five titles on questions of physics and metaphysics (list *Ib*).

Ibn Abī Uṣaybiʿa gives two more lists of Ibn al-Haytham's work, which we shall designate as II and III. List II, which he found attached to list I and in the author's hand, contains twenty-one titles of works composed between 10 February 1027 and 25 July 1028. Ibn Abī Uṣaybiʿa does not say whether he also copied list III from an autograph but simply describes it as a catalogue (*fihrist*) which he found of Ibn al-Haytham's works to the end of 429 A.H./2 October 1038. Nor does he specify the *terminus a quo* of this catalogue. However that may be, two things are remarkable about this last list: consisting of ninety-two titles, it includes all sixty-nine titles ascribed to Ibn al-Haytham by Ibn al-Qifṭī, with two exceptions; and in it are to be found all of Ibn al-Haytham's extant works (not fewer than fifty-five), again with only a very few exceptions. It may also be noted that the order of works in list III almost always agrees with the chronological order of their composition, whenever the latter can be independently determined from internal cross references. Thus, III 2 was written before III 53; III 3 before III 36, III 49, III 60, III 77, and III 80; III 20 before III 21; III 25 before III 31; III 26 before III 38 and III 68; III 42 before III 74; III 53 before III 54; III 61 before III 63; III 63 before III 64; and III 66 before III 77. Item III 17, however, was written before III 16 (see bibliography).

Among the subjects on which Ibn al-Haytham wrote are logic, ethics, politics, poetry, music, and theology (*kalām*); but neither his writings on these subjects nor the summaries he made of Aristotle and Galen have survived. His extant works belong to the fields in which he was reputed to have made his most important contributions: optics, astronomy, and mathematics.

Optics: Doctrine of Light. Ibn al-Haytham's theory of light and vision is neither identical with nor directly descendant from any one of the theories known to have previously existed in antiquity or in Islam. It is obvious that it combines elements of earlier theories—owing perhaps more to Ptolemy than to any other writer—but in it these elements are reexamined and rearranged in such a way as to produce something new. Ibn al-Haytham's writings on optics included a treatise written "in accordance with the method of Ptolemy" (III 27), whose *Optics* was available to him in an Arabic translation lacking the first book and the end of the fifth and last book, and a summary of Euclid and Ptolemy in which he "supplemented the matters of the first Book, missing from Ptolemy's work" (*Ia* 5). These two works are now lost.

• But in his major work, the *Optics* or *Kitāb al-Manāẓir* (III 3),[9] in seven books, Ibn al-Haytham deliberately set out to dispel what appeared to him to be a prevailing confusion in the subject by "recommencing the inquiry into its principles and premises, starting the investigation by an induction of the things that exist and a review of the conditions of the objects of vision." Once the results of induction were established he was then to "ascend in the inquiry and reasonings, gradually and in order, criticizing premises and exercising caution in the drawing of conclusions," his aim in all this being "to employ justice, not to follow prejudice, and to take care in all that we judge and criticize that we seek the truth and not to be swayed by opinions" (Fatih MS 3212, fol. 4a r).

The book is in fact an earnest and assiduous exercise in the method outlined. Its arguments are either inductive, experimental, or mathematical, and it cites no authorities. Experiment (*iʿtibār*) in particular emerges in it as an explicit and identifiable methodological tool involving the manipulation of artificially constructed devices. (In the Latin translation of the *Optics* the word *iʿtibār* and its cognates *iʿtabara* and

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mu'tabir became *experimentum, experimentare,* and *experimentator,* respectively.) Perhaps as a result of its derivation from the astronomical procedure of testing past observations by comparing them with new ones, the method of *i'tibār* often appears aimed at proof rather than discovery. It establishes beyond doubt that which is insecurely suggested by inadequate observations.

The *Optics* is not a philosophical dissertation on the nature of light, but an experimental and mathematical investigation of its properties, particularly insofar as these relate to vision. With regard to the question "What is light?," Ibn al-Haytham readily adopted the view ascribed by him to "the physicists" or natural philosophers *(al-ṭabī'iyyūn)*—not, however, because that view was by itself sufficient, but because it constituted an element of the truth which had to be combined with other elements derived from "mathematicians" *(ta'līmiyyūn)* such as Euclid and Ptolemy. In the resulting synthesis *(tarkīb)* the approach of "the mathematicians" dominated the form of inquiry, while their doctrines were altered, indeed reversed, in the light of those of "the physicists." That "the physicists" were the natural philosophers working in the Aristotelian tradition is clear enough from comparing the view attributed to them by Ibn al-Haytham with expressions and doctrines that had been current in the works of peripatetics from Alexander to Avicenna.

Light, says Ibn al-Haytham, is a form *(ṣūra,* εἶδος) essential *(dhātiyya)* in self-luminous bodies, accidental *('araḍiyya)* in bodies that derive their luminosity from outside sources. Transparency *(al-shafīf)* is an essential form in virtue of which transparent bodies, such as air or water, transmit light. An opaque body, such as a stone, has the power to "receive" or take on and make its own the light shining upon it and thereby to become itself a luminous source. This received light is called accidental because it belongs to the body only as long as the body is irradiated from outside. There are no perfectly transparent bodies. All transparent bodies possess a certain degree of opacity which causes light to be "received" or "fixed" in them as accidental light.

The light which radiates directly from a self-luminous source is called "primary" *(awwal)*; that which emanates from accidental light is called "secondary" *(thānī)*. Primary and secondary lights are emitted by their respective sources in exactly the same manner, that is, from every point on the source in all directions along straight lines. The only difference between these two kinds of light is one of intensity: accidental light is weaker than its primary source and the secondary light deriving from it is weaker still. All radiating lights become weaker the farther they travel. The distinction is made in transparent bodies between the accidentally fixed and the traversing light, and it is from the former that secondary light is emitted. Thus from every point of the sunlit air, or on the surface of an illuminated opaque object, a secondary light, fainter than the light coming to this point directly from the sun, radiates "in the form of a sphere," rectilinearly in all directions. (The picture is interesting since it later appears in the doctrine of the multiplication of species and it is at the basis of Huygens' principle.)

Two other modes of propagation are the reflection of light from smooth bodies and its refraction when passing from one transparent body into another. Unlike an opaque body, a smooth surface does not behave, when illuminated, like a self-luminous object; rather than "receive" the impinging light it sends it back in a determinate direction. In *Optics,* book I, chapter 3, numerous experiments involving the use of various devices—sighting tubes, strings, dark chambers—are adduced to support all of the above statements, and in particular to establish the property of rectilinear propagation for all four kinds of radiation: primary, secondary, reflected, and refracted.

Colors are asserted to be as real as light and distinct from it; they exist as forms of the colored objects. A self-luminous body either possesses the form of color or something of "the same sort as color." Like light, colors radiate their forms upon surrounding bodies and this radiation originates from every point on the colored object and extends in all directions. It is possible that colors should be capable of extending themselves into the surrounding air in the absence of light; but experiments show that they are always found in the company of light, mingled with it, and they are never visible without it. Whatever rules apply to light also apply to colors.

Some time after writing the *Optics,* Ibn al-Haytham remarked in the *Discourse* (III 60) that natural philosophers, in contrast to mathematicians, had failed to supply a definite concept of ray. In book IV of the *Optics* he had in fact tried to remedy the defect by introducing the concept of a physical ray. The underlying idea is that for a body to be able to carry the form of light it must be of certain minimal magnitude. Imagine, then, that a transparent body through which light travels is made progressively thinner by a process of division. (The operation is essentially the same as that of narrowing an aperture through which the light passes.)

Ibn al-Haytham considered that a limit would be reached after which further division would cause the light to vanish. At this limit there would pass through

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the thin body a light of finite breadth which he calls the smallest or least light *(asghar al-ṣaghīr min al-ḍaw')*, a single ray whose only direction of propagation is the straight line extending through its length. A wider volume of light should not, however, be regarded as an aggregate of such minimal parts *(aḍwā' diqāq mutadāmma)*, but a continuous and coherent whole in which propagation takes place along all the straight lines, both parallel and intersecting, that can be imagined within its width. It follows that an aperture will either be wide enough to allow only rectilinear propagation, or too small to let any light pass through; there is no room for the diffraction of light. The result of the new concept is thus an uncompromising formulation of the ray theory of light. (Compare Newton's concept of "least Light or part of Light" which accords with his interpretation of diffraction as a kind of refraction.)[10]

Theory of Vision. As employed by Ibn al-Haytham the language of forms serves merely to express the view that light and color are real properties of physical bodies. He sometimes conducted his discussion without even using the term "form" (as in the greater part of book I, chapter 3) and his experimental arguments would lose nothing of their import if that term were to be removed from them. And yet it was the term "form" that had been closely associated with the intromission theory of vision maintained in the Peripatetic tradition, whereas mathematical opticians had formulated their geometrically represented explanations in terms of "visual rays" issuing from the eye. Ibn al-Haytham adopted the intromission hypothesis as the more reasonable one and took over with it the vocabulary of forms. To this he added, as we saw, a new concept of ray that satisfied the mathematical condition of rectilinearity but was consistent with the physics of forms. His theory of vision (to be described presently) may thus be seen as one chief illustration of the program he outlined in the *Optics* (III 3), in the treatise *On the Halo and the Rainbow* (III 8), and in the *Discourse on Light* (III 60): optical inquiry must "combine" the physical and the mathematical sciences.

In chapter 5 of book I of the *Optics* Ibn al-Haytham described the construction of the eye on the basis of what had been generally accepted in the tradition of medical and anatomical writings derived from Galen's works. But he adapted the geometry of this construction to suit his own explanation of vision. In particular he assumed both surfaces of the cornea opposite the pupil to be parallel to the anterior surface of the crystalline humor, all these surfaces being spherical and having the center of the eye as common center. He placed the center of the eye

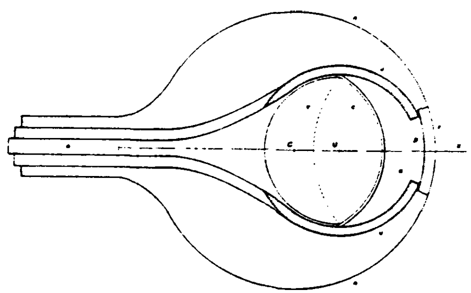

FIGURE 1. A cross section of the eye. Constructed from the text of *Kitāb al-Manāẓir.* After M. Naẓīf.

a, albugineous humor, *al-bayḍiyya; C,* center of eyeball; *c,* crystalline humor, *al-jalīdiyya; n,* exterior surface of conjunctiva, *al-multaḥima; o,* optic nerve, *al-'aṣab al-baṣarī; p,* uveal opening or pupil, *thaqb al-'inabiyya; r,* cornea, *al-qarniyya; U,* center of uvea; *u,* uvea, *al-'inabiyya; v,* vitreous humor, *al-zujājiyya; x,* axis of symmetry.

The axis of symmetry, passing through the middle of the pupil, the center of the uvea, and the center of the eye, goes to the middle of the optic nerve where the eyeball bends as a whole in its socket. The uvea is displaced forward toward the surface of the eye.

behind the posterior surface of the crystalline humor. The latter surface may be plane or spherical, so that the line passing through the middle of the pupil and the center of the eye would be perpendicular to it. (See Figure 1.[11]) The theory of vision is itself expounded in chapters 2, 4, 6, and 8.

Observations (such as the feeling of pain in the eye when gazing on an intense light, or the lingering impression in the eye of a strongly illuminated object) show that it is a property of light to make an effect on the eye, and a property of sight to be affected by light; visual sensation is therefore appropriately explained solely in terms of light coming to the eye from the object. As maintained by natural philosophers this effect is produced by the forms of the light and color in the visible object. But as an explanation of vision this statement in terms of forms is, by itself, "null and void" *(tantaqiḍ wa-tabṭul, destruitur)*.[12]

The problem Ibn al-Haytham posed for himself was to determine what further conditions are needed in order to bring the form of an external object intact into the eye where it makes its visual effect. His solution assumed the crystalline humor to be the organ in which visual sensation first occurs—an assumption which had been current since Galen. The solution also employs the experimentally supported principle which considers the shining object as a collection of points individually radiating their light and color (or the forms of their light and color) rectilinearly in all directions.[13] In consequence of this

principle any point on a visible object may be regarded as the origin of a cone of radiation with a base at the portion of the surface of the eye opposite the pupil. Since this holds for all points of the object, there will be spread over the whole of that portion the forms of the light and color of every one of these points.

Further confusion will result after the majority of these forms have been refracted upon their passage through the cornea. Ibn al-Haytham considered that for veridical perception to be possible it must be assumed that vision of any given object point can occur only through a given point on the surface of the eye, and he defined the latter point as that at which the perpendicular from the object point meets the cornea. It follows from the geometry of the eye that forms coming from all points on the object along perpendiculars to the surface of the eye will pass unrefracted through the pupil into the albugineous humor and again strike the anterior surface of the crystalline at right angles. There will then be produced on the crystalline humor a total form whose points will correspond, one-to-one, with all the points on the object, and it is this "distinct" and erect form which the crystalline humor will sense. Because the effective perpendiculars are precisely those that make up the outward extension of the cone having the center of the eye as vertex and the pupil as base (the so-called "radial cone," *makhrūṭ al-shuʿāʿ*), what we have in the end is the geometry of the Euclidean visual-ray theory.

But now the "mathematicians'" rays are strictly mathematical, that is, they are no more than abstract lines along which the light travels toward the eye—which is enough to save the geometrical optics of the ancients. As for the hypothesis that something actually goes out of the eye, it is clearly declared to be "futile and superfluous"—"Exitus ergo radiorum est superfluus et otiosus."[14] It would be absurd, says Ibn al-Haytham, to suppose that a material effluence flowing out of the eye would be capable of filling the visible heavens almost as soon as we lift our eyelids. If such effluence or visual rays are not corporeal, then they would not be capable of sensation and their function would merely be to serve as vehicles for bringing back something else from the object which itself would produce vision in the eye. But since this function is already fulfilled by the transparent medium through which light and color (or their forms) extend, visual rays are no longer of any use. (In the presence of this decisive argument it is curious that the editor of the Latin translation should misinterpret Ibn al-Haytham's remarks about preserving the geometrical property of the mathematicians' rays as an argument in support of the "Platonic" theory of συναύγεια, combining the intromission and extramission hypotheses.)[15]

Ibn al-Haytham managed to introduce the form of the visible object into the eye—an achievement which had apparently defeated his predecessors. But it should be noted that the "distinct form" he succeeded in realizing inside the eye is apparent only to the sensitive faculty; it is not a visibly articulate image such as that produced by a pinhole camera. In one place he ascribed the privileged role of the perpendicular rays to their superior strength. But there is another dominant idea. As a transparent body the crystalline humor allows non-perpendicular rays to be refracted into it from all points on its surface; as a sensitive body, however, it is especially concerned with those rays that go through it without suffering refraction. Veridical vision is thus due in the first place to the selective or directional sensitivity of the crystalline humor.

The vitreous humor, whose transparency differs from that of the crystalline, has still another property, namely that of preserving the integrity of the form handed down to it at its common face with the crystalline, where refraction of the effective rays takes place away from the axis of symmetry. The sensitive body (visual spirit), issuing along independent and parallel lines from the brain into the optic nerve, finally receives the form from the vitreous body and channels it back along the same lines to the front of the brain where the process of vision is completed. In the optic chiasma, where corresponding lines of the optic nerves join together, the form from the one eye coincides with that from the other, and from there the two forms proceed to the brain as one.

In book VII Ibn al-Haytham introduced what may be considered a generalization of the theory of vision already set out in book I. The form of his inquiry is the same as before: the determination of the conditions that must be assumed in order to accommodate the results of certain indubitable experiments. The experiments described here at first appear to speak against the earlier theory. A small object placed in the radial cone close to one eye, while the other is shut, does not hide an object point lying behind it on the common line drawn from the center of the eye. This means that the object point must in this case be seen by means of a ray falling obliquely, and therefore refracted, at the surface of the eye. Again, a small object placed outside the radial cone, as when a needle is held close to the corner of one eye, can be seen while the other eye is shut. Since no perpendicular can be drawn from the object in this position to any point in the area cut off from the eye-surface

by the radial cone, the object must be seen by refraction.

Briefly stated (and divorced from its rather problematic, though interesting, arguments), the final doctrine intended to take all of these observations into account is that vision of objects within the radial cone is effected both by direct and refracted rays, whereas objects outside the cone are seen only by refraction. Ibn al-Haytham here maintains that sensation of refracted as well as direct forms or rays takes place in the crystalline humor, although (in accordance with the earlier theory) he states that the "sensitive faculty" apprehends them all along perpendiculars drawn from the center of the eye to the objects seen. It is this general doctrine, that whatever we see is seen by refraction,[16] whether or not it is also seen by direct rays, that, according to Ibn al-Haytham, had not been grasped or explained by any writer on optics, ancient or modern.

The main part of Ibn al-Haytham's general theory of light and vision is contained in book I of the *Optics*. In book II he expounded an elaborate theory of cognition, with visual perception as the basis, which was referred to and made use of by fourteenth-century philosophers including, for example, Ockham,[17] and which has yet to receive sufficient attention from historians of philosophy. Book III deals with binocular vision and with the errors of vision and of recognition. Reflection is the subject of book IV, and here Ibn al-Haytham gave experimental proof of the specular reflection of accidental as well as essential light, a complete formulation of the laws of reflection, and a description of the construction and use of a copper instrument for measuring reflections from plane, spherical, cylindrical, and conical mirrors, whether convex or concave. He gave much attention to the problem of finding the incident ray, given the reflected ray (from any kind of mirror) to a given position of the eye. This is characteristic of the whole of the *Optics*—an eye is always given with respect to which the problems are to be formulated. The investigation of reflection—with special reference to the location of images—is continued in book V where the well-known "problem of Alhazen" is discussed, while book VI deals with the errors of vision due to reflection.

Book VII, which concludes the *Optics*, is devoted to the theory of refraction. Ibn al-Haytham gave considerable space to a detailed description of an improved version of Ptolemy's instrument for measuring refractions, and illustrated its use for the study of air-water, air-glass, and water-glass refractions at plane and spherical surfaces. Rather than report any numerical measurements, as in Ptolemy's tables, he stated the results of his experiments in eight rules which mainly govern the relation between the angle of incidence i (made by the incident ray and the normal to the surface) and the angle of deviation d (*zāwiyat al-inʿiṭāf, angulus refractionis*) contained between the refracted ray and the prolongation of the incident ray into the refracting medium. (This concentration on d rather than the angle of refraction r—which being equal to $i - d$ he called the remaining angle, *al-bāqiya*—was also a feature of Kepler's researches.)

His rules may be expressed as follows. Let d_1, d_2 and r_1, r_2 correspond to i_1, i_2, respectively, and let $i_2 > i_1$. It is asserted that

(1) $d_2 > d_1$;
(2) $d_2 - d_1 < i_2 - i_1$;
(3) $\frac{d_2}{i_2} > \frac{d_1}{i_1}$;
(4) $r_2 > r_1$;
(5) In rare-to-dense refraction, $d < 1/2\ i$;
(6) In dense-to-rare refraction, $d < 1/2\ (i + d)$ [$d < 1/2\ r$];
(7) A denser refractive medium deflects the light more toward the normal; and
(8) A rarer refractive medium deflects the light more away from the normal.

It is to be noted that (2) holds only for rare-to-dense refraction, and (5) and (6) are true only under certain conditions which, however, were implicit in the experiments, as Naẓīf has shown.[18] Concluding that "these are all the ways in which light is refracted into transparent bodies," Ibn al-Haytham does not give the impression that he was seeking a law which he failed to discover; but his "explanation" of refraction certainly forms part of the history of the formulation of the refraction law. The explanation is based on the idea that light is a movement which admits of variable speed (being less in denser bodies) and of analogy with the mechanical behavior of bodies. The analogy had already been suggested in antiquity, but Ibn al-Haytham's elaborate application of the parallelogram method, regarding the incident and refracted movements as consisting of two perpendicular components which can be considered separately, introduced a new element of sophistication. His approach attracted the attention of such later mathematicians as Witelo, Kepler, and Descartes, all of whom employed it, the last in his successful deduction of the sine law.

Minor Optical Works. The extant writings of Ibn al-Haytham include a number of optical works other than the *Optics*, of which some are important, show-

ing Ibn al-Haytham's mathematical and experimental ability at its best, although in scope they fall far short of the *Optics*. The following is a brief description of these works.

The Light of the Moon (III 6). Ibn al-Haytham showed here that if the moon behaved like a mirror, the light it receives from the sun would be reflected to a given point on the earth from a smaller part of its surface than is actually observed. He accordingly argued that the moon sends out its borrowed light in the same manner as a self-luminous source, that is, from every point on its surface in all directions. This is confirmed through the use of an astronomical diopter having a slit of variable length through which various parts of the moon could be viewed from an opposite hole in a screen parallel to the slit. The treatise is a beautiful combination of mathematical deduction and experimental technique. The experiments do not, however, lead to the discovery of a new property, but only serve to prove that the mode of emission from the moon is of the same kind as the already known mode of emission from self-luminous objects. Here, as in the *Optics*, the role of experiment is in contrast to its role in the work of, say, Grimaldi or Newton.

The Halo and the Rainbow (III 8). The subject is not treated in the *Optics*. In this treatise Ibn al-Haytham's explanation of the bow fails, being conceived of solely in terms of reflection from a concave spherical surface formed by the "thick and moist air" or cloud. The treatise did, however, become one of the starting points of Kamāl al-Dīn's more successful researches.

On Spherical Burning Mirrors (III 18). In contrast to the eye-centered researches of the *Optics* the only elements of the problems posed in this treatise (and in III 19) are the luminous source, the mirror, and the point or points in which the rays are assembled. Ibn al-Haytham showed that rays parallel to the axis of the mirror are reflected to a given point on the axis from only one circle on the mirror; his remarks imply a recognition of spherical aberration along the axis.

On Paraboloidal Burning Mirrors (III 19). This refers to Archimedes and Anthemius "and others" as having adopted a combination of spherical mirrors whose reflected rays meet in one point. Drawing ably on the methods of Apollonius, Ibn al-Haytham set out to provide a proof of a fact which, he said, the ancients had recognized but not demonstrated: that rays are reflected to one point from the whole of the concave surface of a paraboloid of revolution.

The Formation of Shadows (III 36). That there were many writings on shadows available to Ibn al-Haytham is clear from his reference here to *aṣḥāb al-aẓlāl* (the authors on shadows). Indeed, a long treatise on shadows by his contemporary al-Bīrūnī is extant. Ibn al-Haytham defines darkness as the total absence of light, and shadow as the absence of some light and the presence of another. He made the distinction between umbra and penumbra—calling them *ẓulma* (darkness) or *ẓill maḥḍ* (pure shadow), and *ẓill* (shadow), respectively.

The Light of the Stars (III 48). This argues that all stars and planets, with the sole exception of the moon, are self-luminous.

Discourse on Light (III 60). Composed after the *Optics*, this treatise outlines the general doctrine of light. Some of its statements have been used in the account given above.

The Burning Sphere (III 77). In this work, written after the *Optics*, Ibn al-Haytham continued his investigations of refraction, but, as in III 18 and III 19, without reference to a seeing eye. He studied the path of parallel rays through a glass sphere, tried to determine the focal length of such a sphere, and pointed out spherical aberration. The treatise was carefully studied by Kamāl al-Dīn, who utilized it in his account of the path of rays from the sun inside individual rain drops.

The Shape of the Eclipse (III 80). This treatise is of special interest because of what it reveals about Ibn al-Haytham's knowledge of the important subject of the *camera obscura*. The exact Arabic equivalent of that Latin phrase, *al-bayt al-muẓlim*, occurs in book I, chapter 3 of the *Optics*;[19] and indeed dark chambers are frequently used in this book for the study of such various properties of light as its rectilinear propagation and the fact that shining bodies radiate their light and color on neighboring objects. But such images as those produced by a pinhole camera are totally absent from the *Optics*. The nearest that Ibn al-Haytham gets to such an image is the passage in which he describes the patches of light cast on the inside wall of a "dark place" by candle flames set up at various points opposite a small aperture that leads into the dark place; the order of the images on the inside wall is the reverse of the order of the candles outside.

The experiment was designed to show that the light from one candle is not mingled with the light from another as a result of their meeting at the aperture, and in general that lights and colors are not affected by crossing one another. Although this passage occurs in book I in the context of the theory of vision,[20] the eye does not in Ibn al-Haytham's explanation act as a pinhole camera and it is expressly denied the role of a lens camera. In the present treatise, however,

he approached the question, already posed in the pseudo-Aristotelian *Problemata*, of why the image of a crescent moon, cast through a small circular aperture, appears circular, whereas the same aperture will cast a crescent-shaped image of the partially eclipsed sun. Although his answer is not wholly satisfactory, and although he failed to solve the general problem of the pinhole camera, his attempted explanation of the image of a solar crescent clearly shows that he possessed the principles of the working of the camera. He formulated the condition for obtaining a distinct image of an object through a circular aperture as that when

$$\frac{m_a}{m_s} \le \frac{d_a}{d_s},$$

where m_a, m_s are the diameters of the aperture and of the object respectively, and d_a, d_s the distances of the screen from the aperture and from the object respectively.

Ibn al-Haytham's construction of the crescent-shaped image of the partially eclipsed sun can be clearly understood by reference to Figure 2. (Because Ibn al-Haytham's own diagram shows the crescents but not the circles, the figure shown is that constructed by Nazīf.) It represents the special case in which the two ratios just mentioned are equal. It is assumed that the line joining the centers of the two arcs forming the solar crescent is parallel to the planes of the aperture and the screen, and further that the line joining the center of the sun and the center of the circular hole is perpendicular to the plane of the latter and to the plane of the screen.

The crescents *p, q, r*, are inverted images produced

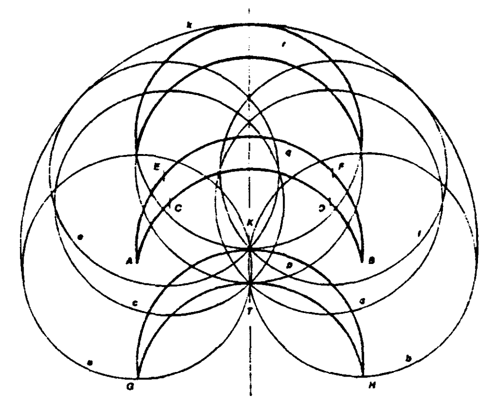

FIGURE 2. Ibn al-Haytham's construction of an inverted image of the partially eclipsed sun.

by three double conical solids of light whose vertices are three different points on the aperture, and whose bases are, on the one side, the shining solar crescent, and, on the other, the inverted image. These solids are each limited by two conical surfaces of which one is convex and the other concave; and in every double solid the convex surface on one side of the aperture corresponds to the concave surface on the other. The middle crescent image *q* is produced by such a double solid having its vertex at the aperture-center; *p* and *r* have their vertices at the extremities of a diameter of the aperture. The circular images are each produced by a single cone whose vertex is a single point on the shining crescent; as many such circles are produced as there are points on the crescent sun.

The center of each circle is therefore the point at which the axis of the cone, passing through the center of the aperture, intersects the screen. It is clear that the centers of all circles will be points on crescent *q*, and that their radii, as well as those of the arcs forming crescents *p, q, r*, will all be equal. The resultant image will therefore be bounded from above by a convex curve of which the upper part is the tangential arc of a circle whose center is the midpoint *K* of the convex arc of crescent *p*, and whose radius is twice the radius of that arc. Although circles of light will occur below arc *GTH*, they will be relatively few.

The sensible overall effect will be, according to Ibn al-Haytham, a crescent-shaped image bordered on the lower side by a sensibly dark cavity. He showed by a numerical example that the cavity will increase or decrease in size according as the ratio $m_a : m_s$ is less or greater than $d_a : d_s$. It is certain that the treatise *On the Shape of the Eclipse* was composed after the *Optics*, to which it refers. It is not impossible that, at the time of writing the *Optics*, Ibn al-Haytham was acquainted with the remarkable explanation revealed in the later work, but of this we have no evidence.

Transmission and Influence of the Optics. Of all the optical treatises of Ibn al-Haytham that have been mentioned, only the *Optics* (III 3) and the treatise *On Paraboloidal Burning Mirrors* (III 9) are known to have been translated into Latin in the Middle Ages, the latter probably by Gerard of Cremona.[21] It is remarkable that in the Islamic world the *Optics* practically disappeared from view soon after its appearance in the eleventh century until, in the beginning of the fourteenth century, the Persian scholar Kamāl al-Dīn composed his great critical commentary on it, the *Tanqīḥ al-Manāẓir*, at the suggestion of his teacher Quṭb al-Dīn al-Shīrāzī.

By this time the *Optics* had embarked on a new career in the West where it was already widely and

avidly studied in a Latin translation of the late twelfth or early thirteenth century, entitled *Perspectiva* or *De aspectibus*. Of the manuscript copies that have been located (no fewer than nineteen[22]), the earliest are from the thirteenth century; but where and by whom the *Optics* was translated remains unknown. The Latin text was published by Frederick Risner at Basel in 1572 in a volume entitled *Opticae thesaurus*, which included Witelo's *Perspectiva*. In both Risner's edition and the Latin manuscripts examined by the present writer (see bibliography) the Latin text wants the first three chapters of book I of the Arabic text (133 pages, containing about 130 words per page, in MS Fatih 3212).

The Latin *Perspectiva* shows the drawbacks as well as the advantages of the literal translation which in general it is. Often, however, it only paraphrases the Arabic, sometimes inadequately or even misleadingly, and at times it omits whole passages. But an exhaustive and critical study of the extant manuscripts is needed before a full and accurate evaluation of the translation can be made. In any case there is no doubt that through this Latin medium a good deal of the substance of Ibn al-Haytham's doctrine was successfully conveyed to medieval, Renaissance, and seventeenth-century philosophers in the West. Roger Bacon's *Perspectiva* is full of references to "Alhazen," or *auctor perspectivae*, whose influence on him cannot be overemphasized. Pecham's *Perspectiva communis* was composed as a compendium of the *Optics* of Ibn al-Haytham.[23] That Witelo's *Opticae libri decem* also depends heavily on *Alhazeni libri septem* has been noted repeatedly by scholars; the cross-references provided by Risner in his edition of the two texts have served as a sufficient indication of that. But Witelo's precise debt to Ibn al-Haytham, as distinguished from his own contribution, has yet to be determined.

The influence of Ibn al-Haytham's *Optics* was not channelled exclusively through the works of these thirteenth-century writers. There is clear evidence that the book was directly studied by philosophers of the fourteenth century[24] and an Italian translation made at that time was used by Lorenzo Ghiberti.[25] Risner's Latin edition made it available to such mathematicians as Kepler, Snell, Beeckman, Fermat, Harriot, and Descartes, all of whom except the last directly referred to Alhazen. It was, in fact, in the sixteenth and seventeenth centuries that the mathematical character of the *Optics* was widely and effectively appreciated.

Astronomy. No fewer than twenty of Ibn al-Haytham's extant works are devoted to astronomical questions. The few of these that have been studied by modern scholars do not appear to justify al-Bayhaqī's description of Ibn al-Haytham as "the second Ptolemy." (The description would be apt, however, if al-Bayhaqī had optics in mind.) Many of these works are short tracts that deal with minor or limited, although by no means trivial, theoretical or practical problems (sundials, determination of the direction of prayer, parallax, and height of stars), and none of them seems to have achieved results comparable to those of, say, Ibn Yūnus, al-Ṭūsī, or Ibn al-Shāṭir. Nevertheless, some of Ibn al-Haytham's contributions in this field are both interesting and historically important, as has sometimes been recognized.

As a writer on astronomy Ibn al-Haytham has been mainly known as the author of a treatise *On the Configuration of the World* (III 1 = Ib 10). The treatise must have been an early work: it speaks of "the ray that goes out of our eyes" and describes the moon as a "polished body" which "reflects" the light of the sun—two doctrines which are refuted in the *Optics* (III 3) and in *The Light of the Moon* (III 6), respectively. The treatise was widely known in the Islamic world,[26] and it is the only astronomical work of Ibn al-Haytham to have been transmitted to the West in the Middle Ages. A Spanish translation was made by Abraham Hebraeus for Alfonso X of Castile (*d.* 1284), and this translation was turned into Latin (under the title *Liber de mundo et coelo*) by an unknown person.

Jacob ben Maḥir (Prophatius Judaeus, *d. ca.* 1304) translated the Arabic text of the *Configuration* into Hebrew, a task which was suggested to him as a corrective to the *Elements of Astronomy* of al-Farghānī, whose treatment of the subject "did not accord with the nature of existing things," as the unknown person who made the suggestion said.[27] The physician Salomo ibn Pater made another Hebrew translation in 1322. A second Latin version was later made from Jacob's Hebrew by Abraham de Balmes for Cardinal Grimani (both of whom died in 1523). In the fourteenth century Ibn al-Haytham's treatise was cited by Levy ben Gerson. Its influence on early Renaissance astronomers and in particular on Peurbach's *Theoricae novae planetarum* has recently been pointed out.[28]

The declared aim of the *Configuration* was to perform a task which, in Ibn al-Haytham's view, had not been fulfilled either by the popularly descriptive or the technically mathematical works on astronomy. The existing descriptive accounts were only superficially in agreement with the details established by demonstrations and observations. A purely mathematical work like the *Almagest*, on the other hand, explained the laws (*qawānīn*) of celestial motions in terms of imaginary points moving on imaginary cir-

cles. It was necessary to provide an account that was faithful to mathematical theory while at the same time showing how the motions were brought about by the physical bodies in which the abstract points and circles must be assumed to exist. Such an account would be "more truly descriptive of the existing state of affairs and more obvious to the understanding."[29]

Ibn al-Haytham's aim here was not therefore to question any part of the theory of the *Almagest* but, following a tradition which goes back to Aristotle and which had been given authority among astronomers by one of Ptolemy's own works, the *Planetary Hypotheses*, to discover the physical reality underlying the abstract theory. The description had to satisfy certain principles already accepted in that tradition: a celestial body can have only circular, uniform, and permanent movement; a natural body cannot by itself have more than one natural movement; the body of the heavens is impassable; the void does not exist. Ibn al-Haytham's procedure was then, for every simple motion assumed in the *Almagest*, to assign a single spherical body to which this motion permanently belongs, and to show how the various bodies may continue to move without in any way impeding one another or creating gaps as they moved.

The heavens were accordingly conceived of as consisting of a series of concentric spherical shells (called spheres) which touched and rotated within one another. Inside the thickness of each shell representing the sphere of a planet other concentric and eccentric shells and whole spheres corresponded to concentric and eccentric circles and epicycles respectively. All shells and spheres rotated in their own places about their own centers, and their movements combined to produce the apparent motion of the planet assumed to be embedded in the epicyclic sphere at its equator. In his careful description of all movements involved Ibn al-Haytham provided, in fact, a full, clear, and untechnical account of Ptolemaic planetary theory—which alone may explain the popularity of his treatise.

A brief look at Ibn al-Haytham's other works will give us an idea of how seriously he took the program he inherited and of its significance for the later history of Islamic astronomy. Perhaps at some time after writing the *Configuration of the World* (III 1 = Ib 10) Ibn al-Haytham composed a treatise (III 61) on what he called the movement of *iltifāf*, that is the movement or rather change in the obliquities (singular, *mayl*) of epicycles responsible for the latitudinal variations of the five planets (*Almagest*, XIII.2). This treatise is not known to have survived. But we have Ibn al-Haytham's reply to a criticism of it by an unnamed scholar. From this reply, the tract called *Solution of the Difficulties [shukūk] Concerning the Movement of Iltifāf* (III 63), we learn that in the earlier treatise he proposed a physical arrangement designed to produce the oscillations of epicycles required by the mathematical theory. The same subject is discussed, among other topics, in the work entitled *Al-Shukūk 'alā Baṭlamyūs (Dubitationes in Ptolemaeum)* (III 64). More than any other of Ibn al-Haytham's writings, this work (almost certainly composed after the reply just mentioned) reveals the far-reaching consequences of the physical program to which he was committed.

The *Dubitationes* is a critique of three of Ptolemy's works: the *Almagest*, the *Planetary Hypotheses* and the *Optics*. As far as the first two works are concerned, the criticism is mainly aimed against the purely abstract character of the *Almagest* (this exclusiveness being in Ibn al-Haytham's view a violation of the principles accepted by Ptolemy himself) and against the fact that the *Planetary Hypotheses* had left out many of the motions demanded in the *Almagest* (a proof that Ptolemy had failed to discover the true arrangement of the heavenly bodies).

Ibn al-Haytham's objection to the "fifth motion" of the moon, described in *Almagest* V.5, is particularly instructive, being nothing short of a *reductio ad absurdum* by "showing" that such a motion would be physically impossible. Ptolemy had assumed that as the moon's epicycle moves on its eccentric deferent, the diameter through the epicycle's apogee (when the epicycle-center is at the deferent's apogee) rotates in such a way as to be always directed to a point on the apse-line (called the opposite point, *nuqṭat al-muḥādhāt*), such that the ecliptic-center lies halfway between that point and the deferent-center. The assumption implied that the epicycle's diameter alternately rotates in opposite senses as the epicycle itself completes one revolution on its deferent. But, Ibn al-Haytham argued, such a movement would have to be produced either by a single sphere which would alternately turn in opposite senses, or by two spheres of which one would be idle while the other turned in the appropriate sense. "As it is not possible to assume a body of this description, it is impossible that the diameter of the epicycle should be directed towards the given point."[30] Whatever one thinks of the argument, the problem it raised was later fruitfully explored by Naṣīr al-Dīn al-Ṭūsī in the *Tadhkira*.[31]

Perhaps most important historically was Ibn al-Haytham's objection against the theory of the five planets, and in particular against the device introduced by Ptolemy which later came to be known as the equant. Ptolemy supposed that the point from which the planet's epicycle would appear to move

uniformly is neither the center of the eccentric deferent nor that of the ecliptic, but another point (the equant) on the line of apsides as far removed from the deferent-center as the latter is from the ecliptic-center. This entailed, as Ibn al-Haytham pointed out, that the motion of the epicycle-center, as measured on the circumference of its deferent, was not uniform, and consequently that the deferent sphere carrying the epicycle was not moving uniformly—in contradiction to the assumed principle of uniformity.

Although the equant had succeeded in bringing Ptolemy's planetary theory closer to observations, the validity of this criticism remained as long as the principle of uniform circular motion was adhered to. To say that the equant functioned merely as an abstract calculatory device designed for the sake of saving the phenomena was an answer which satisfied none of Ptolemy's critics, down to and including Copernicus. Nor was Ptolemy himself unaware of the objectionable character of such devices. In the *Dubitationes*, Ibn al-Haytham points to a passage in *Almagest* IX. 2 where Ptolemy asks to be excused for having employed procedures which, he admitted, were against the rules (*παρὰ τὸν λόγον*, *khārij ʿan al-qiyās*), as, for example, when for convenience's sake he made use merely of circles described in the planetary spheres, or when he laid down principles whose foundation was not evident. For, Ptolemy said, "when something is laid down without proof and is found to be in accord with the phenomena, then it cannot have been discovered without a method of science [*sabīl min al-ʿilm*], even though the manner in which it has been attained would be difficult to describe."[32]

Ibn al-Haytham agreed that it was indeed appropriate to argue from unproved assumptions, but not when they violated the admitted principles. His final conclusion was that there existed a true configuration of the heavens which Ptolemy had failed to discover.

It has been customary to contrast the "physical" approach of Ibn al-Haytham with the "abstract" approach of mathematical astronomers. The contrast is misleading if it is taken to imply the existence of two groups of researchers with different concerns. The "mathematical" researches of the school of Marāgha (among them al-Ṭūsī and al-Shīrāzī) were motivated by the same kind of considerations as those revealed in Ibn al-Haytham's *Dubitationes*.[33] Al-Ṭūsī, for instance, was as much worried about the moon's "fifth movement" and about the equant as was Ibn al-Haytham, and for the same reasons.[34] His *Tadhkira* states clearly that astronomical science is based on physical as well as mathematical premises. From a reference in it to Ibn al-Haytham,[35] made in the course of expounding alterations based on what is now known as the "Ṭūsī couple," it is clear that al-Ṭūsī recognized the validity of Ibn al-Haytham's physical program, although not the particular solutions offered by his predecessor.

The longest of the astronomical works of Ibn al-Haytham that have come down to us is a commentary on the *Almagest*. The incomplete text in the unique Istanbul manuscript which has recently been discovered occupies 244 pages of about 230 words each (see bibliography, additional works, no. 3). The manuscript, copied in 655 A.H./A.D. 1257, bears no title but twice states the author's name as Muḥammad ibn al-Ḥasan ibn al-Haytham, the name found by Ibn Abī Uṣaybiʿa in Ibn al-Haytham's own bibliographies, that is lists I and II. No title in list III seems to correspond to this work, but there are candidates in the other lists. The first title in Ibn al-Qifṭī's list is *Tahdhīb al-Majisṭī* or *Expurgation of the Almagest*. Number 19 in list II is described as "A book which works out the practical part of the *Almagest*." And number 3 in list *Ia* begins as follows: "A commentary and summary of the *Almagest*, with demonstrations, in which I worked out only a few of the matters requiring computation. . . ." The last title is highly appropriate to the work that has survived.

Most commentators on the *Almagest*, Ibn al-Haytham says in the introduction, were more interested in proposing alternative techniques of computation than in clarifying obscure points for the beginner. As an example he mentions al-Nayrīzī who "crammed his book with a multiplicity of computational methods, thereby seeking to aggrandize it." Ibn al-Haytham sought rather to explain basic matters relating to the construction of Ptolemy's own tables, and he meant his commentary to be read in conjunction with the *Almagest*, whose terminology and order of topics it followed. The book was therefore to comprise thirteen parts, but, for brevity's sake and also because the *Almagest* was "well known and available," Ibn al-Haytham would not follow the commentators' customary practice of reproducing Ptolemy's own text. Unfortunately the manuscript breaks off before the end of the fifth part, shortly after the discussion of Ptolemy's theories for the sun and the moon. In the course of additions designed to complete, clarify, or improve Ptolemy's arguments, Ibn al-Haytham referred to earlier Islamic writers on astronomy, including Thābit ibn Qurra (on the "secant figure"), Banū Mūsā (on the sphere), and Ibrāhīm ibn Sinān (on gnomon shadows). All diagrams have been provided and are clearly drawn in the manuscript but the copyist has not filled in the tables.

Mathematics. Ibn al-Haytham's fame as a mathematician has rested on his treatment of the problem known since the seventeenth century as "Alhazen's problem." The problem, as viewed by him, can be expressed as follows: from any two points opposite a reflecting surface—which may be plane, spherical, cylindrical, or the surface of a cone, whether convex or concave—to find the point (or points) on the surface at which the light from one of the two points will be reflected to the other. Ptolemy, in his *Optics*, had shown that for convex spherical mirrors there exists a unique point of reflection. He also considered certain cases relating to concave spherical mirrors, including those in which the two given points coincide with the center of the specular sphere; the two points lie on the diameter of the sphere and at equal or unequal distances from its center; and the two points are on a chord of the sphere and at equal distances from the center. He further cited some cases in which reflection is impossible.[36]

In book V of his *Optics*, Ibn al-Haytham set out to solve the problem for all cases of spherical, cylindrical, and conical surfaces, convex and concave. Although he was not successful in every particular, his performance, which showed him to be in full command of the higher mathematics of the Greeks, has rightly won the admiration of later mathematicians and historians. Certain difficulties have faced students of this problem in the work of Ibn al-Haytham. In the Fatih manuscript, and in the Aya Sofya manuscript which is copied from it, the text of book V of the *Optics* suffers from many scribal errors, and in neither of these manuscripts are the lengthy demonstrations supplied with illustrative diagrams.[37] Such diagrams exist in Kamāl al-Dīn's commentary and in Risner's edition of the medieval Latin translation, but neither the diagrams nor the texts of these two editions are free from mistakes. One cannot, therefore, be too grateful to M. Naẓīf for his clear and thorough analysis of this problem, to which he devotes four chapters of his masterly book on Ibn al-Haytham.

Ibn al-Haytham bases his solution of the general problem on six geometrical lemmas (*muqaddamāt*) which he proves separately: (1) from a given point A on a circle ABG, to draw a line that cuts the circumference in H and the diameter BG in a point D whose distance from H equals a given line; (2) from the given point A to draw a line that cuts the diameter BG in a point E and the circumference in a point D such that ED equals the given line; (3) from a given point D on the side BG of a right-angled triangle having the angle B right, to draw a line DTK that cuts AG in T (and the extension of BA in K), such that $KT:TG$ equals a given ratio; (4) from two points E,D outside a given circle AB, to draw two lines EA and DA, where A is a point on the circumference, such that the tangent at A equally divides the angle EAD; (5) from a point E outside a circle having AB as diameter and G as center, to draw a line that cuts the circumference at D and the diameter at Z such that DZ equals ZG; and (6) from a given point D on the side GB of a right-angled triangle having the angle B right, to draw a line that meets the hypotenuse AG at K and the extension of AB on the side of B at T, such that $TK:KG$ equals a given ratio.[38]

Obviously lemmas (1) and (2) are special cases of one and the same problem, and (3) and (6) are similarly related. In his exposition of Ibn al-Haytham's arguments Naẓīf combines each of these two pairs in one construction. It will be useful to reproduce here his construction for (1) and (2) and to follow him in explaining Ibn al-Haytham's procedure by referring to this construction. It happens that (1) and (2) contain characteristic features of the proposed solution of the geometrical problem involved. In Figure 3, A is a given point on the circumference of the small circle with diameter BG. It is required to draw a straight line from A that cuts the circle at D and the diameter, or its extension, at E, such that DE equals the given segment z.

From G draw the line GH parallel to AB; let it cut the circle at H; join BH. Let the extensions of AG, AB respectively represent the coordinate axes x, y whose origin thus coincides with A. Draw the hyperbola passing through H with x, y as asymptotes. Then, with H as center, draw the circle with radius

$$HS = \frac{BG^2}{z}$$

(HS being the side of a rectangle whose other side is z, and whose area equals BG^2). The circle will, in the general case, cut the two branches of the hyperbola at four points, such as S, T, U, V. Join H with all four points, and from A draw the lines parallel to HS, HT, HU, and HV. Each of these parallels will cut the circle circumscribing the triangle ABG at a point, such as D, and the diameter, or its extension, at another point, such as E. It is proved that each of these lines satisfies the stated condition.

As distinguished from the above demonstration, Ibn al-Haytham proceeded by considering three cases one after the other: (a) the required line is tangential to the circle, that is, A and D coincide; (b) D is on the arc AG; (c) D is on the arc AB. Despite the generality of the enunciation of lemma (1), he does not consider the case in which the line cuts the exten-

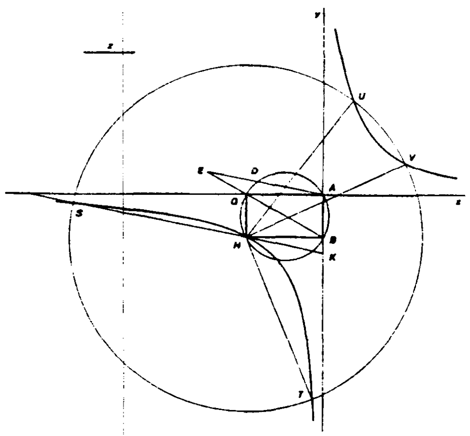

FIGURE 3

sion of BG on the side of B. Similarly in dealing with lemma (2) he separately examines three possibilities in respect of the relation of the circle HS to the "opposite branch" of the hyperbola: (a) the circle cuts that branch at two points; (b) the circle is tangential to it at one point; (c) the circle falls short of it. For finding the shortest line between H and the "opposite branch" of the hyperbola he refers to Apollonius' *Conics*, V. 34. Ibn al-Haytham did not, of course, speak of a coordinate system of perpendicular axes whose origin he took to be the same as the given point A. He did, however, consider a rectangle similar to $ABHG$, and described the sides of it corresponding to AB, AG as asymptotic to the hyperbola he drew through a point corresponding to H in Figure 3. For drawing this hyperbola he referred to *Conics*, II. 4.

Applying the six geometrical lemmas for finding the points of reflection for the various kinds of surface, Ibn al-Haytham again proceeded by examining particular cases in succession. Nazīf shows that the various cases comprised by lemma 4 constitute a general solution of the problem in respect of spherical surfaces, concave as well as convex. With regard to cylindrical mirrors, Ibn al-Haytham considered the cases in which (a) the two given points are in the plane perpendicular to the axis; and (c) the general case in which the intersection of the plane containing the two points with the cylinder is neither a straight line nor a circle, but an ellipse. He described six different cases in an attempt to show that reflection from convex conical surfaces can take place from only one point, which he determined. For concave conical mirrors, he showed that reflection can be from any number of points up to but not exceeding four. And he argued for the same number of points for concave cylindrical mirrors.

Apart from the mathematical sections of the *Optics*, some twenty of the writings of Ibn al-Haytham which deal exclusively with mathematical topics have come down to us. Most of these writings are short and they vary considerably in importance. About a quarter of them have been printed in the original Arabic and about half of them are available in European translations or paraphrases. Some of the more important among these works fall into groups and will be described as such.

List III includes three works (III 39, III 55, and III 56) which are described as solutions of difficulties arising in three different parts of Euclid's *Elements*. There are no manuscripts exactly answering to these descriptions. There exist, on the other hand, several manuscripts of a large work entitled *Solution of the Difficulties in Euclid's Elements*, which does not appear in list III. It therefore seems likely that III 39, III 55, and III 56 are parts of the larger work which is listed below in the bibliography as additional work no. 1.

The object of the *Solution* was to put into effect a rather ambitious program. Unlike earlier works by other writers, it proposed to deal with all or most of the difficulties occasioned by Euclid's book, and not with just a few of them; it examined particular cases and offered alternative constructions for many problems; it revealed the "remote mathematical causes" (*al-ʿilal al-taʿlīmiyya al-baʿīda*) of the theoretical propositions (*al-ashkāl al-ʿilmiyya*)—something which "none of the ancients or the moderns had previously mentioned"; and, finally, it replaced Euclid's indirect proofs with direct ones. In this book Ibn al-Haytham referred to an earlier *Commentary on the Premises of Euclid's Elements* (III 2), and said that he meant the two works to form together a complete commentary on the whole of the *Elements*. This earlier work, restricted to the definitions, axioms, and postulates of the *Elements*, is extant both in the Arabic original and in a Hebrew translation made in 1270 by Moses ibn Tibbon. Ibn al-Haytham's interesting treatment of Euclid's theory of parallels well illustrates his approach in these two "commentaries."

In III 2 Ibn al-Haytham ascribed to Euclid the "axiom" that "two straight lines do not enclose a space [*saṭḥ*]" (his own opinion is that the statement should be counted among the "postulates"). Concerning Euclid's definition of parallel lines as nonsecant lines he remarked that the "existence" of such lines should be proved and, for this purpose, introduced the following "more evident" postulate: if a straight line so moves that the one end always

touches a second straight line, and throughout this motion remains perpendicular to the second and in the same plane with it, then the other end of the moving line will describe a straight line which is parallel to the second. Ibn al-Haytham thus replaced parallelism in Euclid's sense by the property of equidistance, a procedure which had originated with the Greeks and which had characterized many Islamic attempts to prove Euclid's postulate 5.

Like Thābit ibn Qurra before him Ibn al-Haytham based his proof on the concept of motion—which procedure al-Khayyāmī and, later, al-Ṭūsī found objectionable as being foreign to geometry. The crucial step in the deduction of Euclid's postulate is the demonstration of Saccheri's "hypothesis of the right angle" by reference to a "Saccheri quadrilateral." Let AG, BD be drawn at right angles to AB (Figure 4): it is to be proved that perpendiculars to BD from points on AG are equal to AB, and, consequently, perpendicular to AG. From any point G draw GD perpendicular to BD; produce GA to E such that AE equals AG; draw ET perpendicular to DB produced; and join BG, BE. Considering, first, triangles ABG, ABE, then triangles BDG, BTE, it is seen that GD equals ET. Let GD now move along DBT, the angle GDT being always right. Then, when D coincides with B, G will either coincide with A, or fall below it on AB, or above it (occupying the position of H in the figure) on BA produced, according as GD is assumed equal to, less than, or greater than AB. When D reaches T, GD will exactly coincide with ET. During this motion, G will have described a straight line which, on the hypothesis that DG is not equal to AB, would enclose an area, such as $GHEA$, with another straight line, GAE—which is impossible. Finally, by considering in turn triangles BDG, BDA, and AKB, GKD, it is clear that $DGA = BAG =$ a right angle. The Euclidean postulate follows as a necessary consequence.

In the larger commentary, Ibn al-Haytham refor-

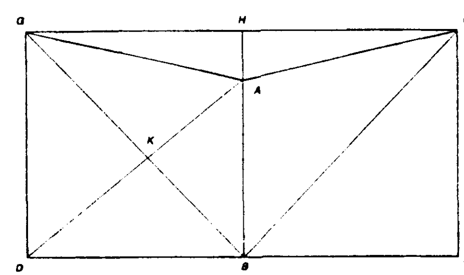

FIGURE 4

mulated postulate 5, stating that two intersecting straight lines cannot both be parallel to a third ("Playfair's axiom"), and referring to the proof set forth in the earlier, and shorter, work. It is to be noted that al-Ṭūsī's criticism (in his own work on the theory of parallels, *Al-Risāla al-Shāfiya*) of Ibn al-Haytham's attempt was based on the remarks in this larger commentary, not on the earlier proof, which al-Ṭūsī said was not available to him.[39]

Ibn al-Haytham wrote two treatises on the quadrature of crescent-shaped figures (*al-ashkāl al-hilāliyya*) or lunes. (Their titles have sometimes been misunderstood as referring to the moon.) The second, and fuller, treatise (III 21), although extant in several manuscripts, has not been studied. From the introduction we gather that it was composed quite some time after the first (III 20, now lost), although the two works appear consecutively in list III. The treatise comprises twenty-three propositions on lunes, of which some are generalizations of particular cases already proved in the earlier treatise, as the author tells us, while others are said to be entirely new. The subject was connected with that of squaring the circle: if plane figures bounded by two unequal circular arcs could be squared, why not the simpler figure of a circle? Ibn al-Haytham put forward such an argument in a short tract on the *Quadrature of the Circle* (III 30), which has been published. The object of the tract is to prove the "possibility" of squaring the circle without showing how to "find" or construct a square equal in area to a given circle.

To illustrate his point, Ibn al-Haytham proves a generalization of a theorem ascribed to Hippocrates of Chios. The proof is reproduced from his earlier work on lunes. In Figure 5, let B be any point on the semicircle with diameter AG; describe the smaller semicircles with AB, BG as diameters; it is shown that the lunes $AEBH$, $BZGT$ are together equal in area to the right-angled triangle ABG. On the basis of Euclid XII.2, which states that circles are to one another as the squares on the diameters, it is easily proved that the semicircles on AB, BG are together equal to the semicircle on the hypotenuse AG. The equality of the lunes to the triangle ABG follows from subtracting the segments AHB, BTG from both sides of the equation. Hippocrates had considered the particular case in which the triangle ABG is isosceles.[40]

Two more works which are closely related are *On Analysis and Synthesis* (*Maqāla fi 'l-taḥlīl wa 'l-tarkīb*, III 53) and *On the Known Things* (*Maqāla fi 'l-Maʿlūmāt*, III 54). The subject matter of the latter work overlaps with that of Euclid's *Data*, which is called in Arabic *Kitāb al-Muʿṭayāt* (Δεδομένα). Ibn al-Haytham's use of *al-maʿlūmāt*, rather than *al-muʿṭayāt*,

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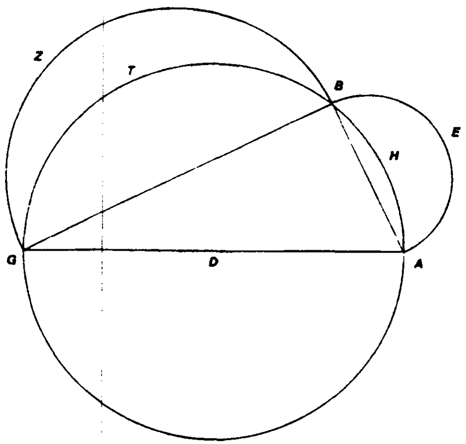

FIGURE 5

has a precedent in the Arabic translation of Euclid's book itself, where *al-ma'lūm* (the known) is regularly employed to denote the given. *On Analysis* is a substantial work of about 24,000 words whose object is to explain the methods of analysis and synthesis, necessary for the discovery and proof of theorems and constructions, by illustrating their application to each of the four mathematical disciplines: arithmetic, geometry, astronomy, and music. It lays particular emphasis on the role of "scientific intuition" (*al-ḥads al-ṣinā'ī*), when properties other than those expressly stated in the proposition to be proved have to be conjectured before the process of analysis can begin.

In describing the relationship of this treatise to the one on *The Known Things* Ibn al-Haytham made certain claims which should be quoted here. The art of analysis, he says, is not complete without the things that are said to be known.

> Now the known things are of five kinds: the known in number, the known in magnitude, the known in ratio, the known in position, and the known in species [*al-ma'lūm al-ṣūra*]. The book of Euclid called *Al-Mu'ṭayāt* includes many of these known things which are the instruments of the art of analysis, and on which the larger part of analysis is based. But that book does not include other known things that are indispensable to the art of analysis . . . nor have we found them in any other book. In the examples of analysis we give in the present treatise we shall prove the known things used, whether or not we have found them in other works. . . . After we have completed this treatise we shall resume the subject in a separate treatise in which we shall show the essence of the known things that are used in mathematics and give an account of all their kinds and of all that relates to them.[41]

The treatise on known things, which is extant, divides in fact into two parts, of which the first (comprising twenty-four propositions) is said to be the invention of Ibn al-Haytham himself. In 1834 L. Sédillot published a paraphrase of the introduction to this work (a discussion of the concept of knowledge) together with a translation of the enunciations of the propositions constituting both parts. There is no study of the work on *Analysis and Synthesis*. The more important of the remaining mathematical works are all available in European translations.

NOTES

1. On al-Fāsī, see Ibn al-Qifṭī, *Ta'rīkh*, pp. 392–394.
2. *Al-Mutawassiṭāt*, or intermediate books, so called because they were studied after the *Elements* of Euclid and before the *Almagest*. They included, for instance, Euclid's *Data*, Theodosius' *Spherics*, and the *Spherics* of Menelaus. See the explanation of Abu'l-Hasan al-Nasawī in al-Ṭūsī, *Majmū' al-Rasā'il*, II (Hyderabad, 1359 A.H. [1940]), *risāla* no. 3, p. 2. The existence of a copy of Apollonius' *Conics* in Ibn al-Haytham's hand (MS Aya Sofya 2762, 307 fols., dated Safar 415 A.H. [1024]) may be taken to confirm the story that he lived on selling copies of scientific texts, although the *Conics* is not one of the books mentioned in the story.
3. The expression "fī ḥudūd" could also mean "about" or "toward the end of."
4. On Bayhaqī's dates see the article devoted to him in *Encyclopaedia of Islam*, 2nd ed.
5. On Qayṣar see A. I. Sabra, "Simplicius's Proof of Euclid's Parallels Postulate," in *Journal of the Warburg and Courtauld Institutes*, 32 (1969), 8.
6. The title is "[Ibn al-Haytham's] Answer to a Geometrical Question Addressed to Him in Baghdad [*su'ila 'anhā bi-Baghdād*] in the Months of the Year Four Hundred and Eighteen."
7. See Galen's *Opera omnia*, C. G. Kühn, ed., XIX (repr. Hildesheim, 1965), 8–61; and F. Rosenthal, "Die arabische Autobiographie," pp. 7–8. Galen's *De libris propriis* was translated into Arabic by Ḥunayn ibn Isḥāq in the ninth century.
8. See Galen's *Opera omnia, ed. cit.*, X (repr. Hildesheim, 1965), 457, ll. 11–15.
9. In the context of Arabic optics *manāẓir* is the plural, not of *manẓar* (view, appearance) but of *manẓara*, that by means of which vision is effected, an instrument of vision. One evidence for this is Ḥunayn ibn Isḥāq's Arabic translation of Galen's *De usu partium*, where *manẓara* and *manāẓir* correspond to ὄψις and ὄψεις, respectively (see Escorial MS 850, fol. 29v). *Al-Manāẓir* had been used as the Arabic title of Euclid's (and Ptolemy's) Ὀπτικά.
10. Newton, *Opticks*, bk. I, pt. 1, def. 1. See A. I. Sabra, *Theories of Light*, pp. 288–289, 310–311, n. 25.
11. The diagram of the eye in Risner's ed. of the Latin text is taken from Vesalius' *De corporis humani fabrica* (D. Lindberg, "Alhazen's Theory of Vision . . . ," p. 327, n. 30). The diagram in MS Fatih 3212 of *Kitāb al-Manāẓir*, bk. I, does not clearly and correctly represent Ibn al-Haytham's descriptions; it can be seen in S. Polyak, *The Retina* (Chicago, 1957), and in G. Nebbia, "Ibn al-Haytham nel millesimo anniversario della nascita," p. 204.
12. MS Fatih 3212, fol. 83r; *Opticae thesaurus. Alhazeni libri VII*, p. 7, sec. 14, l. 26.

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13. The importance of this principle and of its application by Ibn al-Haytham to the problem of vision has been rightly emphasized by Vasco Ronchi in his *Storia della luce*, 2nd ed. (Bologna, 1952), pp. 33–47, trans. into English as *The Nature of Light* (London, 1970), pp. 40–57.
14. *Opticae thesaurus. Alhazeni libri VII*, p. 14, sec. 23, l. 20.
15. *Ibid.*, p. 15, sec. 24: "Visio videtur fieri per συναύγειαν, id est receptos simul et emissos radios."
16. At least some of the Latin MSS have "reflexe." Risner's text, however, correctly reads "refracte" (*bi 'l-in'iṭāf*). See Vescovini, *Studi*, p. 93, n. 10.
17. Vescovini, *Studi*, p. 141.
18. See M. Nazīf, *Al-Ḥasan ibn al-Haytham*, pp. 709–721. Unlike the Arabic, the Latin text in Risner's ed. expresses rule (5) as $d < i$ (*Alhazeni libri VII*, p. 247, ll. 8–11). Both the Arabic MSS and Risner's ed. omit the words "less than" from rule (6), and consequently express this rule as $d = 1/2\ (i + d)$[!]. The correction has been made by Nazīf and is supported by Kamāl al-Dīn's formulation of the rule on the basis of Ibn al-Haytham's autograph (see *Tanqīḥ*, I, 7; II, 134, ll. 10–11).
19. The earliest occurrence of "al-bayt al-muẓlim" is in a ninth-century tract on burning mirrors by 'Uṭārid ibn Muḥammad al-Ḥāsib: Istanbul MS Laleli 2759, fols. 1–20. The tract is based on earlier Greek works including at least one by Anthemius of Tralles, and the term may therefore have been derived from them. See M. Schramm, "Ibn al-Haythams Stellung in der Geschichte der Wissenschaften," pp. 15–16.
20. Bk. I, ch. 5, sec. 29, p. 17 in Risner's ed. of the Latin text. The discussion continues in the Arabic for the greater part of two pages to which nothing corresponds in Risner's text.
21. M. Clagett, "A Medieval Latin Translation of a Short Arabic Tract on the Hyperbola," in *Osiris*, 11 (1954), 361.
22. D. Lindberg, *Pecham and the Science of Optics* (Madison, Wis., 1970), p. 29, n. 69. (See bibliography, "Original Works," no. III 3.)
23. *Ibid.*, p. 20.
24. Vescovini, *Studi*, pp. 137 ff.
25. See especially Vescovini, "Contributo per la storia della fortuna di Alhazen in Italia" (in the Bibliography).
26. W. Hartner, "The Mercury Horoscope . . . ," esp. pp. 122–124.
27. M. Steinschneider, "Notice . . . ," p. 723.
28. Hartner, *op. cit.*, pp. 124, 127 ff.
29. MS India Office, Loth 734, fol. 101r.
30. *Dubitationes* (III 64), *ed. cit.*, p. 19.
31. W. Hartner, "Naṣīr al-Dīn al-Ṭūsī's Lunar Theory," in *Physis*, 11 (1969), 287–304.
32. *Dubitationes* (III 64), *ed. cit.*, p. 39; also p. 33. The English trans. is of Isḥāq's Arabic version quoted by Ibn al-Haytham. The Greek differs only slightly from the Arabic: ". . . οὔτε τὰ ἀναποδείκτως ὑποτιθέμενα, ἐὰν ἅπαξ σύμφωνα τοῖς φαινομένοις καταλαμβάνηται, χωρὶς ὁδοῦ τινος καὶ ἐπιστάσεως εὑρῆσθαι δύναται, κἂν δυσέκθετος ᾖ ὁ τρόπος αὐτῶν τῆς καταλήψεως (Ptolemy, *Syntaxis mathematica*, J. L. Heiberg, ed., II [Leipzig, 1903], 212, ll. 11–14).
33. See E. S. Kennedy, "Late Medieval Planetary Theory," in *Isis*, **57** (1966), 365–378, esp. 366–368.
34. I have consulted the British Museum MSS Add. 23, 394; Add. 23, 397; and Add. 7472 Rich, the last two being al-Nīsābūrī's commentary, *Tawḍīḥ*, on the *Tadhkira*.
35. The reference is very probably to the lost tract on the movement of *iltifāf* (III 61).
36. A. Lejeune, *Recherches sur la catoptrique grecque* (Brussels, 1957), pp. 71–74.
37. Diagrams are supplied in MS Köprülü 952 (see bibliography, "Original Works," under III 3). As far as I know, this MS has not been used in studies of the *Optics*.
38. *Opticae thesaurus. Alhazeni libri VII*, pp. 142–150.
39. See al-Ṭūsī, *Rasā'il*, II (cited in note 2 above), *risāla* no. 8, pp. 5–7. See also A. I. Sabra, "Thābit ibn Qurra on Euclid's Parallels Postulate," in *Journal of the Warburg and Courtauld Institutes*, **31** (1968), 12–32; and A. P. Juschkewitsch [Youschkevitch], *Geschichte der Mathematik im Mittelalter* (Leipzig, 1964), pp. 277–288.
40. Sir Thomas Heath, *A History of Greek Mathematics*, I (Oxford, 1921), pp. 183 ff.
41. Chester Beatty MS 3652, fol. 71r–v.

BIBLIOGRAPHY

Ibn Abī Uṣaybi'a's lists I*a*, I*b*, II, and III of Ibn al-Haytham's works, described in the article, have been published, wholly or in part, more than once in European languages. They have recently been reproduced in a convenient form in Italian trans. by G. Nebbia in "Ibn al-Haytham nel millesimo anniversario della nascita," in *Physis*, 9 (1967), 165–214. Since practically all of Ibn al-Haytham's extant works are included in list III, they will be arranged here according to their numbers in that list. The same numbers can be used to refer to Nebbia's article, where the reader will find a useful bibliography, and to M. Schramm's book, *Ibn al-Haythams Weg zur Physik* (Wiesbaden, 1963), where many of the works listed are discussed. (The titles constituting what Nebbia calls list I*c* are in fact chapter headings of the last work in list I*b*.)

Arabic MSS of Ibn al-Haytham's works are listed in H. Suter, "Die Mathematiker und Astronomen der Araber und ihre Werke," in *Abhandlungen zur Geschichte der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*, **10** (Leipzig, 1900), no. 204, 91–95; and "Nachträge und Berichtigungen zur 'Mathematiker . . . ,'" *ibid.*, **14** (1902), esp. 169–170; H. P. J. Renaud, "Additions et corrections à Suter 'Die Mathem. u. Astr. der Arab.,'" *Isis*, **18** (1932), esp. 204; C. Brockelmann, *Geschichte der arabischen Literatur*, I (Weimar, 1898), 469–470; 2nd ed. (Leiden, 1943), pp. 617–619; supp. I (Leiden, 1937), 851–854. The Istanbul MSS are more fully described in M. Krause, "Stambuler Handschriften islamischer Mathematiker," in *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abt. B, Studien, 3 (1936), 437–532. P. Sbath, in *Al-Fihris: Catalogue de manuscrits arabes*, 3 pts. plus *Supplément* (Cairo, 1938–1940), pt. I, p. 86, cites MSS, belonging to a private collection in Aleppo, of the following works: III 6, III 8, III 48, III 60, III 65, III 67, III 68, and III 82. (I owe this reference to Robert E. Hall.)

In the list of extant original works that follows, reference will be made to Brockelmann and Krause by means of the abbreviations "Br." and "Kr.," followed by the numbers given to Ibn al-Haytham's treatises in these two authors.

Other abbreviations used are the following:

Rasā'il: *Majmū' al-Rasā'il* (Hyderabad, 1357 A.H. [1938]). A collection of eight treatises by Ibn al-Haytham to which a ninth, published at Hyderabad, 1366 A.H. (1947), has been added.

Tanqīḥ: *Tanqīḥ al-Manāẓir* . . ., 2 vols. (Hyderabad, 1347–1348 A.H. [1928–1930]). This is Kamāl al-Dīn al-Fārisī's "commentary" ("*Tanqīḥ*" means revision or correction) on Ibn al-Haytham's *Kitāb al-Manāẓir*. Vol. II has a sequel (*dhayl*) and an appendix (*mulḥaq*) which contain Kamāl al-Dīn's recensions (sing., *taḥrīr*) of a number of Ibn al-Haytham's other optical works.

I. ORIGINAL WORKS.

III 1 (Br. 28). *M(aqāla). f(ī). Hay'at al-'ālam* ("On the Configuration of the World"). A MS that has recently come to light is Kastamonu 2298, 43 fols.; unlike the India Office MS it is incomplete. For Hebrew and Latin MSS see M. Steinschneider, "Notice sur un ouvrage astronomique inédit d'Ibn Haitham," in *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche*, 14 (1881), 721-736, also published as *Extrait . . .* (Rome, 1883); "Supplément," *ibid.*, 16 (1883), 505-513; and *Die hebraeischen Uebersetzungen des Mittelalters und die Juden als Dolmetscher* (Berlin, 1893), II, 559-561; F. Carmody, *Arabic Astronomical and Astrological Sciences in Latin Translation* (Berkeley, Cal., 1955), pp. 141-142; and Lynn Thorndike and Pearl Kibre, *Catalogue of Incipits of Mediaeval Scientific Writings in Latin* (Cambridge, Mass., 1963), cols. 894, 895, 1147 (the last being a Spanish trans. by Abraham Hebraeus).

The Arabic text has not been edited. A Latin version has been published from a MS of the 13th or early 14th century in Millás Vallicrosa, *Las traducciones orientales en los manuscritos de la Biblioteca Catedral de Toledo* (Madrid, 1942), app. II, 285-312; see pp. 206-208. There is a German trans. by K. Kohl, "Über den Aufbau der Welt nach Ibn al Haiṯam," in *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, **54-55** (1922-1923), 140-179.

III 2 (Br. 8, Kr. 14). *M. f. Sharḥ muṣādarāt Kitāb Uqlīdis* ("Commentary on the Premises of Euclid's *Elements*"). Composed before III 53 and before the larger commentary on the *Elements* (see below, "Additional Works," no. 1). MSS of Ibn Tibbon's Hebrew trans. are listed in M. Steinschneider, *Hebraeischen Uebersetzungen* (cited under III 1), II, 509-510. A partial Russian trans. of this work (using a Kazan MS not recorded in Brockelmann) has been published by B. A. Rozenfeld as "Kniga kommentariev k vvedeniyam knigi Evklida 'Nachala.'" in *Istoriko-matematicheskie issledovaniya*, **11** (1958), 743-762.

III 3 (Br. 34, Kr. 15). *Kitāb al-Manāẓir* ("Optics"). All known Arabic MSS of this work are in Istanbul; see Krause. Köprülü MS 952 contains practically the whole of bks. IV, V, VI, and VII. The folios must be rearranged as follows: IV, 108r-133v; V, 2r-v, 74r-81v, 89r-107v, 134r-135v; VI, 3r-47v; VII, 1r-v, 48r-73v, 82r-88v. The reference in Brockelmann to a recension of this work in the Paris MS. ar. 2460 (Br. has 2640) is mistaken: the MS is a recension of Euclid's *Optics* which is attributed on the title page to Ḥasan ibn [Mūsā ibn] Shākir.

I have examined the following Latin MSS: Bruges 512, 113 fols., 13th c.; Cambridge University Library, Peterhouse MS 209 (= 11 · 10 · 63), 111 fols., 14th c.; Cambridge University Library, Trinity College MS 1311 (= 0 · 5 · 30), 165 fols., 13th c.; Edinburgh, Royal Observatory, Crawford Library MS 9 · 11 · 3 (20), 189 fols., dated 1269; Florence, Biblioteca Nazionale, Magliabechi CI.XX.52, 136 fols., incomplete, 15th c.; London, British Museum, Royal 12 G VII, 102 fols., 14th c.; British Museum, Sloane 306, 177 fols., 14th c.; Oxford, Corpus Christi 150, 114 fols., 13th c.; Vienna, Nationalbibliothek 2438, a fragment only from beginning of bk. I, ch. 1 (ch. 4 in Arabic text), fols. 144r-147r, 15th c. Other Latin MSS have been reported in F. Carmody, *Arabic Astronomical and Astrological Sciences*, cited under III 1, p. 140; L. Thorndike and P. Kibre, *Catalogue*, cited under III 1, cols. 774, 803, 1208; and G. F. Vescovini, *Studi sulla prospettiva medievale* (Turin, 1965), pp. 93-94, n. 10.

The only known copy of the fourteenth-century Italian trans. of the *Optics* is MS Vat. Lat. 4595, 182 fols. Like the Latin text it lacks chs. 1-3 of bk. I. It includes an Italian trans. of the *Liber de crepusculis* (see below), fols. 178r-182v.

The Latin text was published in the collective volume bearing the following title: *Opticae thesaurus. Alhazeni Arabis libri septem, nunc primum editi, eiusdem liber de crepusculis et nubium ascensionibus, item Vitellionis Thuringo-Poloni Libri X, omnes instaurati, figuris illustrati et aucti, adjectis etiam in Alhazenum commentariis a Federico Risnero* (Basel, 1572). Concerning the authorship of *De crepusculis*, see below, "Spurious Works."

Kamāl al-Dīn's commentary, the *Tanqīḥ* (cited above), does not reproduce the integral text of the *Optics*, as was at one time supposed. An ed. of the Arabic text of *Kitāb al-Manāẓir* and English trans. are being prepared by the present writer.

III 4 (Br. 51). *M. f. Kayfiyyat al-arṣād* ("On the Method of [Astronomical] Observations").

III 6 (Br. 27). *M. f. Ḍaw' al-qamar* ("On the Light of the Moon"). Composed before 7 Aug. 1031, the date on which a copy was completed by 'Alī ibn Riḍwān (Ibn al-Qifṭī, *Ta'rīkh*, p. 444). Published as no. 8 in *Rasā'il*. There is a German trans. by Karl Kohl, "Über das Licht des Mondes. Eine Untersuchung von Ibn al-Haitham," in *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, **56-57** (1924-1925), 305-398.

III 7 (Br. 22, Kr. 18). *M.* (or *Qawl*) *f. Samt al-qibla bi 'l-ḥisāb* ("Determination of the Direction of the Qibla by Calculation"). A German trans. is C. Schoy, "Abhandlung des Ḥasan ibn al-Ḥasan ibn al-Haiṯam (Alhazen) über die Bestimmung der Richtung der Qibla," in *Zeitschrift der Deutschen morgenländischen Gesellschaft*, **75** (1921), 242-253.

III 8 (Br. 41, Kr. 19). *M. f. al-Hāla wa-qaws quzaḥ* ("On the Halo and the Rainbow"). Completed in Rajab, 419 A.H. (A.D. 1028); see *Tanqīḥ*, II, p. 279. Recension by Kamāl al-Dīn in *Tanqīḥ*, II, 258-279. A shortened German trans. of this recension is E. Wiedemann, "Theorie des Regenbogens von Ibn al Haiṯam," in *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, **46** (1914), 39-56.

III 9 (Br. 42, Kr. 20). *M. f. Mā ya'riḍ min al-ikhtilāf fī irtifā'āt al-kawākib* ("On What Appears of the Differences in the Heights of the Stars").

III 10 = Ia10 (Br. 39, Kr. 16). *M. f. Ḥisāb al-mu'āmalāt* ("On Business Arithmetic").

III 11 (Br. 43, Kr. 21). *M. f. al-Rukhāma al-ufuqiyya* ("On the Horizontal Sundial"). This work refers to a treatise to be written later on "shadow instruments" (*ālāt al-aẓlāl*); the reference may be to III 66.

III 14. *M. f. Marākiz al-athqāl* ("On Centers of Gravity"). This is not extant but has been abstracted by al-Khāzinī in *Mīzān al-ḥikma*; see the Hyderabad ed. (1359 A.H. [1940]), pp. 16-20.

III 15 (Br. 13 a, Kr. 22). *M. f. Uṣūl al-misāḥa* ("On the Principles of Measurement"). A summary of the results of an earlier work or works by Ibn al-Haytham on the subject. Published as no. 7 in *Rasā'il*. German trans. by E. Wiedemann in "Kleinere Arbeiten von Ibn al Haiṯam," in *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, **41** (1909), 16-24.

III 16 (Br. 2, Kr. 23). *M. f. Misāḥat al-kura* ("On the Measurement of the Sphere"). Later in composition than III 17; it may be one of the works referred to in III 15.

III 17 (Br. 14). *M. f. Misāḥat al-mujassam al-mukāfī* ("On the Measurement of the Paraboloidal Solid"). See III 16. Refers to a work on the same subject by Thābit ibn Qurra and another by Wayjan ibn Rustam al-Qūhī. German trans. by H. Suter, "Die Abhandlung über die Ausmessung des Paraboloides von el-Ḥasan b. el-Ḥasan b. el-Haitham," in *Bibliotheca mathematica*, 3rd ser., **12** (1912), 289-332. See also H. Suter, "Die Abhandlungen Thâbit b. Ḳurras und Abû Sahl al-Ḳûhîs über die Ausmessung der Paraboloide," in *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, **48-49** (1916-1917), 186-227.

III 18 (Br. 33, Kr. 10). *M. f. al-Marāya al-muḥriqa bi 'l-dawā'ir* ("On Spherical Burning Mirrors"). Published as no. 4 in *Rasā'il*. German trans. by E. Wiedemann, "Ibn al Haiṯams Schrift über die sphärischen Hohlspiegel," in *Bibliotheca mathematica*, 3rd ser., **10** (1909-1910), 293-307. See also E. Wiedemann, "Zur Geschichte der Brennspiegel," in *Annalen der Physik und Chemie*, n.s. 39 (1890), 110-130, trans. into English by H. J. J. Winter and W. 'Arafāt, "A Discourse on the Concave Spherical Mirror by Ibn al-Haitham," in *Journal of the Royal Asiatic Society of Bengal*, 3rd ser., Science, **16** (1950), 1-16.

III 19 (Br. 33). *M. f. al-Marāya al-muḥriqa bi 'l-quṭū'* ("On Paraboloidal Burning Mirrors"). A hitherto unrecorded MS of the Arabic text is Florence, Biblioteca Medicea-Laurenziana, Or. 152, fols. 90v-97v. It was copied in the 13th century and bears no title or author's name. Here Ibn al-Haytham mentions an earlier treatise of his on how to construct all conic sections by mechanical means (*istikhrāj jamī' al-quṭū' bi-ṭarīq al-āla*); see below, "Additional Works," no. 2. The Arabic text has been published as no. 3 in *Rasā'il*. A medieval Latin trans. as *Liber de speculis comburentibus*, probably made by Gerard of Cremona, has been published together with a German trans. from the Arabic by J. L. Heiberg and E. Wiedemann: "Ibn al Haiṯams Schrift über parabolische Hohlspiegel," in *Bibliotheca mathematica*, 3rd ser., **10** (1909-1910), 201-237. See also E. Wiedemann, "Über geometrische Instrumente bei den muslimischen Völkern," in *Zeitschrift für Vermessungswesen*, nos. 22-23 (1910), 1-8; and "Geschichte der Brennspiegel," cited under III 18. An English trans. is H. J. J. Winter and W. 'Arafāt, "Ibn al-Haitham on the Paraboloidal Focusing Mirror," in *Journal of the Royal Asiatic Society of Bengal*, 3rd ser., Science, **15** (1949), 25-40.

III 20. *Maqāla mukhtaṣara fi 'l-Ashkāl al-hilāliyya* ("A Short Treatise on Crescent-Shaped Figures"). Not extant; see III 21.

III 21 (Br. 1, Kr. 12). *Maqāla mustaqṣāt fi 'l-Ashkāl al-hilāliyya* ("A Longer Treatise on Crescent-Shaped Figures"). Composed after III 20, which it is intended to supersede, and before III 30 (*q.v.*). It may also have been written before the work listed below as "Additional," no. 1 (*q.v.*).

III 22 ([?] Br. 6). *Maqāla mukhtaṣara fī Birkār al-dawā'ir al-'iẓām* ("A Short Treatise on the Birkār of Great Circles"). See Wiedemann, "Geometrische Instrumente . . .," cited under III 19. See III 23. "*Birkār*" is Persian for compass. Ibn al-Haytham explains the theory and construction of an instrument suitable for accurately drawing very large circles.

III 23 ([?] Br. 6). *Maqāla mashrūḥa fī Birkār al-dawā'ir al-'iẓām* ("An Expanded Treatise on the Birkār of Great Circles"). See III 22.

III 25 (Br. 52). *M. f. al-Tanbīh 'alā mawāḍi' al-ghalaṭ fī kayfiyyat al-raṣd* ("On Errors in the Method of [Astronomical] Observations"). Earlier in composition than III 31.

III 26 (Br. 44, Kr. 24). *M. f. anna 'l-Kura awsa' al-ashkāl al-mujassama allatī iḥāṭatuhā muta-sāwiya, wa-anna 'l-dā'ira awsa' al-ashkāl al-musaṭṭaḥa allatī iḥāṭatuhā mutasāwiya* ("That the Sphere Is the Largest of the Solid Figures Having Equal Perimeters, and That the Circle is the Largest of the Plane Figures Having Equal Perimeters"). Composed before III 38 and III 68. Refers to Archimedes' *On the Sphere and the Cylinder*.

III 28 (Br. 29). *Kitāb fī Taṣḥīḥ al-a'māl al-nujūmiyya, maqālatān* ("A Book on the Corrections of Astrological Operations, Two Treatises").

III 30 (Br. 9, Kr. 2). *M. f. Tarbī' al-dā'ira* ("On the Quadrature of the Circle"). Refers to the "book on lunes" (*kitābina fi 'l-hilāliyyāt*), that is either III 20 or III 21. There is an ed. of the Arabic text and German trans. by H. Suter, "Die Kreisquadratur des Ibn el-Haiṯam," in *Zeitschrift für Mathematik und Physik*, Hist.-lit. Abt., **44** (1899), 33-47.

III 31 (Br. 45, Kr. 25). *M. f. Istikhrāj khaṭṭ niṣf al-nahār 'alā ghāyat al-taḥqīq* ("Determination of the Meridian with the Greatest Precision"). Composed after III 25. Points out the relevance of the subject to astrology.

III 36 (Br. 31, Kr. 7). *M. f. Kayfiyyat al-aẓlāl* ("On the Formation of Shadows"). Composed before the "Commentary on the *Almagest*" (see below, "Additional Works," no. 3) and after III 3. A recension by Kamāl al-Dīn al-Fārisī is in *Tanqīḥ*, II, 358-381. A German trans. by E. Wiedemann is "Über eine Schrift von Ibn al Haiṯam: Über die Beschaffenheit der Schatten," in *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, **39** (1907), 226-248.

III 38 (Br. 30, Kr. 26). *M. f. Ḥall shukūk fi 'l-maqāla l-ūlā min Kitāb al-Majisṭī yushakkiku fīhā ba'ḍ ahl al-'ilm* ("Solution of Difficulties in the First Book of the *Almagest* Which a Scholar Has Raised"). This work is to be distinguished from III 64. It was composed after III 26. The name of "the scholar" appears in MS Fatih 3439, fol. 150v to be Abu 'l-Qāsim ibn [?]Ma'dān, who is otherwise unknown to me. The Title in the Fatih MS (*Ḥall Shukūk fī Kitāb al-Majisṭī yushakkiku fīhā ba'ḍ ahl al-'ilm*) does not limit the discussion to book I of the *Almagest*. The text

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in fact discusses, among other things, book V of Ptolemy's *Optics*.

III 39 ([?] Kr. 27). *M. f. Hall shakk fī mujassamāt Kitāb Uqlīdis* ("Solution of a Difficulty in the Part of Euclid's Book Dealing With Solid Figures"). This may be part of the work listed below as "Additional Works," no. 1. But see Krause, no. 27, where reference is made to a work bearing a partially similar title and of uncertain authorship. (I have not examined MS Yeni Cami T 217, 2°, 893 A.H., referred to by Krause.)

III 40 (Br. 3). *Qawl fī Qismat al-miqdārayn al-mukhtalifayn al-madhkūrayn fi 'l-shakl al-awwal min al-maqāla 'l-'āshira min Kitāb Uqlīdis* ("On the Division of the Two Unequal Magnitudes Mentioned in Proposition I of Book X of Euclid's Book"). The subject is closely connected with the so-called "axiom of Archimedes."

III 41 (Br. 23). *Mas'ala fī Ikhtilāf al-naẓar* ("A Question Relating to Parallax"). MS India Office, Loth 734, fols. 120r–120v, specifies that lunar parallax is meant.

III 42 (Br. 17). *Qawl fī Istikhrāj muqaddamat dil' al-musabba'* ("On the Lemma [Used by Archimedes] for [Constructing] the Side of the Heptagon [in the Book at the End of Which He Mentioned the Heptagon]"). Composed before III 74. German trans. by C. Schoy in *Die trigonometrischen Lehren des persischen Astronomen Abu 'l-Raiḥân Muḥ. ibn Aḥmad al Bîrûnî, dargestellt nach al-Qânûn al-Mas'ûdî* (Hannover, 1927), pp. 85-91.

III 43 (Br. 10, Kr. 9). *Qawl fī Qismat al-khaṭṭ alladhī ista'malahu Arshimīdis fī Kitāb al-Kura wa 'l-usṭuwāna* ("On the Division of the Line Used by Archimedes in His Book on the Sphere and Cylinder"). Concerned with prop. 4 of bk. II in Archimedes' work. French trans. by F. Woepcke, *L'algèbre d'Omar Alkhayyâmî* (Paris, 1851), pp. 91-93.

III 44 (Br. 46, Kr. 28). *Qawl fī Istikhrāj khaṭṭ niṣf al-nahār bi-ẓill wāḥid* ("Determination of the Meridian by Means of One Shadow").

III 46 (Br. 26). *M. f. al-Majarra* ("On the Milky Way"). German trans. by E. Wiedemann, "Über die Lage der Milchstrasse nach ibn al Haiṯam," in *Sirius*, **39** (1906), 113-115.

III 48 (Br. 24, Kr. 5). *M. f. Aḍwā' al-kawākib* ("On the Light of the Stars"). Composed before III 49. Published as no. 1 in *Rasā'il*. Abridged German trans. by E. Wiedemann, "Über das Licht der Sterne nach Ibn Al Haitham," in *Wochenschrift für Astronomie, Meteorologie und Geographie*, n.s. **33** (1890), 129-133. English trans. by W. 'Arafat and H. J. J. Winter, "The Light of the Stars—a Short Discourse by Ibn al-Haytham," in *British Journal for the History of Science*, **5** (1971), 282-288.

III 49 (BR. 37). *M. f. al-Athar alladhī [yurā] fī [wajh] al-qamar* ("On the Marks [Seen] on the [Face of the] Moon"). Composed after III 3, III 6, and III 48, to all of which it refers. German trans. by C. Schoy, as *Abhandlung des Schaichs Ibn 'Alî al-Hasan ibn al-Hasan ibn al-Haitham: Über die Natur der Spuren [Flecken], die man auf der Oberfläche des Mondes sieht* (Hannover, 1925).

III 53 (Br. 35). *M. f. al-Taḥlīl wa 'l-tarkīb* ("On Analysis and Synthesis"). Brockelmann lists a Cairo MS. Another is Dublin, Chester Beatty 3652, fols. 69v-86r, dated 612 A.H. (1215). Composed before III 54 (to which it is closely related) and after III 2.

III 54 (Br. 11). *M. f. al-Ma'lūmāt* ("On the Known Things [Data]"). Trans. of the enunciations of its propositions by L. A. Sédillot, in "Du *Traité* des connues géométriques de Hassan ben Haithem," in *Journal asiatique*, **13** (1834), 435-458.

III 55. *Qawl fī Hall Shakk fi 'l-maqāla 'l-thāniya 'ashar min Kitāb Uqlīdis* ("Solution of a Difficulty in Book XII of Euclid's Book"). Possibly a part of the work listed below as "Additional," no. 1.

III 56. *M. f. Hall shukūk al-maqāla 'l-ūlā min Kitāb Uqlīdis* ("Solution of the Difficulties in Book 1 of Euclid's Book"). Possibly part of the work listed below as "Additional Works," no. 1.

III 60 (Br. 32, Kr. 4). *M.* (or *Qawl*) *f. al-Ḍaw'* ("A Discourse on Light"). Composed after III 3. Printed as no. 2 in *Rasā'il*. J. Baarmann published an ed. of the Arabic text together with a German trans. as "Abhandlung über das Licht von Ibn al-Haiṯam," in *Zeitschrift der Deutschen morgenländischen Gesellschaft*, **36** (1882), 195-237. (See remarks on this ed. by E. Wiedemann, *ibid.*, **38** (1884), 145-148.) A Cairo ed. by A. H. Mursī correcting Baarmann's text appeared in 1938. There is now a critical French trans. by R. Rashed: "Le 'Discours de la lumière' d'Ibn al-Haytham," in *Revue d'histoire des sciences et de leurs applications*, **21** (1968), 198-224. A recension is in *Tanqīḥ*, II, 401-407. A German trans. of this recension (*taḥrīr*) is E. Wiedemann, "Ueber 'Die Darlegung der Abhandlung über das Licht' von Ibn al Haiṯam," in *Annalen der Physik und Chemie*, n.s. **20** (1883), 337-345.

III 63 (Br. 19, Kr. 29). *M. f. Hall shukūk ḥarakat al-iltifāf* ("Solution of Difficulties Relating to the Movement of Iltifāf"). A reply to an unnamed scholar who raised objections against an earlier treatise by Ibn al-Haytham (III 61: "On the Movement of Iltifāf") which is now lost. In the reply Ibn al-Haytham revealed an intention he had entertained to write a critique of Ptolemy's *Almagest*, *Planetary Hypotheses* (*Kitāb al-Iqtiṣāṣ*), and *Optics* (MS Pet. Ros. 192, fols. 19v-20r)—almost certainly a reference to III 64.

III 64 (Br. 30). *M. f. al-Shukūk 'alā Baṭlamyūs* ("Dubitationes in Ptolemaeum"). Composed after III 63; see preceding note. There is a critical ed. by A. I. Sabra and N. Shehaby (Cairo, 1971). English trans. of part of this work by A. I. Sabra, "Ibn al-Haytham's Criticism of Ptolemy's *Optics*," in *Journal of the History of Philosophy*, **4** (1966), 145-149.

III 65. *M. f. al-Juz' alladhī la yatajazza'* ("On Atomic Parts"). A unique copy belonging to a private collection in Aleppo is recorded in P. Sbath, *Al-Fihris* (cited above), I, 86, no. 724.

III 66 (Br. 40, Kr. 17). *M. f. Khuṭūṭ al-sā'āt* ("On the Lines of the Hours [i.e., on sundials]"). "*Al-sā'āt*" has sometimes been misread as "*al-shu'ā'āt*" (rays). The treatise refers to a work by Ibrāhīm ibn Sinān, "On Shadow Instruments." See note for III 11 above.

III 67. *M. f. al-Qarasṭūn* ("On the *Qarasṭūn*"). A unique copy belonging to a private collection in Aleppo is recorded in P. Sbath, *Al-Fihris* (cited above), I, p. 86, no. 726.

III 68 (Br. 12, Kr. 11). *M. f. al-Makān* ("On Place"). Later in composition than III 26. Published as no. 5 in *Rasā'il*. A short account is given by Wiedemann in "Kleinere Arbeiten . . .," cited under III 15 above, pp. 1-7.

III 69 (Br. 18). *Qawl fī Istikhrāj a'midat al-jibāl* ("Determination of the Altitudes of Mountains"). A longer title is *Fī Ma'rifat irtifā' al-ashkhāṣ al-qā'ima wa-a'midat al-jibāl wa irtifā' al-ghuyūm* ("Determination of the Height of Erect Objects and of the Altitudes of Mountains and of the Height of Clouds"). A German trans. is H. Suter, "Einige geometrische Aufgaben bei arabischen Mathematiker," in *Bibliotheca mathematica*, 3rd ser., **8** (1907), 27-30. A short account by Wiedemann is in "Kleinere Arbeiten . . .," cited under III 15 above, pp. 27-30.

III 71 (Br. 38). *M. f. A'midat al-muthallathāt* ("On the Altitudes of Triangles"). (An alternate title is *Khawāṣṣ al-muthallath min jihat al-'amūd* ("Properties of the Triangle in Respect of Its Altitude"). Published as no. 9 in *Rasā'il*.

III 73. (Br. 13, Kr. 3). *M. f. Shakl Banū Mūsā* ("On the Proposition of Banū Mūsā [proposed as a lemma for the *Conics* of Apollonius]"). Published as no. 6 in *Rasā'il*. An account of it is in Wiedemann, "Kleinere Arbeiten . . .," cited under III 15 above, pp. 14-16.

III 74 (Br. 48, Kr. 30). *M. f. 'Amal al-musabba' fi 'l-dā'ira* ("On Inscribing a Heptagon in a Circle"). Composed after III 42, to which it refers. As well as referring to Archimedes it mentions al-Qūhī, whose treatise on the subject has been published and trans. by Y. Dold-Samplonius, "Die Konstruktion des regel-mässigen siebenecks nach Abū Sahl al-Qūhī," in *Janus*, **50** (1963), 227-249.

III 75 (Br. 25, Kr. 1). *M. f. Irtifā' al-quṭb 'alā ghāyat al-taḥqīq* ("Determination of the Height of the Pole With the Greatest Precision"). A German trans. is C. Schoy, "Abhandlung des Ḥasan ben al-Ḥasan ben al-Haiṯam über eine Methode, die Polhöhe mit grösster Genanigkeit zu bestimmen," in *De Zee*, **10** (1920), 586-601.

III 76 (Br. 47, Kr. 31). *M. f. 'Amal al-binkām* ("On the Construction of the Water Clock").

III 77 (Br. 33b, Kr. 32). *M. f. al-Kura 'l-muḥriqa* ("On the Burning Sphere"). Written after III 3 and III 66. A recension by Kamāl al-Dīn is in *Tanqīḥ*, II, 285-302. A German trans. of this recension is in E. Wiedemann, "Brechung des Lichtes in Kugeln nach Ibn al Haiṯam und Kamâl al Dîn al Fârisî," in *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, **42** (1910), 15-58, esp. 16-35.

III 78 (Br. 15). *M. f. Mas'ala 'adadiyya mujassama* ("On an Arithmetical Problem in Solid Geometry").

III 79 (Br. 5). *Qawl fī Mas'ala handasiyya* ("On a Geometrical Problem"). A German trans. is in C. Schoy, "Behandlung einiger geometrischen Fragenpunkte durch muslimische Mathematiker," in *Isis*, 8 (1926), 254-263, esp. 254-259.

III 80 (Br. 20, Kr. 8). *M. f. Ṣūrat al-kusūf* ("On the Shape of the Eclipse"). Composed after III 3. A recension by Kamāl al-Dīn is *Tanqīḥ*, II, 381-401. A German trans. of the original text from the India Office MS is in E. Wiedemann, "Über die Camera obscura bei Ibn al Haiṯam," in *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, **46** (1914), 155-169.

III 82 (Br. 21, Kr. 13). *M. f. Ḥarakat al-qamar* ("On the Motion of the Moon"). A vindication of Ptolemy's account of the mean motion of the moon in latitude.

III 83 (Br. 4). *M. f. Masā'il al-talāqī* ("On Problems of Talāqī"). These are problems involving the solution of simultaneous linear equations. There is an account by E. Wiedemann, in "Über eine besondere Art des Gesellschaftsrechnens besondere nach Ibn al Haiṯam," in *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, **58-59** (1926-1927), 191-196.

III 92 (Br. 16). *Qawl fī Istikhrāj mas'ala 'adadiyya* ("Solution of an Arithmetical Problem"). An account is given by Wiedemann in "Kleinere Arbeiten . . .," cited under III 15 above, pp. 11-13.

ADDITIONAL WORKS. These are extant works whose titles do not appear in list III.

Add. 1 (Br. 7, Kr. 6). *Kitāb fī Ḥall shukūk Kitāb Uqlīdis fi 'l-Uṣūl wa-sharḥ ma'ānīh* ("A Book on the Solution of the Difficulties in Euclid's *Elements* and an Explanation of Its Concepts"). This seems to be a different work from Ia 1: *Sharḥ Uṣūl Uqlīdis fi 'l-handasa wa 'l-'adad wa talkhīṣuhu* ("A Commentary on and Summary of Euclid's *Elements of Geometry and Arithmetic*").

The absence of this comprehensive work from list III may perhaps be explained by supposing III 39, III 55, and III 56 to be parts of it. It refers to III 2 and also to "our treatise on crescent-shaped figures," which is either III 20 or III 21. An Istanbul MS that is not recorded in Brockelmann or in Krause is Üniversite 800, copied before 867 A.H. (1462-1463). The MS has 182 fols. but is not complete.

Add. 2. *Kalām fī tawṭi'at muqaddamāt li-'amal al-quṭū' 'alā saṭḥ mā bi-ṭarīq ṣinā'ī* ("A Passage in Which Lemmas Are Laid Down for the Construction of [Conic] Sections by Mechanical Means"). MS Florence, Biblioteca Medicea-Laurenziana, Or. 152, fols. 97v-100r. No author is named, and the "lemmas" follow immediately after a copy of Ibn al-Haytham's III 19 ("On Paraboloidal Burning Mirrors"), which also does not bear the author's name. Since Ibn al-Haytham refers in III 19 to a treatise of his on the mechanical construction of conic sections, it is very likely that the "passage" we have here is a fragment of that treatise which the copyist found joined to III 19.

Add. 3. "Commentary on the *Almagest*," Istanbul MS, Ahmet III 3329, copied in Jumādā II 655 (1257), 123 fols. Probably written after III 36, to which it appears to refer (fol. 90r).

SPURIOUS WORKS. Ibn al-Haytham is not the author of the *Liber de crepusculis*, the work on dawn and twilight translated by Gerard of Cremona and included in Risner's *Opticae thesaurus* (see A. I. Sabra, "The Authorship of the *Liber de crepusculis*," in *Isis*, **58** [1967], 77-85; above, III 3). An astrological work, *De imaginibus celestibus*, Vatican MS Urb. Lat. 1384, fols. 3v-26r, has also been mistakenly ascribed to him (*ibid.*, p. 80, n. 14).

Two more writings are listed in Brockelmann (nos. 49, 50) which may or may not be genuine.

I am grateful to M. Clagett for showing me a microfilm

of MS Bruges 512 (III 3) and to M. Schramm for showing me microfilms of the following MSS: Kastamonu 2298 (III 1), Üniversite 800 (Add. 1), and Ahmet III 3329 (Add. 3). For the last three MSS and for other Arabic MSS not hitherto recorded, see the appropriate volume of F. Sezgin, *Geschichte des arabischen Schrifttums* (Leiden, 1967–).

II. SECONDARY LITERATURE. Sources for the biography of Ibn al-Haytham are Ibn al-Qifṭī, *Ta'rīkh al-ḥukamā'*, J. Lippert, ed. (Leipzig, 1903), pp. 155–168 (see corrections of this ed. by H. Suter in *Bibliotheca mathematica*, 3rd ser., **4** [1903], esp. 295–296); ʿAlī ibn Zayd al-Bayhaqī, *Tatimmat ṣiwān al-ḥikma*, M. Shafīʿ, ed., fasc. I: Arabic text (Lahore, 1935), 77–80 (analysis and partial English trans. of this work by M. Meyerhof in *Osiris*, **8** [1948], 122–216, see esp. 155–156); Ibn Abī Uṣaybiʿa, *Ṭabaqāt al-aṭibbā'*, A. Müller, ed. (Cairo–Königsberg, 1882–1884), II, 90–98 (German trans. in E. Wiedemann, "Ibn al-Haiṯam, ein arabischer Gelehrter," cited below); Ṣāʿid al-Andalusī, *Ṭabaqāt al-umam*, L. Cheikho, ed. (Beirut, 1912), p. 60 (French trans. by R. Blachère [Paris, 1935], p. 116). The account in Abu'l-Faraj ibn al-ʿIbrī, *Ta'rīkh mukhtaṣar al-duwal*, A. Ṣālḥānī, ed. (Beirut, 1958), pp. 182–183, derives from Ibn al-Qifṭī. In addition to the works by Suter (*Mathematiker*) and Brockelmann (*Geschichte*) already cited, see M. Steinschneider, "Vite di matematici arabi, tratte da un' opera inedita di Bernardino Baldi, con note di M.S.," in *Bullettino di bibliografia e di storia delle scienze matematiche et fisiche*, **5** (1872), esp. 461–468, also printed separately (Rome, 1874); M. J. de Goeje, "Notice biographique d'Ibn al-Haitham," in *Archives néerlandaises des sciences exactes et naturelles*, 2nd ser., **6** (1901), 668–670; E. Wiedemann, "Ueber das Leben von Ibn al Haiṯam und al Kindī," in *Jahrbuch für Photographie und Reproduktiontechnik*, **25** (1911), 6–11 (not important).

The literary relationship of Ibn al-Haytham's autobiography to Galen's *De libris propriis* is discussed by F. Rosenthal in "Die arabische Autobiographie," in *Studia arabica I*, Analecta Orientalia, no. 14 (Rome, 1937), 3–40, esp. 7–8. There is a discussion of the autobiography in G. Misch, *Geschichte der Autobiographie*, III, pt. 2 (Frankfurt, 1962), 984–991. Lists Ia and III of the works of Ibn al-Haytham are translated from Ibn Abī Uṣaybiʿa in F. Woepcke, *L'algèbre d'Omar Alkhayyāmī* (Paris, 1851), pp. 73–76; but see H. Suter's corrections in *Mathematiker*, pp. 92–93. There is a German trans. of Ibn al-Haytham's autobiography and of Lists I–III in E. Wiedemann, "Ibn al Haiṯam, ein arabischer Gelehrter," in *Festschrift [für] J. Rosenthal* (Leipzig, 1906), pp. 169–178. M. Schramm discusses the chronological order of some of Ibn al-Haytham's works in *Ibn al-Haythams Weg zur Physik* (Wiesbaden, 1962), pp. 274–285.

The most complete study of Ibn al-Haytham's optical researches is M. Naẓīf, *Al-Ḥasan ibn al-Haytham, buḥūthuhu wa-kushūfuhu al-baṣariyya* ("Ibn al-Haytham, His Optical Researches and Discoveries"), 2 vols. (Cairo, 1942–1943)—reviewed by G. Sarton in *Isis*, **34** (1942–1943), 217–218. Based on the extant MSS of *Kitāb al-Manāẓir* and on Ibn al-Haytham's other optical works, this voluminous study (more than 850 pages) is distinguished by clarity, objectivity, and thoroughness. It is particularly valuable as a study of the mathematical sections of Ibn al-Haytham's works. M. Schramm, *Ibn al-Haythams Weg zur Physik*, is the most substantial single study of Ibn al-Haytham in a European language, and it has the merit of drawing on MS sources not previously available. In analyzing Ibn al-Haytham's attempt to combine Aristotelian natural philosophy with a mathematical and experimental approach, Schramm illuminates other important treatises of Ibn al-Haytham besides the *Optics*.

The question of mathematizing Aristotelian physics is also discussed in S. Pines, "What Was Original in Arabic Science," in A. C. Crombie, ed., *Scientific Change* (London, 1963), pp. 181–205, esp. 200–202. It is taken up afresh by R. Rashed in "Optique géometrique et doctrine optique chez Ibn al Haytham," in *Archive for History of Exact Sciences*, **6** (1970), 271–298. For further discussions of the concept of experiment in Arabic optics generally and in the work of Ibn al-Haytham in particular, see M. Schramm, "Aristotelianism: Basis and Obstacle to Scientific Progress in the Middle Ages," in *History of Science*, **2** (1963), 91–113, esp. 106, 112; and "Steps Towards the Idea of Function: A Comparison Between Eastern and Western Science of the Middle Ages," *ibid.*, **4** (1965), 70–103, esp. 81, 98; A. I. Sabra, "The Astronomical Origin of Ibn al-Haytham's Concept of Experiment," in *Actes du XIIe Congrès international d'histoire des sciences*, Paris, 1968, III A (Paris, 1971), 133–136.

General accounts mainly based on the *Optics* are J. B. J. Delambre, "Sur l'*Optique* de Ptolémée comparée à celle qui porte le nom d'Euclide et à celle d'Alhazen et de Vitellion," in *Historie de l'astronomie ancienne*, II (Paris, 1817), 411–432; E. Wiedemann, "Zu Ibn al Haiṯams Optik," in *Archiv für Geschichte der Naturwissenschaften und der Technik*, 3 (1910–1911), 1–53, an account of Kamāl al-Dīn's revision (*Tanqīḥ*) of Ibn al-Haytham's *Optics*, based on a Leiden MS; includes an abbreviated trans. of *Optics*, bk. I, chs. 1–3, as reported by Kamāl al-Dīn; L. Schnaasse, *Die Optik Alhazens* (Stargard, 1889); V. Ronchi, "Sul contributo di Ibn al-Haitham alle teorie della visione e della luce," in *Actes du VIIe Congrès international d'histoire des sciences* (Jerusalem, 1953), pp. 516–521; and *The Nature of Light*, a trans. of *Storia della luce* (2nd ed., Bologna, 1952) (London, 1970), pp. 40–57; H. J. J. Winter, "The Optical Researches of Ibn al-Haitham," in *Centaurus*, **3** (1953–1954), 190–210, which includes accounts of treatises other than the *Optics*.

Studies of particular aspects of Ibn al-Haytham's optical work are A. Abel, "La sélénographie d'Ibn al Haitham (965–1039) dans ses rapports avec la science grecque," in *Comptes rendus, IIe Congrès national des sciences* (Brussels, 1935), pp. 76–81 (concerned with III 49); J. Lohne, "Zur Geschichte des Brechungsgesetzes," in *Sudhoffs Archiv für Geschichte der Medizin und der Naturwissenschaften*, **47** (1963), 152–172, esp. 153–157; R. Rashed, "Le modèle de la sphère transparente et l'explication de l'arc-en-ciel: Ibn al-Haytham, al-Fārisī," in *Revue d'histoire des sciences et de leurs applications*, **23** (1970), 109–140; E. Wiedemann, "Ueber den Apparat zur Untersuchung und Brechung des

Lichtes von Ibn al Haiṯam," in *Annalen der Physik und Chemie*, n.s. **21** (1884), 541–544; "Über die Erfindung der Camera obscura," in *Verhandlung der Deutschen physikalischen Gesellschaft*, **12** (1910), 177–182; and "Über die erste Erwähnung der Dunkelkammer durch Ibn al Haiṯam," in *Jahrbuch für Photographie und Reproduktiontechnik*, **24** (1910), 12–13; J. Würschmidt, "Zur Theorie der Camera obscura bei Ibn al Haiṯam," in *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, **46** (1914), 151–154; and "Die Theorie des Regenbogens und das Halo bei Ibn al Haiṯam und bei Dietrich von Freiberg," in *Meteorologische Zeitschrift*, **13** (1914), 484–487. Apart from Vescovini's *Studi* (see below) there is one account of Ibn al-Haytham's psychological ideas as expounded in bk. II of the *Optics*: H. Bauer, *Die Psychologie Alhazens auf Grund von Alhazens Optik dargestellt*, in the series Beiträge zur Geschichte der Philosophie des Mittelalters, 10, no. 5 (Münster in Westfalen, 1911). The physiological aspect of vision is discussed in M. Schramm, "Zur Entwicklung der physiologischen Optik in der arabischen Literatur," in *Sudhoffs Archiv für Geschichte der Medizin und der Naturwissenschaften*, **43** (1959), 289–316, esp. 291–299.

The following are concerned with the transmission of Ibn al-Haytham's optical ideas to the West; they include comparisons with Hebrew and Latin medieval, Renaissance, and seventeenth-century writers: M. Steinschneider, "Aven Natan e le teorie sulla origine della luce lunare e delle stelle, presso gli autori ebrei del medio evo," in *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche*, **1** (1868), 33–40; E. Narducci, "Nota intorno ad una traduzione italiana fatta nel secolo decimoquarto, del trattato d'*Ottica* d'Alhazen, matematico del secolo undecimo, e ad altri lavori di questo scienziato," *ibid.*, **4** (1871), 1–48; and "Giunte allo scritto intitolato 'Intorno ad una traduzione italiana, fatta nel secolo decimoquarto, dell' *Ottica* di Alhazen,'" *ibid.*, pp. 137–139; A. I. Sabra, "Explanation of Optical Reflection and Refraction: Ibn al-Haytham, Descartes and Newton," in *Actes du Xe Congrès international d'histoire des sciences*, Ithaca, 1962 (Paris, 1964), I, 551–554; and *Theories of Light From Descartes to Newton* (London, 1967), pp. 72–78, 93–99 (concerned with the theories of reflection and refraction); G. F. Vescovini, *Studi sulla prospettiva medievale* (Turin, 1965), which reveals the influence of Ibn al-Haytham's *Optics* on the development of empiricist theories of cognition in the fourteenth century; and "Contributo per la storia della fortuna di Alhazen in Italia: Il volgarizzamento del MS. Vat. 4595 e il 'Commentario terzo' del Ghiberti," in *Rinascimento*, 2nd ser., **5** (1965), 17–49; D. Lindberg, "Alhazen's Theory of Vision and Its Reception in the West," in *Isis*, **58** (1968), 321–341; and "The Cause of Refraction in Medieval Optics," in *British Journal for the History of Science*, **4** (1968), 23–38. See also G. Sarton, "The Tradition of the *Optics* of Ibn al-Haitham," in *Isis*, **29** (1938), 403–406.

The following are studies relating to Ibn al-Haytham's astronomical works, particularly his treatise on *The Configuration of the World* (III 1): M. Steinschneider, "Notice sur un ouvrage astronomique inédit d'Ibn Haitham," in *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche*, **14** (1881), 721–736; and "Supplément à la 'Notice sur un ouvrage inédit d'Ibn Haitham,'" *ibid.*, **16** (1883), 505–513—*Extrait du Bullettino* . . ., containing the "Notice" and the "Supplément" (Rome, 1884), includes many corrections of the earlier publications; E. Wiedemann, "Ibn al Haiṯam und seine Bedeutung für die Geschichte der Astronomie," in *Deutsche Literaturzeitung*, **44** (1923), 113–118; P. Duhem, *Système du monde*, II (Paris, 1914), 119–129; W. Hartner, "The Mercury Horoscope of Marcantonio Michiel of Venice, a Study in the History of Renaissance Astrology and Astronomy," in A. Beer, ed., *Vistas in Astronomy* (London–New York, 1955), pp. 84–138, esp. 122–127; S. Pines, "Ibn al-Haytham's Critique of Ptolemy," in *Actes du Xe Congrès international d'histoire des sciences*, Ithaca, 1962 (Paris, 1964), I, 547–550 (concerned with Ibn al-Haytham's criticism of the equant in the *Dubitationes in Ptolemaeum*, III 64); M. Schramm, *Ibn al-Haythams Weg zur Physik*, esp. pp. 63–69, 88–146.

For discussions of "Alhazen's problem," see P. Bode, "Die Alhazensche Spiegelaufgabe in ihrer historischen Entwicklung nebst einer analytischen Lösung des verallgemeinerten Problems," in *Jahresbericht des Physikalischen Vereins zu Frankfurt am Main*, for 1891–1892 (1893), pp. 63–107; M. Baker, "Alhazen's Problem. Its Bibliography and an Extension of the Problem," in *American Journal of Mathematics*, **4** (1881), 327–331; M. Naẓīf, *Al-Ḥasan ibn al-Haytham* . . ., pp. 487–589; J. A. Lohne, "Alhazens Spiegelproblem," in *Nordisk matematisk tidskrift*, **18** (1970), 5–35 (with bibliography).

A general survey of Ibn al-Haytham's work in various fields is M. Schramm, "Ibn al-Haythams Stellung in der Geschichte der Wissenschaften," in *Fikrun wa fann*, no. **6** (1965), 2–22. See also M. Naẓīf and P. Ghalioungui, "Ibn al Haitham, an 11th-Century Physicist," in *Actes du Xe Congrès international d'histoire des sciences*, Ithaca, 1962 (Paris, 1964), I, 569–571. No. 2 of the Publications of the Egyptian Society for the History of Science (Cairo, 1958) includes articles in Arabic by M. Naẓīf, M. Madwar, M. ʿAbd al-Rāziq, M. Ghālī, and M. Ḥijāb on various aspects of Ibn al-Haytham's thought. Some of these articles are reprints of previous publications. For a detailed table of contents see *Isis*, **51** (1960), 416.

Many of the European translations of Ibn al-Haytham's works, cited in the first part of the bibliography, include historical and critical notes.

III

Explanation of Optical Reflection and Refraction: Ibn-al-Haytham, Descartes, Newton

The theory of optical reflection and refraction, developed in the 11th century by Ibn-al-Haytham (Alhazen), had a long period of popularity in Europe after his *Optics* was translated into Latin, probably at the beginning of the 13th century. This theory, which reached its culmination in the 17th century, is both historically important and philosophically instructive. I will, however, concentrate in this brief outline on three characters. These are: Ibn-al-Haytham, the founder of the theory; Descartes, who developed it and extended its application; and Newton, whose work represents a terminal point as well as a decisive transformation. One aim of this account is to bring out the significance of Ibn-al-Haytham's investigations in understanding Descartes' deduction of the sine law within a mechanical theory of light[1].

Although, for Ibn-al-Haytham, a complete theory of light would consist of a physical part (answering questions as to the *nature* of light rays, of transparency, of opacity, etc.) and a mathematical part (answering questions as to *how* light is propagated, reflected, refracted, etc.[2]), he has chosen to speak of light in his writings almost exclusively as a mathematician. Some general points clearly emerge from his texts: he denies that light is a body, but states that it requires a body (a medium) for its transmission; the "movement of light", directed from the object to the eye, is not of infinite speed, it is "extremely quick", "too quick to be perceived by sense", and it is "easier" and "quicker" in rarer media.

To explain reflection, he appeals to mechanical analogies. A small iron sphere is observed to be reflected more strongly by harder bodies, the reason being that hard bodies possess a "force of repulsion" *(quwwat al-mudâfa'a* or *al-mumâna'a : eiectio, repulsio)* which increases with hardness. To account for the rebounding of the sphere in the case of a perpendicular fall on to an iron sheet, he makes three points: (1) the incident movement is "abolished" at impact; (2) a new movement is "acquired from the force of repulsion itself"; (3) the reflected movement is also said to depend on the incident movement.

A problem, not viewed by Ibn-al-Haytham, is how to account for (3) in the presence of (1). If elasticity were introduced, it would be possible to answer by saying that the incident movement is, in fact, imparted to the reflecting body which, being elastic, would re-transfer it to the sphere, thus causing it to return in accordance with (2). We may say that Ibn-al-Haytham was moving in this direction, but he did not expressly mention elasticity.

Considering the case of oblique incidence, he resolves the impact force *(i'timâd)* of the sphere into two "parts" or components, one tangential to the reflecting surface at the point of impact, and the other perpendicular to that tangent. The tangential component remains unaffected by impact, while the perpendicular component is reversed in accordance with the preceding considerations. The equality of the angles of incidence and of reflection follows; and the same arguments are applied to optical reflection where "smoothness" replaces "hardness" in the mechanical picture.

In the analogy for refraction, Ibn al-Haytham imagines the sphere to strike a thin slate which covers a wide hole in the iron sheet. If the sphere is thrown on to the slate perpendicularly, it may break the slate and pass through, while it would only slide on the slate if thrown at it obliquely with equal force and from an equal distance. This means that "the movement along the perpendicular is easier and stronger, and that of [all] the inclined movements those nearer the perpendicular are easier than those that are farther from it." He uses this conclusion to explain refraction from a rare into a dense medium. Because of the greater resistance offered by the denser body, the light cannot continue in the direction of incidence; it therefore takes a course in which the movement will be easier, that is, it is deflected towards the normal.

Ibn-al-Haytham seems to abandon this idea of *easiest course* when he turns to refraction from a dense into a rare medium. Here he appeals to the parallelogram method which he applied to reflection. While he says nothing about what happens in this case to the perpendicular velocity, he asserts that the tangential velocity is increased. He gives no reason for this assertion, but it seems to have been suggested by the assumption that the total speed has become greater in the rare medium, together with the fact that now the refraction is *away from* the normal.

Almost every writer on optics from the 13th up to the 17th century tried to apply these ideas of Ibn-al-Haytham's. But Descartes, to whom Ibn-al-Haytham's *Optics* was available in Risner's edition of 1572, was the first to make full use of them. He adopted both Ibn-al-Haytham's model and mathematical method. A minor change was Descartes' replacement of the iron sphere by a tennis ball, and of the thin slate by a frail cloth. Nevertheless, Descartes' explanation of reflection reveals an essentially different approach. Contrary to the usual accounts of his treatment, Descartes does *not* compose the reflected movement of the reversed perpendicular component and the unaffected tangential component. Rather, he assumes the constancy of the latter and of the actual speed. Having eliminated elasticity, the impinging ball is not supposed to act on the hard surface, nor does the surface re-act upon it. This amounts to a purely kinematical explanation which contrasts with Ibn-al-Haytham's dynamical approach, but which is in perfect agreement with Descartes's general view of physics. Within this framework, Descartes' treatment of reflection gives rise to certain unanswerable objections which his contemporary Fermat promptly pointed out [3].

Descartes' explanation of refraction was successful in so far as he formulated two assumptions implying the sine law which he was the first to publish (*Dioptrique*, 1637) as a consequence of theoretical considerations. One assumption stated the idea, already expressed by Ibn-al-Haytham, that the velocity of light is a property of the medium it is traversing. The other asserted the unchangeability, in refraction, of the velocity component parallel to the interface. This assumption, in fact, follows from Ibn-al-Haytham's model, since it is a collision model in which refraction is understood as a

surface action that can only affect the perpendicular component. It could not have been accepted by Ibn-al-Haytham, however, because it yields the result, which he rejected *a priori*, that the velocity is greater in the denser medium. In this sense it may be said that Ibn-al-Haytham's correct belief, regarding the relative velocity of light in different media, stood in the way of his arriving at the refraction law by his proposed method. On the other hand, this method was well adapted to Descartes' opposite belief which he had held since 1619-1621.

Newton's theory of refraction provided a convincing dynamical interpretation of the two assumptions which Descartes had failed to explain satisfactorily. Refraction, being a change of direction, indicates the existence of a force which can only act perpendicularly since a vertical ray passes through the refracting surface without being deflected. Thus, the tangential velocity remains constant, and this implies, as in Descartes, that the velocity increases or decreases according as the refraction is towards or away from the normal. This explanation, given by Newton in the *Principia* as a proposition belonging to particle dynamics [4], in fact committed him to a corpuscular view of light. At the same time, his analysis of partial reflection led him to abandon the simple picture of mechanical collision employed by Ibn-al-Haytham and Descartes [5]. It is interesting to note that the same problem was pointed out in the 14th century by Ibn-al-Haytham's commentator, Kamâl-al-Dîn al-Fârisî. Whereas, however, Kamâl-al-Dîn rejected Ibn-al-Haytham's "force of repulsion" as an explanation of reflection and would have sought the solution in a wave interpretation of light, Newton postulated that the force acting at the surface of a transparent body will repel the ray or attract it, thus causing it to be reflected or refracted, depending on whether the ray is in a "fit" of easy reflection or of easy transmission. That is to say, the force of repulsion remains, but it can also act as a force of attraction.

Ibn-al-Haytham's conception of light, as something transmitted successively in a transparent body, would seem to be a wave-like conception. This conception itself he did not mathematize; rather, he superimposed a model to which the parallelogram method could be directly applied. Descartes' conception was somewhat similar to that of Ibn-al-Haytham. Descartes failed, however, in his attempt to remain within the domain of geometry by kinematizing Ibn-al-Haytham's model. In Newton's theory the model coincides with reality; the sphere becomes the light corpuscle. The original continuist conception finally disappears, and there only remains the dynamical picture for which the parallelogram method was originally introduced.

References and Notes

[1] There is no edition of the Arabic text of Ibn-al-Haytham's *Optics (Al-Manâzir)*, manuscripts of which exist in Istanbul. A commentary on Ibn-al-Haytham's work by Kamâl-al-Dîn al-Fârisî (died c. 1320) was published in Hyderabad (*Tanqih al-manâzir*, etc., 1928-30, 2 vol.) Ibn-al-Haytham's text is reproduced by Kamâl-al-Dîn but without always being quoted verbatim. A modern analytical study of Ibn-al-Haytham's *Optics*, based on the Istanbul MSS and Kamâl-al-Dîn's commentary, was published in Cairo by Professor Mustafâ Nazîf (*Al-Hasan Ibn-al-Haytham. His researches and discoveries in optics*, Cairo, 1942-3, 2 vol., in Arabic)

I wish to express my indebtedness to this study, by far the most complete in any language.

[2] Cf. "Abhandlung über das Licht von Ibn al-Haitam" (Arabic text edited with German translation by J. Baarmann), *Zeitschrift der Deutschen Morgenländischen Gesellschaft* 36 (1882), p. 197.

[3] See my forthcoming *Theories of Light from Descartes to Newton* (The Oldbourne Library of the History of Science), London, 1964.

[4] Newton, *Principia*, Bk. I, Sec. XIV, Prop. XCIV.

[5] Newton, *Opticks*, Bk. II, Pt. III, Prop. VIII.

IV

Ibn Al-Haytham's Criticisms of Ptolemy's *Optics*

I

Ptolemy's *Optics* has survived in a Latin translation made in the twelfth century from an Arabic version of the Greek text.[1] Both the Greek original and the Arabic translation have been lost. Of the five parts (*maqālāt: sermones*) which the Greek text originally comprised, the extant Latin version has preserved only parts II–IV and a fragment from the beginning of part V. There is no evidence that the first part ever existed in Arabic, and it may have been missing in the Greek text which formed the basis of the Arabic translation. In any case we know that this part was no longer available to Arabic scholars in the eleventh century. This we infer from the title of a work which the mathematician, astronomer, and physicist, Ibn al-Haytham, better known in the west as Alhazen (died *ca.* 1039), wrote before 417 H. (A.D. 1026). The fifth item in an autobibliography including mathematical works which Ibn al-Haytham had composed up to that date, reads as follows: "A book in which I summarized the science of optics (*ʿilm al-manāẓir*) from the two books of Euclid and Ptolemy, to which I added (*wa-tammamtuhu*) the matters of the first part (*maqāla*) that is missing from Ptolemy's book." [2] Unfortunately, this summary has not survived; nor has another work of Ibn al-Haytham's, entitled "A treatise on optics according to Ptolemy's method." [3]

We do possess, however, a piece of writing by Ibn al-Haytham which is directly concerned with Ptolemy's optical work. This is a brief discussion of Ptolemy's *Optics* which Ibn al-Haytham wrote some time after the composition of his great work, *al-Manāẓir*.[4] This discussion has not been edited or translated, and it is my aim here to present an English translation of it, based on the edition now being prepared by Mr. Nabīl Shihābī of Alexandria and myself.

The discussion is contained in a work whose title suggests the critical vein in which it was written: *al-Shukūk ʿalā Baṭlamyūs* (Doubts about Ptolemy).[5] This

[1] The Latin translation of Ptolemy's *Optics* was first published by Gilberto Govi, *L'Ottica di Claudio Tolomeo* . . . (Turin: 1885); then by Albert Lejeune, *L'Optique de Claude Ptolémée, dans la version latine d'après l'arabe de l'émir Eugène de Sicile. Texte critique et exégétique* (Louvain: 1956). References will be to Lejeune's edition.

[2] Ibn Abī Uṣaybiʿa, *ʿUyūn al-anbā'* . . . , ed. A. Müller, II (Cairo: 1882), 93–94; F. Woepcke, *L'Algèbre d'Omar al-Khayyāmī* (Paris: 1851), pp. 73–74, n ***. A recent attempt to reconstruct the first part of Ptolemy's *Optics* from available material is to be found in A. Lejeune, *Euclide et Ptolémée* (Louvain: 1948), pp. 15 ff. For new light on the transmission of the fifth part, see note 16 below.

[3] Ibn Abī Uṣaybiʿa, *loc. cit.*, p. 98.

[4] The Latin translation of *Kitāb al-manāẓir*, made probably at the beginning of the thirteenth century, was published by F. Risner in *Opticae thesaurus:* Alhazeni *libri septem* . . . (Basle: 1572). For Ibn al-Haytham's theory of vision expounded in *al-Manāẓir*, see E. Wiedemann, "Zur Geschichte der Lehre vom Sehen," *Annalen der Physik und Chemie*, Neue Folge, Band XXXIX (1890), 470–474; H. Bauer, *Die Psychologie Alhazens, auf Grund von Alhazens Optik dargestellt* (Münster: 1911).

[5] Listed in Ibn Abī Uṣaybiʿa, *loc. cit.*, p. 98, and by Ibn al-Qifṭī, *Ta'rīkh* . . . , ed. Müller and Lippert (Leipzig: 1903), p. 168. For a report on the part of this work dealing with Ptolemy's astronomy, see S. Pines, "Ibn al-Haytham's Critique of Ptolemy," *Actes du dixième congrès internationale d'histoire des sciences* (Ithaca: 1962), Paris: 1964, I, 547–550.

work consists of a brief introduction explaining the author's aim (viz., to point out the errors which even as great a mathematician as Ptolemy has committed), followed by a criticism of some of Ptolemy's views expressed in three of his works: the *Almagest*, the *Planetary Hypothesis*, and the *Optics*. It has been preserved in two MSS, one in the Bodleian Library at Oxford (Arch. Seld. A32, fols. 162^{b}–184^{b}), the other at the Municipal Library of Alexandria (no. 2057d, 18 fols.).[6] The latter is undated, but obviously more recent than the Bodleian MS, which was transcribed before 633 H. (A.D. 1235–1236), the date at which it came into the possession of a certain Yaḥyā ibn Muḥammad ibn al-Labūdī.[7] Neither MS has the complete text. The Bodleian MS breaks off (fol. 182^{b}) in the middle of the discussion about the *Planetary Hypothesis* and re-joins the text (fol. 183^{a}) shortly after the beginning of the discourse about the *Optics*. On the other hand, a long passage of the Bodleian MS (viz., fols. $174^{a}5$–$176^{a}2$) is missing from the Alexandria MS. Thus the two MSS complement one another.

In the following translation the letters A and B refer to the Alexandria and Bodleian MSS respectively. Since the interest of the translated passage is partly philological, I have included transliterations of important Arabic words and expressions in brackets. One might, in this way, be able to determine what Arabic words corresponded to their equivalents in the Latin version of Ptolemy's *Optics*. Also included in brackets are expressions implied by the text, though not explicitly stated in it; square brackets include additions by the translator.

II

Translation of the Section Dealing With Ptolemy's *Optics* in Ibn Al-Haytham's Treatise "Doubts About Ptolemy"

[A: fol. 17^{v}] As for his [Ptolemy's] book on *Optics* (*al-Manāẓir*), there are places in it which collapse when the matters discussed in them are examined. At the beginning of the second *maqāla* of his book on *Optics* he enumerates the things (*al-maʿānī*)[7a] which sight (*al-baṣar*) perceives, and says that sight cognizes (*yaʿrifu*) body, magnitude, colour, shape, position, movement and rest.[8] Now these things are seven; but we have shown in the second *maqāla* of our book on *Optics* (*al-Manāẓir*) that the things perceived by sight are twenty-two species, some of which are genera having under them several species.[9] For example, sight perceives (*yudriku*)

[6] The Alexandria MS is absent from all lists of Ibn al-Haytham's works. (The Institute for Arabic Manuscripts at Cairo has a microfilm of it [No. 331]; see Fu'ād Sayyid, *Fihris al-makhṭūṭāt almuṣawwara*, III [Sciences], 3 [Mathematics: Arithmetic-Algebra-Geometry] [Cairo: 1960], pp. 90–91.) C. Brockelmann confuses the *Shukūk ʿalā Baṭlamyūs* with another work, *Hall shukūk . . . min kitāb al-Mijisṭī, etc.:* see work No. 30 in *Geschichte der arabischen Litteratur*, I (Weimar: 1898), 470; 2nd ed. (Leiden: 1943), p. 619; Supplementband I (Leiden: 1937), 853.

[7] Bodl. MS Arch. Seld. A32, fol. 1^{r}.

[7a] The word maʿnā (pl. maʿānī) has a variety of meanings: idea, notion, concept, signification, etc. It may also be used more generally, as it is here, to mean: aspect, property, condition, circumstance, thing. The context alone determines which one of these is intended.

[8] Ptolemaei *Optica*, ii, p. 12: "Dicimus ergo quod uisus cognoscit corpus, magnitudinem, colorem, figuram, situm, motum, et quietum."

[9] Alhazeni *Optica*, ii, cap. 2, theor. 15, p. 34 [Arabic: Istanbul MS Fātiḥ 3213, fol. 34^{v}]: "Intentiones particulares [*al-maʿānī al-juz'iyya*], quae comprehenduntur [*tudraku*] sensu uisu, sunt multae, sed generaliter diuiduntur in 22: & sunt lux, color, remotio, situs, corporeitas, figura, magnitudo, continuum, discretio & separatio, numerus, motus, quies, asperitas, leuetas, diaphanitas, spissitudo, umbra, obscuritas, pulchritudo, turpitudo, consimilitudo, & diuersitas in omnibus intentionibus particularibus, & in omnibus formis compositis ex omnibus intentionibus particularibus." Cf. *ibid.*, pp. 34–67.

the distance (*al-bu'd*) between it and the visible object (*al-mubṣar*), and knows (*ya'lamu*) it to be a distance; and it perceives transparency (*al-shafīf*) in transparent bodies, and opacity (*al-kathāfa*) which is the contrary of transparency; and it perceives shadow and darkness; and it perceives separateness (*al-tafarruq*) and continuity (*al-ittiṣāl*)—all this is detailed and explained in the *maqāla* which we have mentioned.

The fact that he limited himself to seven things out of twenty and more is a proof that he erred and that the induction (*istiqrā'*) he made of objects of vision was inadequate.

• • •

He also says [the following] in the course of his discourse about visual illusions (*aghlāṭ al-baṣar*), at the place where he mentions the board (*al-lawḥ*) in which he draws the differently coloured lines[10]: when the eye (*al-baṣar*) gazes at the middle object (*al-shakhṣ al-mutawassiṭ*) supposed to be in the middle of the board, i.e., at the point where the two diameters intersect, then the two lines—which are the intersecting diameters and which are also the axes belonging to the eyes—are seen as [B: fol. 183ʳ] one line coinciding with the common axis, viz., the line standing at right angles on the middle of the line joining the two centres of vision.

Now this is a manifest error, evidenced both by reasoning (*al-qiyās*) and fact (*al-wujūd*: existence). I mean [it is an error] that the two lines coinciding with the axes of vision should be seen as one line; for the extremities of these two lines are at the centres of vision; again, the extremities of these two lines never meet on the common axis, but always remain apart; further, these two lines intersect at the middle object; therefore, they always intersect and they are never seen united (*mujtami'ayn*), though in vision (*fī-l-ru'ya*) they approach one another while still intersecting.

What made Ptolemy fall into this error is [the following]. When he supposed on the axes two objects, and gazed with both eyes at the middle object, he found the two objects united at the common axis.[11] But this only happens when the two supposed objects are near the middle object. When, however, they are near the eyes, they do not become one object, but approach one another while (remaining) apart. The cause of this is that the visual axes [A: fol. 18ʳ] approach one another in vision when one gazes at the middle object, the distance between them becoming less than the real distance. Thus if the objects are near the middle object, namely the point of intersection, the two objects meet; and if the objects are near the eyes, they approach one another but do not unite, unless the eyes, or one of them, deviate so that both axes fall on one of the objects (placed) upon the axes, in consequence of which that object and the axes are seen as one; further, the two objects are seen by the rays (falling) outside the axes as two; therefore, the two objects are seen [B: fol. 183ᵛ] as three. But if the eyes gaze at the middle object, and if the objects (placed) upon the axes are near the eyes, then these two objects are seen as four, two divergent (*mutabā'idayn*) and two approaching one another without being united.

There is also factual evidence for what we mentioned. For if these lines are examined (*u'tubirat*) with the (help of) the board described by Ptolemy, and in which he drew the lines he mentioned, and if the eyes are placed at the extremities of the two diameters intersecting at the middle object, gazing at the middle object, then the matter will be found to be as we mentioned. I mean that the two axes will be found to approach one another and intersect at the middle object, and the two objects that are near the middle object will be found united while the two objects that are far from the middle object will be found to approach one another while (remaining) apart—unless one of the eyes deviates from the middle object; further, the extremities of the axes will always be found at the eyes.

Thus both fact and reasoning testify that what he said about the uniting of the two axes at the common axis is false and impossible. This consideration (*hādhā-l-ma'nā*) of his is the principle (*al-aṣl*) which he asserted for visual illusions; he then built upon it the illusions concerning the places of visible objects. Since this consideration has been shown to be false, all that he built upon it concerning visual illusions is invalidated, its cause not being assured.

[10] Ptolemaei *Optica*, iii, Prop. 20, pp. 109 ff. Cf. Lejeune, *Euclide et Ptolémée*, pp. 160 ff.
[11] Ptolemaei *Optica*, iii, Prop. 16, pp. 102 ff. Cf. Lejeune, *Euclide et Ptolémée*, pp. 148 ff.

He also talked about mirrors: first he talked about plane mirrors (*al-marāyā al-musaṭṭaḥa*), then about convex mirrors (*al-marāyā al-muḥaddaba*). In one of the propositions (sing. *shakl*) on convex mirrors he says that if the eye (*al-baṣar*) perceives in convex mirrors an arc (*qaws*) that is concave towards the mirror, and (if) the extremities of the arc are on the mirror, then the concavity (*taq'īr*) of the arc may be seen as convex or straight or concave.[12] He then drew two rays reflected to two points of the arc, and drew two lines from the centre of the mirror to these two points, and produced the first two rays until they met these two lines: the places where they meet thus become [B: fol. 184ʳ] the images (sing. *khayāl*) of the two points on the arc. He then says that these two points and the first two points which are the extremities of the arc, (and) which are on the surface of the mirror, are all either on a straight line [A: fol. 18ᵛ] or on a curved (*muqawwas*) line whose convexity is towards the eye.

He did not, however, show that it is possible for these points to be on a straight line, or on a convex or a concave line. And unless he shows this possibility, and (also) shows every one of the positions (*awḍā'*) through which each one of the images is produced, he cannot judge the concave arc to be seen in these three positions, nor can he judge this proposition (*qaḍiyya*) to be true. Even if there were also factual evidence for what he maintains, he would not be permitted to make the judgement that the points he mentioned have to be in the three positions he distinguished—unless he shows the possibility of every one of these positions by demonstration.[13]

In addition to this circumstance, he put the arc between the eye and the mirror, and placed the eye in the plane of the arc, as he made the arc cut the perpendicular joining the centre of the circle and the eye; and he placed the extremities of the arc on the surface of the mirror. But if, as he supposed, the arc is (placed) between the eye and the mirror, and the extremities of the arc are on the surface of the mirror, then the eye cannot perceive the arc in the mirror.[14] For an object of vision cannot be perceived by the eye unless it is either in an opaque body, or in a transparent body with some opacity in it. Therefore, if the arc which he considered be in an opaque body, then the eye cannot perceive it in the mirror, since it (the arc) intervenes between the eye and the mirror. And if it be in a transparent body, then the two rays would be refracted (*yan'aṭifān*) in (this) transparent body before they reach the mirror, and they would not go through (it) [B: fol. 184ᵛ] in straight lines. Thus the position which he asserted for the image is invalidated.

He adopted the same consideration in the proposition (*al-shakl*) preceding this one, namely that in which he assumed the convexity (*taḥdīb*) of the arc to be towards the mirror.[15]

• • •

He also says [the following] at the end of the fifth maqāla: let three containers (sing. *āniya*) be made of pure and clear glass; let the first be made in the shape of a cube; let the second be cylindrical-convex; and let the third have a cylindrical-concave surface. He then says: let them be filled with water; let rulers (*masāṭīr*) be inserted into them; and let their images be examined.[16]

[12] Ptolemaei *Optica*, iii, Prop. 34, pp. 142 ff. For a discussion of this Proposition, see A. Lejeune, *Recherches sur la catoptrique grecque, d'après les sources antiques et médiévales* (Brussels: 1954), pp. 93 f.
[13] As A. Lejeune has remarked of Ptolemy's treatment of this problem, "Les moyens géométriques mis en oeuvres sont évidemment en disproportion avec la complexité du problème" (*Catoptrique grecque*, p. 94). In his *Optics*, Ibn al-Haytham altered the conditions of the problem by assuming the arc to be concentric with the surface of the mirror; but he complicated the problem even more by placing the eye outside the plane of the arc (Alhazeni *Optica*, vi, 4, theors. 11–15, pp. 199–201). The complication was again out of proportion with the means employed; see Muṣṭafā Naẓīf, *al-Hasan ibn al-Haytham, buḥūthuhu wa-kushūfu al-baṣariyya*, II (Cairo: 1943), §§ 179–180, pp. 643–649. See following note.
[14] It is the physicist in Ibn al-Haytham who objects here, and it is likely that he had this objection in mind when, previously in his own treatment of the problem in the *Optics*, he chose to place the eye outside the plane of the arc; see preceding note.
[15] Ptolemaei *Optica*, iii, Prop. 33, pp. 140 ff. Cf. Lejeune, *Catoptrique grecque*, pp. 92 f.
[16] Ptolemaei *Optica*, v, Prop. 96, pp. 260 ff. Cf. Lejeune, *Catoptrique grecque*, pp. 172 f. Ibn

Now he had shown before, that glass is more dense (*aghlaẓ*: more gross) than water and air. From this it follows that when the ray reaches the glass from the air, it is refracted because the glass is denser than air; then when this ray reaches the water it is refracted again because water is more subtle (*alṭaf*) than glass. [Therefore,] not all the rays reaching the water will be perpendicular to the surface of the instrument, and thus pass through straight on. There will be only one perpendicular among them.[17] And since these rays have been refracted twice, the images which he asserted for the rulers become invalid.

We have also found in his book on *Optics* a number of propositions which he proved (*bayyanahā*) by invalid demonstrations, even though the assertions are in themselves true, their truth being possible to establish by demonstrations other than his. We have therefore reinterpreted these propositions for him and have not mentioned them.

That is the end of what we have to say about the book on *Optics*.

al-Haytham says that Ptolemy discusses this experiment "at the end of the fifth part" of the *Optics*. In the Latin version the discussion is also to be found near the end of the fifth part which, as remarked before, is not complete. This indicates that, as in the case of the first part, the text of the fifth part of Ptolemy's book was mutilated *before* Ibn al-Haytham's time.

[17] The text seems to be confused here; the following are the readings in both MSS. B: *bal laysa yakūnu fīhā ʿamūdā* [*sic*] *wa-yakūnu ʿamūdā wāḥidā faqaṭ* A: *bal laysa yakūnu fīhā ʿamūdā sic*] *wa-yakūnu fīhā ʿamūd wāḥid faqaṭ*.

The Authorship of the *Liber de crepusculis*, an Eleventh-Century Work on Atmospheric Refraction

THIS ARTICLE AIMS to prove that the *Liber de crepusculis*, which has always been attributed to Ibn al-Haytham (Alhazen, d. *c.* 1039) since it was first published in 1542, is in fact the work of the Andalusian mathematician Abū ʿAbd Allāh Muḥammad ibn Muʿādh, who lived in the second half of the eleventh century. It will be shown that whereas no good reason has been given for ascribing this work to Ibn al-Haytham, there exists positive evidence for Ibn Muʿādh's authorship. This evidence has so far been completely overlooked.

The *Liber de crepusculis* (hereafter *Ldc*) is a short work containing an estimation of the angle of depression of the sun at the beginning of the morning twilight and at the end of the evening twilight, and an attempt to calculate on the basis of this and other data the height of the atmospheric moisture responsible for the refraction of the sun's rays. Hence its fuller title is *De crepusculis matutino et vespertino*, which in manuscripts is sometimes coupled with or replaced by *De nubium ascensionibus*. The existence during the thirteenth to the sixteenth centuries of many manuscript copies of the *Ldc* and its several sixteenth-century editions testify to the interest it attracted in the Latin Middle Ages and in the Renaissance. The reasonably accurate value of 18° which it assumed for the angle described has been found remarkable by modern commentators.

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The *Ldc* was first printed in 1542,[1] then in 1572,[2] and in 1573.[3] In all of these editions it was ascribed to Ibn al-Haytham, and the translator was stated to be Gerard of Cremona. As was the custom of the period, however, no manuscripts were cited and no reasons given for regarding Ibn al-Haytham as the author. We have therefore to consider whether any such reasons exist.

The first thing to be noted is that the attribution of the *Ldc* to Ibn al-Haytham can derive no support from the Arabic sources hitherto known to us. This appears from the following observations: (1) No copy of the Arabic original is known to be extant. (2) Ibn al-Haytham drew up at least two autobibliographies, the first in Dhu'l-Ḥijja, 417 A.H., when he was sixty-three (lunar) years old, and the second in Jumādā II, 419 A.H.[4] Ibn Abī Uṣaybi'a, who has quoted these catalogues from autographs of Ibn al-Haytham,[5] also reproduces a third catalogue covering the period (from 419 A.H.?) to the end of 429 A.H.[6] He does not tell us that he copied this third list from an original in Ibn al-Haytham's hand, but this list also seems to have been derived from an autobibliography.[7] None of these catalogues includes a title bearing a resemblance to the Latin version. And this cannot be due to a change introduced by the Latin translator, for we shall see that we can reconstruct the original Arabic title from data yielded by some of the Latin manuscripts. (3) The title is also absent from other lists of Ibn al-Haytham's writings that have been preserved in the works of Arabic biobibliographers, such as Ibn al-Qifṭī and Ḥājjī Khalīfa. (4) As far as is known, the extant writings of Ibn al-Haytham do not contain an allusion to a work devoted to the subject matter of the *Ldc*.

We therefore have to turn to the Latin tradition of our text. Is there a Latin manuscript tradition which ascribes the *Ldc* to Ibn al-Haytham? Do Latin manuscripts of this work exist which bear Ibn al-Haytham's name? Before we can answer these questions, it must be pointed out that in the Latin versions of works by Ibn al-Haytham there was no standard form of rendering his name. It will thus be necessary to look first at his name as it appears in manuscripts of works whose attribution to Ibn al-Haytham is not in doubt.

1 Petri Nonii Salaciensis *De crepusculis liber unus, nunc recens et natus et editus. Item* Allacen Arabis *uetustissimi, de causis Crepusculorum Liber unus, a* Gerardo Cremonensi *iam olim Latinitate donatus, nunc uero omnium primum in lucem editus* (Olyssipone, 1542).

2 *Opticae thesaurus.* Alhazeni Arabis *Libri septem, nunc primum editi. Eiusdem liber De crepusculis et nubium ascensionibus. Item* Vitellonis Thuringopoloni *Libri X. Omnes instaurati, figuris illustrati et aucti, adiectis etiam in Alhazenum commentarijs, a* Federico Risnero (Basileae, 1572).

3 Petri Nonii Salaciensis *De arte atque ratione navigandi libri duo. Eiusdem in theoricas Planetarum* Georgij Purbachij *annotationes, et in Problema mechanicum* Aristotelis *de motu navigij ex remis annotatio una. Eiusdem de erratis* Orontij Finœi *Liber unus. Eiusdem de Crepusculis Lib. I. Cum libello* Allacen *de causis Crepusculorum* (Conimbricae, 1573). E. Narducci mentions another Basel edition of 1592, which I have not seen. See his "Intorno ad una Traduzione Italiana. Fatto nel Secolo Decimoquarto, del Trattato d'Ottica d'Alhazen. . . ." *Bollettino di Bibliographia e di Storia delle Scienze Matematiche e Fisiche*, 1871, *4*:14–15.

4 Ibn Abī Uṣaybi'a, *'Uyūn al-anbā' fī 'abaqāt al-aṭibbā'* ed. A. Mueller (Vol. 2, Cairo, 1882), pp. 91–97.

5 *Ibid.*, p. 91, line 22, p. 97, line 2.

6 *Ibid.*, pp. 97–98.

7 It should be noted, however, that the first person is completely absent from the third list, in contrast to the first two catalogues.

For the sake of comparison between Latin and Arabic versions, I shall give first a list of the different forms in which the name appears in the Arabic tradition. In the list that follows, each of these names is preceded by a number for ease of reference.

N1: al-Ḥasan ibn al-Hasan ibn al-Haytham, Abū 'Alī
Ibn al-Qifṭī, *Ta'rīkh alḥukamā'*, ed. A. Mueller and L. Lippert (Leipzig, 1903), p. 165.

N2: Abū 'Alī Muḥammad ibn al-Ḥasan ibn al-Haytham
Ibn Abī Uṣaybi'a, *'Uyūn al-anbā' . . .* , ed. A. Mueller (Vol. 2, Cairo, 1882), p. 90.

N3: Abū 'Alī al-Ḥasan ibn Ḥusayn ibn al-Haytham
Ḥājjī Khalīfa, *Kashf al-ẓunūn*, ed. Gustav L. Flügel (Vol. 1, Leipzig, 1835), p. 382.

N4: al-Ḥasan ibn al-Ḥasan ibn al-Haytham
Autograph,[8] Istanbul MS Aya Sofya 2762, fol. 135^{r}.

The following are the Latinized versions of Ibn al-Haytham's name as they appear in manuscripts of the *Optics* (*al-Manāẓir*), known in the Latin tradition as *Perspectiva*, or *De aspectibus*:

Alhacen filii Alhaycan [al-Ḥasan ibn- al-Haytham]
MS Royal Observatory,[9] Edinburgh, 13th century, fol. 2^{r}.

halhacen filii halhaycen filii aycen [al-Ḥasan ibn al-Haytham ibn Haytham]
Ibid., fol. 186^{r}.

haten filij hucaym filij hayten [Ḥasan ibn Ḥusayn ibn Haytham]
MS Trinity College, Cambridge, 1311, 13th century, fol. 165^{r}.

achen filij hucaym [Ḥasan ibn Ḥusayn]
MS Bibliothèque Publique de Bruges, 512, 13th century, fol. 1^{r}, line 1.[10]

filij alhacen [Ibn al-Ḥasan]
Ibid., fol. 1^{r}, line 2.

8 See facsimile reproduction in Matthias Schramm, *Ibn al-Haythams Weg zur Physik* (Wiesbaden: Franz Steiner Verlag, 1963), opposite page x. The MS is a copy in Ibn al-Haytham's hand of Bks. 1–7 of the Banū Mūsā version of Apollonius' *Conica*; it is dated 415 A.H. See Max Krause, "Stambuler Handschriften islamischer Mathematiker." *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, B (Studien), 1936, *3*: 448–449.

9 This MS is one of the earliest and best of the *Optics De aspectibus*, and it also contains the *Ldc*. It is not included in Lynn Thorndike and Pearl Kibre's *Catalogue of Incipits of Mediaeval Scientific Writings in Latin* (Cambridge, Mass.: Mediaeval Academy of America, 1963) but was mentioned by Charles Singer in "Steps Leading to the Invention of the First Optical Apparatus," *Studies in the History and Method of Science*, ed. Charles Singer (Oxford: Clarendon Press, 1921), Vol. 2, pp. 392–393, n. 1.

The colophon at the end of the *Ldc* gives the date at which the MS was transcribed; it runs: "Ego magister Guido dictus de Grana correxi diligenter istos duos libros, scilicet, perspectiuam Alhacen, et librum de ascensionibus nubium; juxta exemplar magistri Johannis Londoniensis qui ipsemet diligenter correxit ut dicitur. Completa fuit correctio horum librorum anno domini M. cc. lx. nono [1269], quinto ydus maii, scilicet, in vigilia penthecostes" (fol. 189^{r}).

10 L'Abbé A. de Poorter, *Catalogue des manuscrits de la Bibliothèque Publique de la ville de Bruges* (Gembloux Paris, 1934), No. 512, p. 599.

achen filij hucaym filij aycen [Ḥasan ibn Ḥusayn ibn Haytham]
Ibid., fol. 113ᵛ.
Alacen [al-Ḥasan]
MS British Museum, London, Royal 12. G.VII, 14th century, fol. 104ᵛ.
haten filii hucaym filii haycen [Ḥasan ibn Ḥusayn ibn Haytham][11]
Nationalbibliothek, Vienna, 2438, 15th century, fol. 144ʳ.

Other forms of Ibn al-Haytham's name associated with Latin translations of some of his other works have been recorded by scholars. For example, a fourteenth-century manuscript of his astronomical work, *Fī hay'at al-'ālam* (On the Configuration of the World), bears the name Aboali ibin Heitam [Abū 'Alī ibn Haytham].[12] A fifteenth-century manuscript of the same work has Abulhazen Abenlaytan [Abu'l-Ḥasan ibn al-Haytham].[13] Another fifteenth-century manuscript, entitled *De imaginibus celestibus*, has Ali ibn Ilhaytim ['Alī ibn al-Haytham].[14] And there is, of course, the well-known version Alhazen [al-Ḥasan] which we find, for example, in Roger Bacon and on the title page of Risner's edition of the *Optics*.

One can easily see that all of these versions, and others that are more or less similar, represent attempts at approximating one part or another of Ibn al-Haytham's name as we know it from the Arabic tradition; this will have clearly appeared from comparison with the corresponding Arabic forms which I have added in brackets. If we now examine manuscripts of the *Ldc*, we find an entirely different situation; the name which manuscripts of this work bear, if any at all, is quite unlike any that we come across elsewhere in Latin or Arabic copies of Ibn al-Haytham's works. The following are examples taken from manuscripts belonging to the thirteenth, fourteenth, and sixteenth centuries respectively:

Incipit liber abomadhi malfagar id est de crepusculo matutino et uespertino et ssafac verba eius ostendere quid sit crepusculum
MS Royal Observatory, Edinburgh, 13th century, fol. 186ʳ.
Incipit liber de crepusculis matutino et uespertino quem fecit abbomadhi[15] translatus a magistro cremonensi toleti de arabico in latinum
MS Biblioteka Jagiellońska, Cracow, 596, early 14th century, p. 99.

[11] Note that the form "al-Ḥasan ibn Ḥusayn," occurring in Ḥājjī Khalīfa (N3) but proved apocryphal by Ibn al-Haytham's autograph (N4), is very strongly represented in the Latin tradition.

[12] José María Millás Vallicrosa, *Las traducciones orientales en los manuscritos de la Biblioteca Catedral de Toledo* (Madrid: Privately printed, 1942), p. 312.

[13] M. Steinschneider, "Notice sur un ouvrage astronomique inédit d'Ibn Haitham," *Bollettino di bibliografia e storia delle scienze matematiche e fisiche*, 1881, *14*: 731 = p. 13 of the *Extrait* (Rome, 1883).

[14] Thorndike and Kibre, *op. cit.*, 1407. [Since writing this article I have had the opportunity to look at the Vatican MS Urb. Lat. 1384 and was able to determine that the work referred to by Thorndike and Kibre, *op. cit.*, is not by Ibn al-Haytham. The author's name as stated at the beginning of the accompanying Arabic text (fol. 3ᵛ) is Abū 'Alī al-Ḥasan ibn al-Ḥātam (thus vowelled). Added in galley proofs.]

[15] This could also be read Ahhomadhi or Abhomadhi. It was rendered by M. Curtze as Ali Homadi, with the explanation that Ali Homadi is the same as Ibn al-Haytham; but no reason was given to justify the transcription or the identification. See Euclidis *Opera omnia*, ed. J. L. Heiberg and H. Menge; *Supplementum*: Anaritii *in decim libros priores Elementorum Euclidis Commentarii*, ed. M. Curtze (Leipzig: Teubner, 1899), p. xi and n. 2.

Incipit liber Abhomadi Malfegeyr de crepusculis. Liber Abhomadi Malfegeyr, id est in crepusculo matutino, in saffae, id est in uesperino crepusculo. . . . . Explicit liber Abhomadj de crepusculis[16]
MS Bibliothèque Nationale, Paris, 10260, 16th century, fols. 194ʳ, 199ʳ.
Incipit liber Abhomady malfegeyr de crepusculis. Liber Abhomady, malfegeyr, id est in crepusculo matutino, et saffac, id est in uespertino crepusculo. . . . Explicit liber Abhomady de crepusculis[17]
MS Vatican, 2975, 16th century, fols. 202ʳ, 208ʳ.

It was first noted by Lucien Leclerc in 1876 that the word *malfegeyr* (or *malfagar)* was but a modification of the Arabic *al-fajr* (dawn) and that *ssafac* (*saffae*, *saffac*, *soffe*) represented the Arabic *shafaq*, meaning evening twilight.[18] If we now conjecture the *m* in *malfegeyr* to be a trace of the Arabic *mā* (what), then the full Arabic title (or incipit?) would seem to have been *Ma'l-fajr wa'l-shafaq* (What is Dawn and Twilight? or On the Nature of Dawn and Twilight).

But what about Abhomadi (or abomadhi, or abhomady, etc.)? According to Leclerc, this is just another variant of Ibn al-Haytham's name; and he suggests that it was derived from Abū 'Alī Muḥammad, in the form N2, by omitting "'Alī Mu-" and combining the rest into one word.[19]

A year later, in 1877, F. Wuestenfeld published a similar conjecture; but starting himself from the form *Alhomadi*, which he found in Bibliothèque Nationale MS 7310 (4), he proposed as its origin a shorter part of N2, namely: 'Alī Muḥammad.[20] So here again we would have to assume that the coining of the name *Alhomadi* was achieved at the expense of the first syllable in "Muḥammad." Another modification implied in both suggestions would be the replacement of the first *a* in Muhammad by an *o*.

The artificiality of these derivations is rather obvious; and it is certain that Leclerc and Wuestenfeld would not have suggested them if it had not been already taken for granted that Ibn al-Haytham was the author of the *Ldc*. But their view was soon adopted by M. Steinschneider[21] and is now universally accepted.

One of the most recent writers to endorse the attribution of the *Ldc* to

[16] Quoted by Axel A. Bjørnbo in "Alkindi, Tideus und Pseudo-Euklid: drei optische Werke," edited and interpreted by Bjørnbo and Sebastian Vogl, *Abhandlungen zur Geschichte der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*, 1912, *26*:140.

[17] Quoted by Bjørnbo, *ibid.*, p. 144.

The only exception to these MSS that I know of is the 14th-century Bodleian MS Digby 104 which has a fragment of the *Ldc* on pp. 80ʳ–81ʳ. At the top of the first page, the author and title are indicated thus: "Allaten: de cr'pusculis." But this is written in a different hand and may have been added at a later date. In any case it was already in the 14th century that the *Ldc* came to be ascribed to Ibn al-Haytham; see passage quoted from Oresme(?) below.

[18] L. Leclerc, *Histoire de la médecine arabe* (Paris: E. Leroux, 1876), Vol. 2, p. 418.

[19] *Ibid.*

[20] F. Wuestenfeld, *Die Übersetzungen arabischer werke in das Lateinische seit dem XI. Jahrhundert* (Göttingen: Dieterich'sche Verlags-Buchhandlung, 1877), p. 66 (reprinted from *Abhandlungen der Kgl. Gesellschaft der Wissenschaften zu Göttingen*, XXII).

[21] M. Steinschneider, "Notice sur un ouvrage astronomique inédit d'Ibn Haitham," *loc. cit.*, p. 729 = p. 11 of the *Extrait*: "Dans mes notes aux *Vite*, ecc. *di Baldi* (page 83), j'ai demandé, qui soit *Abhomadius Malfegeir*, au-

Ibn al-Haytham is Matthias Schramm;[22] and he accordingly seeks to assign to this work a place in the development of Ibn al-Haytham's thought. He is aware of the lack of evidence in the Arabic sources, but he believes the style of the *Ldc* to be conformable to that of Ibn al-Haytham.[23] Even apart from the positive considerations that follow, it would be difficult to maintain such a belief in the absence of the Arabic text of the *Ldc*.

The preceding observations, showing a complete want of convincing evidence in favor of regarding Ibn al-Haytham as author of the *Ldc*, are enough to cast serious doubt on this assumption. But the matter need not remain in doubt; for, fortunately, we have one piece of evidence which decides the question in a positive manner. This is a manuscript in the Bibliothèque Nationale which contains four Hebrew translations of Arabic works.[24] The second of these, folios 7ʳ–9ᵛ, is a treatise *On the Dawn*, attributed in the manuscript to the judge (*ha-shofeṭ*) Abū 'Abd Allāh Muḥammad ibn Mu'ādh, and like all other works in the same volume it was translated by Samuel (Miles) ben Jehuda of Marseilles (fl. *c.* 1335).[25] An examination of this treatise shows that it is a translation of the same work which we have in the Latin of Gerard of Cremona under the title *Liber de crepusculis*. The full title runs as follows (fol. 7ʳ):

teur d'un ouvrage: *de crepusculis*, dans un manuscrit de la bibliothèque nationale de Paris. Ce n'est autre que *Alhazen* (ou Ibn Heitham), comme nous apprend Lucien Leclerc (*Hist. de la médecine arabe*, II, 416, cf. Wuestenfeld, *Die Übersetzungen arabischer Werke*, etc. p. 66)." See *Vite di matematici arabi tratte de un'opera inedita di Bernardino Baldi. Con note di M. Steinschneider*, Rome, 1874, p. 83, n.*** (reprinted from *Boll. bibliogr. Sci. mat.*, 1872, *5*). See also Steinschneider's *Die hebraeischen Übersetzungen des Mittelalters* (Berlin: Kommissionsverlag des Bibliographischen Bureaus, 1893), p. 575, n. 271; *Die europaeischen Übersetzungen aus dem Arabischen bis Mitte des 17. Jahrhunderts* (Graz: Akademische Druk- und Verlagsanstalt, 1956), Sec. A (first published in 1904), pp. 22–23.

[22] See his very interesting book *Ibn al-Haythams Weg zur Physik*, *ed. cit.*, p. 279.

[23] Dr. Schramm writes: "Dass die Schrift, die bisher nur in der lateinischen Version des Gerhard von Cremona als 'Liber de crepusculis et nubium ascensionibus' bekannt ist, auch im 3. Schriftenverzeichnis nicht auftritt, spricht aus diesem Grund keineswegs gegen die Echtheit; auch sonst stimmt der Stil der ganzen Abhandlung aufs beste mit dem von Ibn al-Haytham bekannten ueberein" (*op. cit.*, pp. 278–279).

This decision allows Dr. Schramm to argue against the view (put forward by A. Jourdain, in *Recherches critiques sur l'âge et l'origine des traductions latines d'Aristote . . .* , Paris: Joubert, 1843, p. 123) that the Latin translation of Ibn al-Haytham's *Optics* (*al-Manāẓir*, *Perspectiva, De aspectibus*) was made by Gerard of Cremona. He finds this view sufficiently refuted by the difference in terminology between the *Ldc* (which we know was translated by Gerard) and the Latin translation of the *Optics* (*op. cit.*, p. 279, n. 1). This argument loses its force in the light of the present article; for if the *Ldc* is not by Ibn al-Haytham then the terminological differences noted in the Latin translations might in fact be due to differences between the underlying Arabic texts. This matter may now be studied further with reference to the Hebrew manuscript referred to in the following discussion. But the question whether Gerard of Cremona or someone else translated the *Optics* remains undecided.

[24] See H. Zotenberg and S. Munk, *Catalogues des manuscrits hébreux et samaritains de la bibliothèque impériale* (Imprimerie Impériale, 1866), No. 1036, p. 189; Steinschneider, *Die hebraeischen Übersetzungen*, § 357, pp. 574–575.

[25] The first work in this collection, also by Ibn Mu'ādh, treats of the total solar eclipse which took place on "the last day of the year 47 in the hijra (3 July 1079)," Zotenberg and Munk, *op. cit.*, p. 189. The last two treatises are by other authors. See Steinschneider, *Die hebraeischen Übersetzungen*, p. 575; George Sarton, *Introduction to the History of Science* (Baltimore: Williams & Wilkins, for the Carnegie Institution of Washington, 1931), Vol. 2, p. 342; Heinrich Hermelink, "Tabulae Jahen," *Archive for History of Exact Sciences*, 1964, 2, No. 2: 108–112.

> Iggeret be'ammud ha-shaḥar u-mah sibbato we'illat higgaluto u-leḳiḥat ha-hora'ah mi-mennah 'al 'alīyat ha-'edim ha-mugbahim me-ha-'ares. (A treatise on the column of dawn, its cause, and the reason for its appearance—from which [cause] is shown the height of the vapors rising from the earth.)

The treatise itself then begins thus:

> Amar ha-mashneh ha-shofeṭ Abū 'Abd ha-'el Muḥammad Ibn Mu'ādh: Nirṣeh le-va'er ba-ma'amar ha-zeh mah 'ammud ha-shaḥar u-mah ha-sibbah ha-meḥayyevet le-higgaluto we-nudrag mi-zeh el takhlit mah she-ya'aleh mi-sheṭaḥ hora'ah mi-mennah 'al 'alīyat ha'edim ha-mugbahim me-ha-'areṣ. (A treatise shaḥar we-'ammud ha-'erev mitdammey ha-temunah "a" mi-sheneyhem mi-meruṣat or ha-shemesh we-ha-'aḥer me-ḥazarato.

Disregarding the first sentence (said the master, *ha-mashneh*, the Judge Abū 'Abd Allāh Muḥammad ibn Mu'ādh), these lines exactly correspond to the opening sentences of the *Ldc*:

> Ostendere uolo in hoc tractatu quid sit crepusculum, et quae caussa necessario faciens eius apparitionem; inde uero progrediar ad cognoscendum ultimum, quod eleuatur a superficie terrae, de uaporibus subtilibus ascendentibus ex ea. Dico ergo, quod crepusculum matutinum et crepusculum uespertinum sunt similis figurae; unum namque eorum ex accessione luminis solis, et alterum ex ipsius recessione contingit.[26]

The first sentence of the Hebrew version, to which nothing corresponds in the printed Latin text, immediately explains the problematic names (Abomadhus, etc.) which, as we have seen, are to be found in some manuscripts of the *Ldc*. These forms now appear to be attempts at rendering the name Ibn Mu'ādh, the real author of the treatise; the similarity between the Arabic name and these Latinized versions is quite clear.[27]

Two questions remain to be considered: (1) Prior to the 1542 publication of the *Ldc*, when exactly and by whom was it first ascribed to Ibn al-Haytham? (2) What was the basis for this ascription?

The first question is not easy to answer. Roger Bacon refers to our treatise, but without naming the author.[28] For identification of the latter as Ibn al-Haytham, the time of Nicole Oresme would seem to be the *terminus ante quem*. This appears from the following passage which occurs in a work entitled *De visione stellarum*, probably written by Oresme or one of his circle:

[26] *Opticae thesaurus*, *ed. cit.*, *De crepusculis*, p. 283.

[27] The Arabic underlying Abomadhus, etc., would seem to be Abū Mu'ādh, rather than Ibn Mu'ādh, the more correct appellation. Now Abū is a part of Ibn Mu'ādh's full name, but its proper place does not immediately precede Mu'ādh. Our equation Abomadhus, etc. = Ibn Mu'ādh might thus appear to share some of the *ad hoc* character of the derivation proposed by Leclerc. But in fact this is not so. Steinschneider lists the following names by which the same author was known, quite independently of the *Ldc*: Aben Moat, Abumaad, Abumadh (*Die hebraeischen Übersetzungen*, p. 575). The third of these is almost identical with the name appearing on the 13th-century Royal Observatory MS of the *Ldc*, and the second is not much different.

[28] Roger Bacon, *Opus majus*, ed. J. H. Bridges (Oxford: Clarendon Press, 1897), Vol. 1, pp. 229–230.

. . . Unde si tanta spissitudo aeris esset inter lunam et stellas fixas quanta est corpulentia intermedii celi stelle non possent a nobis videri.

Tertio ex tractatu Alasen de crepusculis apparet quod propter aeris spissitudinem reflectentem radios solis, fiunt(?) crepuscula.[29]

With regard to the second question, at least one plausible answer is at hand. Presumably because of the affinity in subject matter, Ibn al-Haytham's *Optics* is found in some manuscripts immediately followed by the *Ldc*. In some instances, this is done without indicating the name of the author of the second work.[30] It seems not unlikely that someone who faced this situation in one or more manuscripts simply assumed the two works to be by the author of the first, and better-known, work. It is relevant to note in this connection that when the two works occur together in the same manuscript, and names of authors are provided for both, as in the thirteenth-century Royal Observatory manuscript, and the fourteenth-century Cracow manuscript,[31] these names are different, and not the same.

About Ibn Mu'adh himself very little is known. Ibn Bashkuwāl (d. 1183) mentions a certain "Muḥammad Ibn Yūsuf ibn Aḥmad ibn Mu'ādh al-Juhanī of Cordova, surnamed Abū 'Abd Allāh," a Koranic scholar who had "some knowledge of Arabic [philology] and of inheritance laws (*farḍ*) and arithmetic," who was born in 379 A.H. (A.D. 989/90) and lived in Egypt from 403 A.H. to 407 A.H. (A.D. 1012/13–1016/17).[32] H. Suter[33] thought it likely that this man was identical with the author of two mathematical works which survive in the original Arabic. The first of these, attributed in the unique manuscript copy at the Escorial Library to Abū 'Abdallāh Muhammad ibn Mu'ād of Cordova, is a treatise on spherical trigonometry: *Kitāb majhūlāt qisiyy al-kura*.[34] The second, which has been published together with a study and English translation, is a discussion of Euclid's conception of ratio, and it bears the

[29] MS Firenze, Biblioteca Nazionale, Conv. Sopp. San Marco, J.X. 19, fol. 35v.

[30] One interesting example is the 14-century British Museum MS Royal 12 G VII. The *Ldc* begins, with no title, after the *Optics* (*De aspectibus*) ends on p. 102v, col. B. The explicit of the second treatise simply reads (fol. 104v): "Explicit liber etc." Following this someone wrote in a different hand on a separate line: "Alacen in scientia perspectiua [sic]." The writer apparently was ignorant of the title of the *Ldc* but assumed this work to be a contribution to the "science of perspective" by the author of the *De aspectibus*.

The following manuscript copies of the *Ldc* do not bear any indication of authorship (the volumes in which they occur do not include the *Optics*): Bodleian MS Ashmole 341, 13th century, fols. 76v–81r; Bodleian MS Digby 215, fols. 96v–98v; MS Peterhouse 209 (now at University Library, Cambridge), fols. 112r–114r.

[31] See above, pp. 79–80 and Bjørnbo, *op. cit.*, p. 133, where the explicit of a fragment of the *Optics* occurring in the Cracow MS is quoted as follows: "Explicit VII liber Alhaceni de aspectibus."

[32] *Al-Ṣila* (*Bibliotheca arabico-hispana*, Madrid, 1883), Vol. 2, No. 1060, pp. 480–481.

[33] H. Suter, "Die Mathematiker und Astronomen der Araber und ihre Werke." *Abhandlungen zur Geschichte der mathematischen Wissenschaften*, 1900, *10*:96.

[34] Michael Casiri, *Bibliotheca arabico-hispana escurialensis* (Madrid, 1760), Vol. 1, No. 955, p. 382. The title given by Casiri, *Istikhrāj maqādīr al-qisiyy al-wāqi'a 'alā ẓahr al-kura* (Determination of the Magnitudes of the Arcs on the Surface of the Sphere), is taken from the incipit; see H. Derenbourg and H.-P.-J. Renaud, *Les manuscrits arabes de l'Escurial* (Paris, 1941), Vol. 2, Fasc. 3, No. 960, p. 94. [The Escorial manuscript is, after all, not unique. There exists another copy in a codex at the Biblioteca Medicea Laurenziana, Florence, which also includes another work of Ibn Mu'ādh, but not the *Ldc*. I hope to publish a description of this volume in the future. Added in proof.]

name: al-qāḍī [the judge] Abū 'Abd Allāh Muḥammad ibn Mu'ādh al-Jayyānī[35] (that is, of Jaen in Southern Spain). Whether or not we accept this conjectured identification (which in itself is not impossible),[36] it does seem very plausible that the author of these two mathematical works was the same as the author of the previously mentioned treatise on the total solar eclipse of July 1079 and the one on the dawn, both of which survive in the Hebrew translation of Samuel ben Jehuda,[37] as well as the *Tabulae Jahen*, translated into Latin by Gerard of Cremona.[38] We now have to add to this list the Latin version of the treatise on the dawn, the *Liber de crepusculis* which Gerard also translated from the Arabic. Finally we may assume that it was again the same Abū 'Abd Allāh Muḥammad ibn Mu'adh al-Jayyānī whom Averroës (d. 1198) had in mind when he referred to the "Andalusian mathematician Ibn Mu'ādh" as one of those who considered the angle a distinct kind of magnitude to be added to the traditional three: body, surface, and line.[39] Averroës found the doctrine odd, but he regarded the man who held it as "an advanced and high-ranking mathematician."[40]

[35] E. B. Plooij, *Euclid's Conception of Ratio and His Definition of Proportional Magnitudes as Criticized by Arabian Commentators* (Rotterdam: W. J. van Hengel, 1950).

[36] Sarton preferred to leave the question open (*Introduction* . . . Vol. 2, p. 342).

[37] See n. 25 above. Cf. Steinschneider, *Die hebraeischen Übersetzungen*, p. 575; C. Brockelmann, *Geschichte der arabischen Litteratur*, Supplementband 1 (Leiden, 1937), p. 860; D. José A. Sánchez Pérez, *Biografias de matemáticos árabes que florecieron en España* (Madrid: Impr. de E. Maestre, 1921), No. 130, p. 112, No. 148, p. 123.

[38] See Hermelink, *op. cit.*

[39] Averroës, *Tafsīr Mā ba'ad aṭ-ṭabī'at*, ed. M. Bouyges (Beirut, 1942), Vol. 2, p. 665. The sentence referring to Ibn Mu'ādh is missing from the Latin translation; see *Aristotelis Metaphysicorum libri XIII. Cum Averrois . . . Commentariis* (*Aristotelis Opera cum Averrois Commentariis*, Vol. VIII, Venetiis, 1562) (reproduced photographically, Frankfurt am Main, 1962), fol. 137vH–I.

[40] *Tafsīr* . . . , p. 665.

THE ASTRONOMICAL ORIGIN OF IBN AL-HAYTHAM'S CONCEPT OF EXPERIMENT

Probably the most striking feature of the *Optics* of Ibn al-Haytham is the structure of its arguments. One can say, with hardly any exaggeration, that every piece of reasoning in this book is either a mathematical demonstration, or an experimental proof, or an inductive generalization. Mathematical demonstrations are not only applied to problems of geometrical optics, but also sometimes constitute an essential part of an experimental proof. Experiment and induction work together, but, significantly, with separate roles. An induction may be an inspection or review of particular cases aiming to establish the uniformity of certain existing states of affairs. An experiment always has the function of test or proof, and to perform this function it may involve manipulation of an artificially constructed apparatus. Induction again generalizes a result obtained by a series of related experiments.

Ibn al-Haytham's word for induction is the regular term which in the Arabic translations of Aristotle and in related literature, such as Avicenna's *al-Shifā*ᵓ, corresponds to the Greek ἐπαγωγή — namely *istiqrā*ᵓ ; and what he means by this word derives from Aristotelian usage. His term for experiment, however, is something of a problem (1). In Arabic works of peripatetic and Galenic ancestry the most commonly used word to convey the meaning of experience is *tajriba*, which in these works corresponds to ἐμπειρία. Ibn al-Haytham ignores this word — and quite understandably : a reader of his *Optics* would soon realize the difficulty of trying to subsume its experimental arguments under a theory of ἐμπειρία such as we find formulated in the *Posterior Analytics* or in Galen's *Medical Experience*. Instead of *tajriba* Ibn al-Haytham employs the nomen verbis *i*ᶜ*tibār*, together with the verb *i*ᶜ*tabara*, and the nomen agentis *mu*ᶜ*tabir* — all derivatives of the triliteral verb ᶜ*abara*, to go through or traverse. A classical Arabic dictionary might explain the meaning of the eighth form, *i*ᶜ*tabara*, as « to draw an inference about one thing from another », but the medieval Latin translator of the *Optics*, guided no doubt by how these words are repeatedly used in the book, did not hesitate to render *i*ᶜ*tabara* by *experimentare* (or *experiri*), *i*ᶜ*tibār* by *experimentum* (or *experimentatio*), and *mu*ᶜ*tabir* by *experimentator*.

The problem of the origin of the term *iʿtibār* is interesting inasmuch as it necessarily forms part of the question about the origin of the idea connected with this term. For it is not the fact that a large number of experiments are described by Ibn al-Haytham which we find remarkable in his book, but rather the fact that he consciously and systematically operates with a concept of experiment which he associates with cognates of one and the same root. It would not be difficult to find prototypes of many of his experiments in previous writings, particularly in Ptolemy's *Optics* which, it should be noted, has *experimentum* five times and *experiri* once. But what seems to have acquired a new emphasis in the *Optics* of Ibn al-Haytham is the explicit use of experimentation as a paradigmatic type of proof to be distinguished from other types of proof occurring besides it. What this book surprises us with is its clear and exclusive commitment to a small number of definite forms of argument of which experimentation is one.

It is this aspect of *iʿtibār* as used by Ibn al-Haytham, namely its appearance in optics as a separate, articulated methodological concept, that I propose to explain by relating it to a previously established procedure in observational astronomy. My evidence will consist of a compressed account of how this term was first introduced, and then widely used, in Arabic astronomical literature.

In *Almagest* vii. 1 Ptolemy discusses evidence for the doctrine that the zodiacal stars do not change their position with respect to those that are outside the zodiac — a doctrine which implied that the slow eastward motion responsible for producing the precession of the equinoxes was common to all stars. Hipparchus, so Ptolemy tell us, finally adopted the doctrine, but only on the basis of a few and unreliable observations derived from Aristyllus and Timocharis. Ptolemy argued, however, that the doctrine could be put on a firmer basis by comparing (ἐκ τῆς συγκρίσεως) his own observations with the more reliable ones of Hipparchus. And, as an easily accessible test of the matter (τῆς προχείρου πείρας ἕνεκεν), he selected from among the star configurations described by his predecessor, a few which, he said, would be more readily recognizable and thus would lend themselves more easily to such a comparison. Since no discrepancy was apparent, the result of this test was seen by Ptolemy as a sufficient proof of the conjecture made by Hipparchus two hundred and sixty years earlier. But in order to allow his successors to make a similar examination by reference to their own observations, Ptolemy went on to describe some more configurations which he was the first to record.

The two key words which interest us here are σύγκρισις (comparison) and πεῖρα (test or proof). Al-Ḥajjāj, in his Arabic translation of the *Almagest* (completed in 827-8), translated the first by *iʿtibār*, the second by *tajriba* [2]. Isḥāq b. Ḥunayn, in his later translation, preferred to render σύγκρισις as *muqāyasa* (comparison), and for πεῖρα he wrote the conjunction : *al-miḥna wa'l-iʿtibār* [3]. Now the first word in this conjunction means trial, test or proof, and therefore, it ought to have been enough by itself to convey the meaning of πεῖρα. But to Isḥāq this must not have seemed so. There are two possibilities. He may have wished to use the two conjuncts as near synonyms, and in fact this is how they came to be used more or less generally in subsequent literature. But since Isḥāq himself used *iʿtibār* elsewhere in his translation as corresponding to σύγκρισις, it is likely to suppose that by joining this word to *miḥna* he was making an attempt to spell out an idea which clearly underlies many of Ptolemy's arguments, viz. the idea of performing a test or proof by means of a comparative examination of two sets of observations that are (preferably) separated by a long interval of time. The idea was in fact so strongly suggested in the *Almagest* that it became a guiding principle of astronomical research in Islam. In the second half of the ninth century al-Battānī claimed that he was following the example of Ptolemy who, he says, as well as treating astronomy as a mathematical science founded on arithmetical and geometrical demonstrations, " urged that *miḥna* and *iʿtibār* be made after him, saying that his own observations may after a long time need to be corrected just as he himself had to correct Hipparchus and others of his rank' [4]. These actual words do not occur in the *Almagest* (Hartner), but the thought they express is certainly not alien to it, and they repeat the conjunction we have found in Isḥāq's translation. Already in the first half of the ninth century, the astronomers of al-Maʾmūn had put the implicit recommendations of Ptolemy into practice, and on the basis of new observations had prepared the often quoted *Zīj al-mumtaḥan*, or Tested Tables [5]. (Note that *mumtaḥan* is a cognate of Isḥāq's *miḥna*.) In the work of the eleventh-century astronomer, al-Bīrūnī, there are unmistakable echoes of Ptolemy's concern for examining and improving previous observations ; and in one of his treatises he conjoins *iʿtibār* and *imtiḥān* [6]. Finally, Ibn al-Haytham himself used *iʿtibār* in an astronomical context, in a direct reference to Ptolemy ; he wrote in his *al-Shukūk ʿalā Baṭlamyūs* : " The movements of the planets asserted by (Ptolemy in the *Planetary Hypotheses*) are the same as those which he asserted in the *Almagest*, for he had proved them by observations and *iʿtibār*' [7]. (It may also be noted that the distinguished *Zīj* prepared in the twelfth century by ʿAbd al-Raḥmān al-Khāzinī bears the title *al-muʿtabar*, of which the most natural translation would be " the Tested Tables " [8].) I hope these examples will be enough to show that when, in the eleventh century, Ibn al-Haytham wrote the *Optics*, the term *iʿtibār* already had an established usage among researchers working in the tradition of Ptolemaic astronomy.

Scientists in Islām were traditionally divided into two main groups : the so-called naturalists (*ṭabīʿiyyūn*) and the mathematicians (*taʿlīmiyyūn*) Avicenna and Averroes were " naturalists " ; al-Battānī, Ibn al-Haytham

and al-Bīrūnī were " mathematicians ". A concrete illustration of what this division might imply can be obtained by comparing Ibn al-Haytham's *Optics* with the long discussion on vision which we find in the *De anima* of his contemporary Avicenna. As far as doctrine is concerned the two books share the same point of view : they both reject the visual-ray theory which had been upheld by mathematicians such as Euclid and Ptolemy, and they both adopt the " Aristotelian " view of vision as taking place through a form reaching the eye from the object. But the similarity would also end there. In structure and mode of treatment the two books could not be wider apart. For whereas Avicenna relies on Aristotle both for his view of the nature of light and for his general approach to the subject, it would be in vain to look in the works of Aristotle and Alexander for a paradigm of Ibn- al-Haytham's method of argumentation. A study of Ibn al-Haytham's and Avicenn'as terminology would be instructive, but here it will be enough to observe that Avicenna never uses *i ᶜtibār* as a technical term. The two authors, though contemporaries, obviously went to different schools.

To operate consciously and systematically with a concept of experiment *as a distinct method of proof*, and not merely to perform or refer to experiments, was no doubt a significant landmark in the history of experimental science. I have outlined an argument to the effect that the appearance of this concept in the work of Ibn al-Haytham was not a development within the framework of Aristotelianism, but a result of taking over into optics an idea which had had an established career in astronomy.

NOTES

(1) The problem has recently been discussed at length by Matthias Schramm in his *Ibn al-Haythams Weg zur Physik*, Wiesbaden, 1963, and in *History of Science*, vols. 2 (1963) and 4 (1965).

(2) Leiden University Library MS. Or. 680, fol. 104ᵛ.

(3) British Museum MS. Add. 7475, fol. 1ᵛ.

(4) *Al-Zīj al-Ṣābi ͻ*, ed. Nallino, p. 7 ; see also p. 209.

(5) The introduction to the Escorial MS. 927 states that the '*Zīj* of Ptolemy' was to be preferred to other *zijes* because of the strength of the mathematical demonstrations supporting it and because Ptolemy ' urged that observations be made after him... " (*wa-wajadnāhu ya ͻ muru bi'l-raṣdi ba ᶜ dahu...*).

(6) *Ifrād al-maqāl...*, in *Rasā ͻ il al-Bīrūnī*, Hyderabad 1948, p. 20.

(7) Bodleian Library MS. Arch. Seld. A.32, fol. 181ᵛ.

(8) British Museum MS. Or. 6669.

THE PHYSICAL AND THE MATHEMATICAL IN IBN AL-HAYTHAM' S THEORY OF LIGHT AND VISION

[Reproduced, with corrections and updated references, from *The Commemoration volume of Bīrūnī International Congress*, published by the High Council of Culture and Arts, Tehran, 1976, pp. 439-78. Originally delivered at the International Conference on History and Philosophy of Science, Jyväskylä, Finland, 1973. References to *The Optics of Ibn al-Haytham, Books I-III: On Direct Vision*, 2 vols, London, 1989, are by Book, chapter and paragraph numbers.]

The *Optics* of Ibn al-Haytham was one of the most remarkable achievements of Arabic science. Written in the first half of the eleventh century, it constituted by far the most important contribution to the study of light and vision in the period between Ptolemy and Kepler. Not only did this book represent a definite step forward from the point reached by optical research in Greek antiquity, but, presenting as it did a new approach to the study of light and a theory of vision which is not identifiable with any earlier theory, it can claim, with somewhat exaggerated modesty, to have produced a little revolution. This paper does not aim to describe the details of Ibn al-Haytham's doctrine of light and vision, nor to estimate the extent and nature of his influence, but rather to explore the meaning of a methodologically interesting aspect of his work. For, as frequently happens when new departures take place in scientific research, Ibn al-Haytham presented his results in conjunction with a methodological programme which, in his own words, aimed to "combine the physical and the mathematical sciences". What did he mean by this programme? In what sense was it new, and to what aspects of his work was it related? These are the questions with which the following remarks will be concerned.

I

The Preface to the *Optics* of Ibn al-Haytham shows him as being aware of having set out to do something new. Far from being naively optimistic, however, the Preface was couched in cautious, modest, and surprisingly sophisticated terms. Early investigators, he said, had spared no effort in their diligent inquiries into the manner of visual perception, but their views on the nature of vision had widely diverged. Many reasons were

responsible for this. Some of these were general: truth itself was obscure, human beings naturally suffered from a certain cloudiness of mind, methods of investigation varied, and the senses—our instruments for acquring knowledge—were not immune from error. "The path of investigation [was], therefore, obliterated, and the inquirer, however diligent, not infallible. Consequently, when inquiry concerns subtle matters, perplexity grows, views diverge, opinions vary, conclusions differ, and certainty becomes difficult to obtain" (*Optics*, I, 1[1]). Other difficulties were due to the obscurity and complexity of the subject itself.

Ibn al-Haytham distinguished mainly two doctrines of vision which he attributed to "physicists" and "mathematicians", respectively. The physicists' view was that "vision is effected by a form (*ṣūra*) which comes from the visible object to the eye and through which sight perceives the form (*ṣūra*) of the object". Mathematicians, on the other hand, had continued to disagree throughout the ages about the principles of optics, though they had devoted more attention to this subject than any other group. In general, however, "they agree that vision is effected by a ray which issues from the eye to the visible object and by means of which sight perceives the object; that this ray extends in straight lines whose extremities meet at the centre of the eye; and that each ray through which a visible object is perceived has as a whole the shape of a cone the vertex of which is the centre of the eye and the base is the surface of the object" (*Optics*, I, 1[3]).

The doctrines respectively held by the physicists and the mathematicians, he remarked, were clearly opposed to one another. "Now, for any two differing doctrines, it is either the case that one of them is true and the other false; or they are both false, the truth being other than either of them; or they both lead to one thing which is the truth." If the third possibility is the case, then "each of the groups holding those two doctrines would have failed to complete its inquiry and, unable to reach the end, has stopped short of it. Alternatively, one of them may have reached the end but the other has stopped short of it, thus giving rise to the apparent difference between the two doctrines, although the end would have been the same had the investigation been pushed further" (*Optics*, I, 1[5]).

It is my belief that, in Ibn al-Haytham's view, this third possibility was in fact the case, that he believed the doctrines of the physicists and the mathematicians to contain each a part of the truth which ought to be included in any final doctrine, and that he viewed his own achievement as a synthesis of the opposed doctrines described here. That this synthesis was not a mere juxtaposition will become clear later, but let us first read him describe the way in which he hoped to achieve his aim:

> "the nature of our subject being confused, in addition to the continued disagreement throughout the ages among its investigators, and because the manner of vision has not been ascertained, we thought it appropriate that we should direct our attention to this subject as much as we can, and seriously apply ourselves to it....We should, that is, recommence the inquiry into its principles and premisses, beginning our investigation with an inspection of the things that exist and a survey of the conditions of the objects of vision. We should distinguish the properties of particulars, and gather by induction what pertains to the eye when vision takes place and what is found in the manner of [visual] sensation to be uniform, unchanging, manifest and not subject to doubt. After which we should ascend in our inquiry, gradually and orderly, criticizing premisses and exercising caution in regard to conclusions—our aim in all that we make subject to induction and review being to employ justice, not to follow prejudice, and to take care in all that we judge and criticize that we seek the truth and not to be swayed by opinions. We may in this way eventually come to the truth that gratifies the heart, and gradually and carefully reach the end at which certainty appears; while through criticism and caution we may seize the truth that dispels disagreements and dissolves doubtful matters. For all that, we are not free from that human turbidity which is in the nature of man; but we must do our best with what we possess of human power. From God we derive support in all things" (*Optics*, I, 1[6]).

The book Ibn al-Haytham wanted to write, and did write, is not a compilation of views, observations, theorems and problems relating to light and vision; it is not an *opticae thesaurus*, though, ironically, this is the title by which it has been generally known [through misunderstanding of the title of the 1572 collective volume which contained the Latin translation of the *Optics*]. Nor was it Ibn al-Haytham's purpose to write a textbook, be it large and comprehensive, for the use of students. His was the ambitious aim of re-examining the fundamental propositions of the various optical theories which had been handed down to him, and his purpose was to reestablish the science of optics on a new basis. His book was thus a novel and systematic inquiry into the properties of light and the manner of vision, an inquiry which was to be conducted in accordance with the demands of both physics and mathematics as he understood them, and in which the conclusions would be carefully gleaned from no other source than the duly ascertained facts of observation.

With regard to the character of this inquiry, Ibn al-Haytham wrote:

> "our inquiry requires the combination of the natural and the mathematical sciences. It is dependent on the natural sciences because vision is one of the senses and these belong to the natural things. It is dependent on the mathematical sciences because sight perceives shape, position, magnitude, movement and rest, in addition to its being characterized by straight lines;

> but since it is the mathematical sciences that investigate these things, the inquiry into our subject truly combines the natural and the mathematical sciences" (*Optics*, I, 1[2].

We shall see that Ibn al-Haytham expressed himself on this subject in at least two more ways which differ from the above description and from each other. Here, in the *Optics*, it is sight or vision that is taken as the term of reference. *Vision*, we are told, is a natural faculty but the *objects of vision* have geometrical properties. Again, it is a *condition of vision* that only those objects are seen that lie on uninterrupted straight lines stretching from the eye, and this is a condition which involves the geometrical concept of "straight line". Therefore, "our subject", i.e. *vision*, must be both physical and mathematical. It is also with reference to their *doctrines of vision* that the distinction is made here between the physicists and the mathematicians. As we have seen, the former are identified as those who held the view that "forms" came from the object to the eye, the latter as holding the opposite doctrine of "visual rays". It would, therefore, appear that by "physicists" Ibn al-Haytham here meant Peripatetics, that by "mathematicians" he meant writers like Euclid and Ptolemy, and by "combining" physics and mathematics he meant a synthesis of elements drawn from the doctrines of vision held by those two groups. That he should be inclined to describe his programme in these terms in the *Optics* is quite natural, for it is in this book that we have the first *mathematical* theory of vision that is based on the Peripatetic doctrine of forms.

II

Before we examine this theory as it is presented in the *Optics* let us have a look at the other two formulations of the character of optical inquiry. The first occurs in a treatise on *The Halo and the Rainbow* which Ibn al-Haytham wrote about ten years before he died. The opening passage in this treatise reads as follows:

> "Everything whose nature (*ḥaqīqa*) is made subject of inquiry must be investigated in a manner (*naḥw*) conformable (*mujānis*) to its kind (*naw'*): if the thing is simple (*basīṭ*), then (it must be investigated) by a simple reasoning (*naẓar*), and if composite, then by a composite (*murakkab*) reasoning. Now among the things which men have aspired to know about, and which have given rise to much perplexity of thought, are the two effects known as the halo and the rainbow. These effects always exist in dense air and always maintain a regular shape. As for the halo, it always has the shape of a circle, unless something interferes with it that alters it. The rainbow, on the other hand, always has the shape of a segment (*qiṭ'a*) of a circle. Thus, since their subject is air, their investigation (*naẓar*) must be physical, and since their shape is round, they must also be investigated mathematically.

> That is why the inquiry (*naẓar*) by means of which the nature (*ḥaqīqa*) of these two effects is investigated comes to be composed of a physical and a mathematical (examination). Let us then investigate their nature (*ḥaqīqa*) in a manner (*naḥw*) which satisfies the requirements of physical things (*al-umūr al-ṭabī'iyya*), a manner which, moreover, accords with what is found to exist regarding these two effects" (quoted in *Optics I-III*, vol. ii, p. 4).

These words are best understood against the background of Aristotle's corresponding views. In a well-known passage in the *Physics* (*Phys*. II.2, 193b22–194a15) Aristotle asks the question wherein the mathematician differs from the physicist, seeing that physical bodies contain solids, planes, lengths and points, which are what the mathematician studies. It would be absurd, he says, if the physicist were required to know what the sun and moon, for example, are, while expecting him to leave out their shape to be studied by the mathematician. The physicist and the mathematician are, however, interested in shape (and in the other mathematical properties of natural bodies) in two different ways. The former considers shape as a boundary of natural objects, whereas the latter separates it in thought and considers it as multi-dimensional continuity. Optics, as one of the more physical of the mathematical sciences (*ta physikōtera tōn mathēmatōn*), well illustrates the opposite process to that of the abstractive mathematician. For geometry treats of a physical line but not *qua* physical, whereas optics considers a mathematical line not *qua* mathematical but *qua* physical. And since nature means form and matter, Aristotle went on, the physicist must examine both of these in his study of natural bodies. Having read these lines one may find it baffling to read in *Metaphysics* XIII.3 (1078a14-17) about optics and harmonics that "neither considers its objects *qua* sight (*opsis*) or *qua* sound, but *qua* lines and numbers." The puzzle seems to be connected with the idea of subordination, the relationship which, according to Aristotle, holds between arithmetic and geometry on the one hand, and harmonics, optics, astronomy and mechanics, on the other. It is only because optics falls under geometry, says Aristotle, that geometrical proofs are possible in it (cf. *Anal. Post.* I.7, 75b14–17; also I.9, 76a9–13, 22-25; I.13, 78b34-79a6). Thus while the student of optics is concerned with the behaviour of physical rays (whether of sight or light) he can apply geometry to them only by regarding them as straight lines. And the same consideration applies to the theory of the rainbow in relation to optics: "as optics is related to geometry, so is another science related to optics, namely the theory of the rainbow; it is for the physicist to know the fact as regards the rainbow, whereas to know the cause is for the student of optics, either simply as such or so far as he is concerned with the mathematics of it" (*Anal. Post.* I.13, 79a10-16, quoted by Sir Thomas Heath, *Mathematics in Aristotle*, Oxford, 1949, p. 12).

What is absent from these and similar formulations in Aristotle is any explicit use of the concept of "composition" which figures in all [three] of Ibn al-Haytham's characterizations of optical investigation. This, at least, is how Averroes saw the difference between the standpoints of Aristotle and Ibn al-Haytham with regard to the relationship of physics to mathematics (or optics) in the study of the phenomena of the halo and the rainbow. In his *Middle Commentary (Expositio media)* on Aristotle's *Meteorology*, Averroes argues that since these phenomena have physical bodies as their subject, but appear in those bodies with a definite shape and from a definite position, "their investigation must be, in one way, physical, in another, mathematical (*secundum unum modum Naturalis, secundum alium Mathematica*) (*Aristotelis Opera cum Averrois Commentariis*, Venetiis, 1562, repr. Frankfurt am Main, 1962, vol. V, 448v L).

This is close enough to Ibn al-Haytham's statement in his treatise *On the Halo and the Rainbow* which Averroes describes in the same *Commentary* as well-known (*tractatu famoso*) (ibid., 451vE). Following Aristotle, however, Averroes goes on to insist on interpreting the dual character of this investigation in terms of the idea of subordination: when mathematical or optical theorems are employed in a physical study of the rainbow (as they are in Averroes' own treatment), they do not, so to speak, occur on the same level as the physical propositions involved; they are rather used as postulates or hypotheses (*tanquam suppositionibus et fundamentis positis: 'alā jihat al-muṣādara wa al-aṣl al-mawḍū'*) that serve as principles of demonstration. Mathematics and physics remain distinct modes of inquiry, though the former may provide the causes of the facts ascertained in the latter. Averroes then concludes by directly condemning Ibn al-Haytham for having described his procedure as a "combination" of mathematical and physical inquiries: "*Et ille quod congregavit has duas speculationes iam erravit, ut fecit Avenetan; nam speculatio de hoc est duarum diversarum artium. Neque ingreditur id quod declaratum est de hoc ex scientia perspectiva in hac scientia, ita quod haec scientia considerat de his causis alio modo et facit ipsas principia demonstrationis* (ibid., 451vE)".

Averroes was a faithful follower of Aristotle who was extremely sensitive to any departure from the teaching of the master, however small. But just how significant is the difference he notes here between the accounts of Aristotle and Ibn al-Haytham of their study of the rainbow? Does Ibn al-Haytham's account in terms of "combination" represent a serious departure from the Aristotelian analysis in terms of subordination? To answer this question it must be noted that Ibn al-Haytham was not offering a different sort of explanation of the rainbow from the one previously attempted in Aristotle's *Meteorology*. For Aristotle's treatment was no less mathematical in character than that of Ibn al-Haytham, and both treatments were conducted in accordance with the language and conventions of the geometrical optics of Euclid and Ptolemy, being entirely formulated in terms of visual rays emanating from the eye. If a difference existed between them, it would be a difference in their interpretation of what they were doing, not in the *manner* of their actual

treatment of the rainbow. Now, as interpretation of the physico-mathematical study of the rainbow, I do not find Ibn al-Haytham's account significantly different from that of Aristotle. It seems to me that what Ibn al-Haytham wanted to describe in terms of composition could well be described in terms of subordination. In fact I find the Aristotelian analysis in terms of subordination more illuminating. For, as well as indicating the heterogeneous character of the study of the rainbow, it tells us something about how the mathematical and the physical propositions are related in such a study. Compared to Aristotle's attempt at some sort of logical analysis, Ibn al-Haytham's abrupt assertion about the composite nature of his inquiry looks somewhat opaque. He appears to be satisfied with making an observation which, for Aristotle, posed a problem in need of clarification.

III

The last statement of Ibn al-Haytham's view of the respective roles of physics and mathematics in the study of optics comes from his *Discourse on Light*, a summary account of his doctrine of light which he wrote after the *Optics*.

The statement constitutes the first paragraph of the *Discourse:*

> "Discussion of the nature (*māhiyya*) of light belongs to the natural sciences, and the discussion of the manner (*kayfiyya*) of radiation of light depends (*muḥtāj*) upon the mathematical sciences on account of the lines on which the lights extend. Again, discussion of the nature of the ray belongs to the natural sciences, and discussion of its shape (*shakl*) and structure (*hay'a*) belongs to the mathematical sciences. And similarly with regard to the transparent bodies through which the lights pass: discussion of the nature of their transparency belongs to the natural sciences, and discussion of how (*kayfiyya*) light extends through them belongs to the mathematical sciences. Therefore, the discussion of light and of the ray and of transparency must be composed of (*yajibu an yakūna murakkaban min*) the natural and the mathematical sciences (quoted in *Optics I-III*, vol. ii, p. 5).

Thus, for the third time, we have the idea of "composition", but the terms in which the distinction is made between physics and mathematics differ from those employed in the *Optics* and in *The Halo and the Rainbow*. There is no reference here to vision, and this can be explained simply by the fact that the *Discourse*, as distinguished from the *Optics*, is solely concerned with light. Nor is the compositeness of inquiry derived here from the compositeness of its object, the latter being in one respect physical and in another mathematical. Physics and mathematics are not

distinguished here by being assigned the different roles of studying different aspects of the same object, but rather by the *different kinds of questions* for which they provide answers. Physics, we are told, is concerned with essences, whereas mathematics is concerned with behaviour. In consequence, a physical doctrine is about the *nature* of, for example, light and transparency, whereas a mathematical doctrine governs the *manner* in which light behaves in transparent bodies. A complete investigation must therefore "combine" the physical and the mathematical sciences. But this time combination cannot be interpreted in terms of subordination. For, in the Aristotelian analysis of optics, physics and mathematics are said to provide "the fact" and "the reason" respectively, and are thus related to one another as the That and the Why; here they are related as the What and the How respectively, and as such they appear to stand, so to speak, alongside one another, rather than one above the other. The question now is: what, if any, is the effect of one on the other?

At this point it will be helpful to compare Ibn al-Haytham's views on the relation of physics to mathematics in the study of astronomy. Mathematical astronomy meant for him an inquiry whose aim was to show how the apparently irregular motions of the planets were brought about by the combination of uniform circular motions. Such an inquiry was mathematical inasmuch as it was conducted solely in terms of abstract points and circles, thus admitting the direct application of geometrical and numerical procedures. On the other hand, physics, which Ibn al-Haytham was deeply concerned to preserve along with the mathematical theory of the *Almagest,* consisted of a body of propositions about the *nature* of the heavens. The propositions were remarkably few in number: that a natural body cannot of itself have more than one natural movement; that a natural body cannot have a non-uniform movement; that the body of the heavens is impassible; and that the void does not exist. It followed that every one of the motions assumed in the mathematical theory must be understood as existing in a solid spherical body which rotated with uniform speed in its own place about its own diameter. This requirement and the physical propositions from which it derived constituted a testing ground for the mathematical hypotheses: if a mathematical hypothesis *could not* be verified when translated into physical terms, then it was to be rejected. It was by this method of testing by translation to the physical realm that Ibn al-Haytham was led to reject some of the hypotheses expounded in the *Almagest* (cf. *al-Shukūk 'alā Baṭlamyūs*, ed. A. I. Sabra and N. Shehaby, Cairo, 1971, pp. 15-19; and the chapter "An eleventh-century refutation of Ptolemy's planetary theory" in this volume). The equant hypothesis, for example, was false because it implied that the deferent sphere moved with non-uniform speed, thus violating a fundamental *physical* principle. A hypothesis was not to be accepted simply because it saved the phenomena.

No such jurisdiction is assigned to the "physical" doctrine of light, either in the *Discourse* or in the *Optics.* In neither of these works does the "mathematical" study of the behaviour of light seem inhibited by the "physical" assertions. Like those making up the physical doctrine of the heavens, these assertions are very few: that light is an esential form in self-luminous bodies; that it is an accidental form in bodies that derive their light from outside; that transparency is an essential form in virtue of which transparent bodies transmit light; and that a ray of light is an essential form which extends rectilinearly in transparent bodies. The answer to the question as to what "essential form" means is: "Every thing [or property: *ma'nā*] which exists in a natural body and is one of the things that constitute the essence (*māhiyya*) of that body is called an essential form (*ṣūra jawhariyya*), because the essence (*jawhar*) of every body is constituted of the totality of those things that inhere in that body and that are inseparable from it as long as its essence (*jawhar*) remains unchanged" (*Discourse*, in *Majmū' Rasā'il Ibn al-Haytham,* Hyderabad, Dn., A.H. 1357, no. 2, p. 2). Obviously these statements tell us no more than that some natural bodies are of themselves luminous, whereas others only shine when irradiated by an outside source, that some bodies (called transparent) transmit light, whereas others (called opaque) do not, and that light extends rectilinearly in transparent bodies. Far from being jurisdictional, or justificatory, the "physical" doctrine of light is merely descriptive and classificatory. As we shall see, however, this does not mean that the part it played was negligible.

IV

In an autobiographical account written [perhaps] at the age of sixty, Ibn Haytham tells us that early on in his intellectual career he became convinced that true knowledge was to be obtained only in "doctrines whose matter was sensible and whose form was rational" (cf. *Optics I-III,* vol. ii, p. xiii, n.12), and he adds that such doctrines were exemplified in the writings of Aristotle. While it would be wrong, I think, to deny or minimize the influence of Aristotle on him, it would be more seriously incorrect to regard this influence as crucially formative or as the key to the understanding of his own work. Ibn al-Haytham worked mainly in the tradition of Greek mathematicians, not in the tradition of Aristotle and his followers. With the exception of his study of the rainbow, the true exemplars of his optical (and astronomical) researches are to be found in the writings of Ptolemy, not in those of Aristotle. The paradigm for Ibn al-Haytham's *Optics* was clearly the *Optics* of Ptolemy, and it was the example of the latter's method of inquiry more than any Aristolelian theory of science or method which guided his investigations.

When Ptolemy's *Optics* was translated into Arabic it lacked the first Book in which, it is gathered, Ptolemy had expounded a theory of the radiation of light [as convincingly argued in A. Lejeune, *Euclide et Ptolémée, deux stades de l'optique géométrique grecque*, Louvain, 1948]. This historical accident had rather important consequences. It meant that

later students of the subject had to add to the body of Ptolemaic ideas a theory of light propagation which it was their own task to discover. This was what in fact happened in Islam when Ibn al-Haytham formulated such a theory in Chapter 3 of Book I of his *Optics*. When this book was itself translated into Latin, probably in the beginning of the thirteenth century, it suffered the same fate as its Greek predecessor: it, too, lacked the first three chapters. And, as a result, the Latin writers were again faced with the task of constructing their own doctrine of propagation, which they did by developing the theory of "multiplication of species". It would be interesting to compare the efforts of Ibn al-Haytham and of the later European philosophers in dealing with one and the same problem-situation (and I believe the history of Latin medieval optics could profitably be explored from this angle). Here, however, I shall only describe Ibn al-Haytham's procedure in Chapter 3 of Book I as an illustration of what he meant by a "mathematical" study of optical phenomena.

The propositions set forth in that chapter are generally concerned with rectilinear propagation as a property of all kinds of light and colour, whether they emanate from self-luminous objects or the atmosphere or an illuminated opaque body, or whether they are reflected or refracted. They include the important principle (which I propose to call "Alhazen's principle"), stating that light and colour emanate in straight lines *from every point* on the surface of a shining body (whether or not it is self-luminous) in all directions.

The most striking thing about Ibn al-Haytham's arguments in support of these propositions is that they consistently adhere to a regular pattern. A proposition about the manner of radiation of light (or colour) is first stated with reference to certain observations indicating this manner of radiation. A series of experiments are then described in which the lights from various sources (the sun, the moon, the atmosphere, opaque bodies) are subjected to examination under certain, artificially constructed, conditions. The light, for example, may be made to pass through an aperture in the wall of a darkened chamber with the help of a stretched string or by intercepting its path with an opaque object at various points. The result of this examination under control "proves" the initial proposition. And since tests have been performed on all known kinds of light, the proposition is then stated universally. In some cases, more elaborate experiments are possible, and these result in establishing the proposition in question "necessarily and beyond doubt". No propositions are asserted unless they are obtained by inductive generalizations from the uniform results of observations and experiments. The resemblance [between this pattern and] the style of Newton's *Opticks* is unmistakable.

In this systematic application of the articulated concepts of observation, experiment and induction, the appearance of experiment as an identifiable methodological tool with a separate function, comes here as something of a surprise. Not that experiments similar to those of Ibn al-Haytham had never been performed and described before. But I think it would be true to say that his *Optics* is the first work in which a concept of control experiment, as distinguished from the Aristotelian (and Galenic) *empeiria*, first emerges. By this I mean that we have in this book a name (*i'tibār*) which is exclusively applied to a procedure of testing that is distinct from mere repetition of observations. We also have a verb (*i'tabara*) and a *nomen agentis* (*mu'tabir*), both of which derive from the same root as *i'tibār*. In the Latin translation of the *Optics* the three cognate terms are rendered as *experimentatio, experimentare* and *experimentator*, respectively.

But although Ibn al-Haytham systematically operates with a concept of experiment, he nowhere formulates a *theory of experiment*. It is remarkable that in the Preface to the *Optics*, where he describes the limited forms of argument which his subsequent inquiry would utilize, he mentions "inspection of particulars" and induction (*istiqrā'*), but not *i'tibār*. This seems to support an argument I outlined elsewhere, that, rather than having invented an entirely new form of proof, he had taken over into optics a concept of testing which was often applied in astronomy and associated in this field with the term *i'tibār* (see the chapter "The astronomical origin of Ibn al-Haytham's concept of experiment" in this volume). Some transformation was bound to take place in the process of this transfer. Thus, whereas testing in astronomy consisted in comparing sets of observations (*i'tibār* translated the Greek *synkrisis* and, hence, testing, *peira*, by means of comparison), it was possible in optics to construct and manipulate apparatuses. But the idea of proof [as the chief goal of such an examination] remained, and, as a result, the experiments in the *Optics* are as a rule designed to prove what is suggested by observation rather than lead to the discovery of new properties.

V

The "physical" question "What is light?" is nowhere asked or explicitly answered in the *Optics*. However, expressions like "essential light" and "accidental light" are readily used in it in accordance with the meaning given to them in the doctrine outlined in the *Discourse* and attributed to natural philosophers. Thus the light in self-luminous bodies is called "essential", whereas the light "fixed" in transparent or opaque bodies when illuminated by a shining object is called "accidental". By themselves, these terms do not alter the phenomenological or "mathematical" inquiry into the behaviour of light. Sometimes, however, the underlying physical doctrine comes to the surface. The very existence of the category of "accidental" light is an illustration of this.

We may represent Ibn al-Haytham's thought as follows: It is observed that opaque bodies, like stones, do not behave like mirrors; that is, they do not reflect the light falling upon them in a determinate direction. They rather behave like self-luminous bodies in that the light emanates from every point on their surfaces in all directions. And yet, they are not self-luminous: they shine and are visible only as long as they are irradiated by

a luminous source. To account for this empirical observation, Ibn al-Haytham introduces the concept of "accidental light": opaque bodies take possession of the light shining upon them and having made it their own, they in turn shine as if they were self-luminous. In this they differ from mirrors which, owing to their smoothness, simply *push back* the light impinging upon them.

The trouble with this explanation is that it remains on the descriptive level; it only succeeds in stating the fact that illuminated opaque bodies behave like self-luminous bodies though they are not in fact self-luminous. But another alternative was open to Ibn al-Haytham. In his treatment of optical reflection in Book IV he makes a distinction between smooth and rough surfaces in mechanical terms. The parts of smooth surfaces are so closely packed together, and so evenly arranged, that their "resistance" causes the light striking them to be reflected in a specific direction. On the other hand, the parts of rough surfaces are neither compact nor similarly situated. Some of the light falling on them will therefore be dissipated within their gaps, and the rest will be thrown back in multiple directions. The account in terms of "accidental" light as a specific kind of light was a poor substitute for this clear picture in terms of mechanical reflection. Had he applied this picture consistently he would have done away altogether with "accidental light" as a distinct category.

Another illustration of the presence of the "physical doctrine" may be taken from the manner in which Ibn al-Haytham argues for his principle of point-by-point radiation. As usual, the argument starts on the observational level. The divergence of light from shining objects as it passes through small apertures shows that it issues from "every part" of the object and not from a particular part of it; and the same is already suggested by observations on the rising and setting sun and moon and their eclipses. The case of fire (a burning wick) allows an experimental examination with the help of a narrow sighting tube perpendicularly attached to one side of a large copper sheet in which a small circular hole has been made. An opaque object placed at the free end of the tube will show the light issuing from various parts of the shining object as the latter is slowly moved on the other side of the sheet close to the circular hole. The same thing is observed when the tube is made narrower or inclined to the surface of the sheet. It follows that "from every part of every self-luminous body, light radiates in every straight line that extends from that part" (*Optics*, I, 3[19]).

But is this also true of *all* parts, however small? Here is Ibn al-Haytham's argument for answering this question in the affirmative.

> "This property being manifest in the case of the larger parts of self-luminous bodies, their smaller parts—even when extremely small and as long as they preserve their form—must also be luminous; light will radiate from these parts in the same manner as it does from the larger ones, even though the conditions of smaller parts may be imperceptible. For this property is natural to self-luminous bodies and inseparable from their essence. Now small and large parts have the same nature as long as they preserve their form. Therefore, the property that belongs to their nature must exist in each part (whether small or large) provided that the part maintains its nature and form. Now the sun and the moon and the [other] heavenly bodies are not made up of aggregated parts; rather, each is a single continuous body whose nature is one and undifferentiated. Nor does one place in them differ in nature from another. Similarly, fire is not an aggregate of parts, but a continuous body; each place in it is similar in nature to the others, and the nature of its smaller parts is similar to that of the larger parts, as long as the smaller parts preserve the form of fire" (*Optics*, I, 3[20]).

Thus the analysis of the shining object into punctiform elements (as Vasco Ronchi has aptly described Ibn al-Haytham's principle), though based on empirical evidence, is ultimately supported by physical considerations involving the concepts of "nature", "essence" and "form". By virtue of these considerations Ibn al-Haytham is ultimately able to formulate his principle in terms of points rather than parts: light "issues from every point on the luminous body in every straight line that can be imagined to extend in the air from that point" (*Optics*, I, 3[21].

VI

As a result of concentrating on the meaning of the words "physics" and "mathematics" as they are actually used by Ibn al-Haytham, we have failed so far to refer to certain important passages in the *Optics* which, otherwise, would have figured prominently in a disussion of the mode of inquiry in this book. I have in mind the treatment of reflection and refraction in Books IV and VII where Ibn al-Haytham offers an experimental examination of these phenomena followed by a theoretical explanation involving the application of mathematics to the behaviour of light treated as a kind of motion. The prototype for this experimental investigation exists in Ptolemy's *Optics*. Ibn al-Haytham describes in much detail the construction of improved versions of instruments used by Ptolemy, and arrives by their means at formulations of the basic laws of reflection and some rules of refraction. The latter govern certain relations (inequalities) between the angles of incidence (made by the incident ray and the normal to the surface) and the refracted angles (contained between the refracted rays and the extension of the incident rays into the medium of refraction). He then offers an explanation or reason (*'illa*) for the manner in which reflection and refraction take place.

I shall not describe here the details of this explanation (interesting though they are) but only indicate some of its main features. It is based on comparing the behaviour of light to the motion of projectiles, and it

makes use of the parallelogram of motions to account for the deflection of light (and of projectiles) at resisting or yielding, reflecting or refracting surfaces. The comparison had been used in connection with reflection by writers on optics in antiquity, and the parallelogram representation of motion can also be found in the Aristotelian *Mechanica*, in Hero of Alexandria and, perhaps, even in Ptolemy. Ibn al-Haytham's application of the parallelogram method to the mechanical and optical situations was, however, much more complete and elaborate than can be found in any earlier writer. He analyzed the incident motion into two perpendicular "parts" or components which he examined separately in order to construct the paths of reflection and refraction. Part of the sophistication of his treatment derives from the fact that he was able to borrow and explore dynamical concepts, like the concepts of *i'timād* (*conatus*, inclination, impulsion) and *tawlīd* (engendering), which had already been developed in the Mu'tazilite school of *kalām*. (His explanation of the rebound of projectiles, for example, cannot be completely understood without reference to the concept of *tawlīd*, a fact which has so far escaped the notice of all his commentators, including the present writer [see A. I. Sabra, *Theories of Light from Descartes to Newton*, 2nd ed., Cambridge, 1981, pp. 69-78, 93-99].

It is certain that all these considerations made a strong impression on the founders of modern physics, to whom Ibn al-Haytham's major work was available in Risner's edition of the medieval Latin translation. Both Kepler and Descartes adopted the mechanical analogy together with the mathematical method of analyzing and composing motions in accordance with the parallelogram schema. And Descartes, in particular, was able to employ the parallelogram method in a physico-mathematical explanation of refraction which successfully yielded the sine law. Ibn al-Haytham's researches thus formed part of the history of this discovery which, in the work of Descartes, was meant to illustrate the power of a new approach to physical inquiry. Should we not, therefore, regard Ibn al-Haytham's treatment of reflection and refraction as the prime example of his own programme of combining physics and mathematics?

The answer to this question must, I believe, be negative. It is quite unlikely that he had his explanation of these phenomena in mind when he talked of combining the physical and the mathematical sciences. The explanation was quite untypical of what he did in the rest of the book, and, of course, there is no suggestion in Ibn al-Haytham's writings that light itself was mechanical in nature. What distinguished Descartes' use of mechanical comparisons from all similar comparisons in the works of his predecessors was that he meant them to exemplify a universal and thoroughgoing mechanical programme of explanation, and he accordingly offered descriptions of mechanical situations representing not only optical reflection and refraction but also rectilinear propagation and the production of colours. In Ibn al-Haytham's work the Peripatetico-mathematical programme and the mechanical picture suddenly appearing as explanations in Bks IV and VII are divorced from one another, with no link between them. The comparisons developed a line of thought which

had started in antiquity and later came into their own when, in the seventeeth century, they were incorporated into a new mechanistic philosophy. But they did not become paradigmatic for Ibn al-Haytham in the same way as they did later, and his project was conceived to deal with problems different from those which they [later] suggested.

VI

Ibn al-Haytham's theory of vision combines elements from three ancient doctrines: the doctrine of forms favoured by natural philosophers working in the Aristolelian tradition, the doctrine of visual rays adopted by mathematicians like Euclid and Ptolemy, and the doctrine of idols (*eidōla*) attributed to the Greek atomists. He himself viewed his own theory as a synthesis of the "physical" doctrine of forms and the "mathematical" approach of the visual-ray theory, and he nowhere refers to the atomists' idols. But there is some evidence to suggest that the latter played a part in bringing about the synthesis.

The Peripatetic doctrine, explicitly formulated by Alexander of Aphrodisias and ascribed by him to Aristotle (*De amina liber cum mantissa*, in *Comm. Arist. Graeca*, Suppl. 2, p. 141), went little beyond the assertion that we see an object when its form (*eidos*) reaches our eyes through the intervening transparent medium. As Ibn al-Haytham's Peripatetic contemporary, Avicenna, explained in his *De anima*, this did not mean that the form (*ṣūra*) was somehow torn off the object (for "no one has ever said that" ed. F. Rahman, Oxford, 1960, p.141), but that the eye received into itself a form similar to the form of the object. The role assigned to light in this process was either to turn the medium from a potentially to an actually transparent medium, or to actualize colour (the proper object of vision) by illumination. And there was debate as to whether the medium, by conveying the form to the eye, suffered any kind of alteration. It would be in vain, however, to look in Peripatetic writings for any clear explanation of the manner in which a form copying the visible features of an object came to be impressed in the eye.

The geometrical approach traditionally associated with the visual ray theory had certain obvious merits. It based the inquiry on considerations of lines and angles and allowed the application of deductive procedures. But it was a limited approach which reduced the eye to a geometrical point and thereby made a physiological account of vision irrelevant.

As far as the theory of idols is concerned there are passages in Avicenna which strongly suggest the possibility of an influence of that theory on Ibn al-Haytham. In one passage, for example, he ascribes to the proponents of idols or phantoms (*aṣḥāb al-ashbāḥ*, *auctoribus simulacris*) the view that "the phantom (*shabaḥ*, *simulacrum*) falls upon the section in the imaginary cone at the surface of the crystalline (eye lens) behind which the vertex (of the cone) lies" (ibid. p. 124; *Liber de anima I-II-III*, ed. S. Van Riet, Leiden, 1972, p. 227). Avicenna then criticizes in the same passage the explanation of the apparent variation in the size of an object by the angle subtended by the visual rays at the

centre of vision, saying that this explanation suited the theory of idols rather than that of visual rays. He thus distinguishes between the [atomists'] theory of idols on the one hand, and the theories of visual rays and of forms on the other. Now if, in place of the *shabaḥ* at the intersection of the crystalline surface with the visual cone, we put a *ṣūra*, which, like the idol, proceeds from the object, we get an essential part of Ibn al-Haytham's own theory of vision. But in this theory, the substitution involved the use of a principle [Alhazen's principle of point-by-point radiation] which has so far failed to appear in a similar role in any preceding treatment of vision.

The word "form" appears in the *Optics* without justification or explanation. The book begins by talking about "light" and "colour", but soon these words become interchangeable with "form of light" and "form of colour". This can be explained by the assumption that Ibn al-Haytham was simply following a well-estabished Peripatetic usage. In practice, however, the adoption of the vocabulary of forms does not seem to have meant for him anything other than that light and colour were real properties of physical objects, and his experimental *method of inquiry* was not in any way affected by it. But the effect of the theory of forms on his *doctrine of vision* was quite different. To adopt that theory meant subscribing to an intromission hypothesis, distinct from the extramission hypothesis of the visual-ray theory, and it implied the rejection of the view that something corporeal (a material effluence or a stream of atoms, for example) actually flowed from the visible object. This rejection is implicit in the *Optics*; there are no arguments in this book against views such as those attributed to Empedocles or to the Greek atomists. But that vision occurs as a result of an effect of light and colour on the eye is a doctrine which Ibn al-Haytham holds only on the basis of a series of observations and experiments to which he devoted a whole chapter. To be sure, some of these observations (the pain in the eye caused by gazing at the sun, the temporary loss of vision, the after-images, etc.) can be found in the writings of Peripatetics. But Ibn al-Haytham's manner of arguing was quite different. Whereas Avicenna, for example, would establish the intromission doctrine by raising numerous and [in his view] insurmountable *shukūk* or *aporiai* against all possible variants of rival theories, Ibn al-Haytham *directly infers* the same doctrine from the observational evidence he has meticulously described and carefully arranged.

Ibn al-Haytham finally asserts, then, that "the eye senses the light and colour that are in the surface of a visible object through the form which reaches it from that light and colour", and he does not fail to remind us that "this notion is the settled view of physicists regarding the manner of vision" (*Optics*, I, 1[6]). But whereas "physicists" had been satisfied with describing vision as the perception of forms, this description was, for Ibn al-Haytham, "nothing unless something more was added to it" (ibid.). This "something more" was an explanation of how the forms entered the eye.

And since the explanation, being based on the principle of point radiation, was in terms of rectilinear rays which could be treated geometrically, the result was truly a combination of the physical and the mathematical sciences.

Let me try to make this account a little more concrete without going into the details of Ibn al-Haytham's theory of vision; it will be enough to see how he guided the form of an object to the anterior surface of the crystalline lens, for, according to his theory, what happens beyond that surface largely falls within the domain of psychology.

It is clear from Ibn al-Haytham's schematic description of the eye (Bk I, ch. 5) that he was mainly concerned with the geometry of its construction. The uveal sphere, containing, in this order, the albugineous, crystalline and vitreous humours, is eccentric to the centre of the eyeball, being displaced forward towards the eye-surface. The cornea (or the part of it in front of the pupil) and the anterior surface of the crystalline are, however, concentric with the centre of the eye. A straight line goes through the middle of the pupil, the uveal centre and the centre of the eye towards the middle of the optic nerve, somewhat like a fixed line of apsides. It follows that straight lines drawn from the eye's centre through various points in the pupil will all be perpendicular to the crystalline's surface and to the cornea. The outward extensions of these lines produce the visual cone of the ancient geometrical theory.

Now imagine a shining object within the visual cone. Light and colour (or their forms) will issue from every point on the surface of that object rectilinearly in all directions. Some of these "rays" will fall perpendicularly on the cornea and, having passed through the albugineous humour, will again strike the crystalline surface at right angles. The points of intersection of these rays with the crystalline's surface will have a one-to-one correspondence with all points on the surface of the object. The mosaic of *point forms* gathered on the crystalline's surface will thus be a *total form* representing the whole "form of the object".

As for the forms which arrive at the front surface of the crystalline after having been refracted upon entering the cornea, their effect is eliminated by force of a hypothesis regarding the directional sensitivity of the crystalline humour: only those forms are sensed which pass through the crystalline surface on lines that make up the visual cone. In other words, the crystalline *as a sensitive body* is not capable of sensing the light and colour that pass along any other lines. To wipe off the *ad hoc* character of this explanation, Ibn al-Haytham points to parallel situations [in the natural world]: heavy bodies naturally fall only in straight lines directed towards the centre of the earth; and light [left to itself] only extends in straight lines; similarly [he maintained] there exist certain privileged directions in the lens *as a sensitive body*. It is, therefore, important to note that the form which Ibn al-Haytham succeeds in constructing inside the eye is, to use Kepler's apt distinction (in *ad Vitellionem Paralipomena*, Frankfurt, 1604, p. 193) an *ens rationale* [a

mental entity] and not an [actual] *pictura*. Unlike the image cast by a pinhole camera or a lens camera, and unlike the impression made by a material *eidōlon*, it is apparent only to the perceptive faculty and cannot be viewed otherwise.

What, then, becomes of the doctrine of visual rays in consequence of the above account? Ibn al-Haytham is anxious to show us both what is sound and what is unsound in it. He argues as follows: If vision occurred only through something that issued from the eye, then that thing would either be a body or not. If it were a body, then when we open our eyes, a corporeal substance would flow out of them and fill the entire space facing them. "But this is quite impossible and quite absurd" (*Optics*, I, 1[56]). If, on the other hand, the thing that issued from the eye were not corporeal, then it would not be capable of perceiving the visible object, "for sensation belongs only to animate bodies" (ibid.), and the function of that thing would be rather to convey something of the object to the eye in which perception would take place.

> "And since it has been shown that the air and the transparent bodies receive the form of the visible object and convey it to the eye and to every body opposite the object, then that which is thought to convey to the eye something of the visible object is the air and the transparent bodies placed between the eye and the object. But if the air and the transparent bodies convey to the eye something of the visible object at all times and in any event (provided that the eye faces the object) without the need for something that issues forth from the eye, then the reason that led those who hold the doctrine of the ray to maintain their doctrine ceases to exist. For they were led to assert that doctrine by their belief that vision is effected only through something that extends between the eye and the object for the purpose of conveying something of the object to the eye. But if the air and the transparent bodies placed between the eye and the object convey to the eye something of the object without the need for anything to issue from the eye; and, moreover, if these bodies extend [all the way] between the eye and the object; then the need to affirm the existence of anything else through which something is conveyed to the eye no longer exists, and there no longer exists a reason for their saying that a conjectured entity conveys to the eye something of the object. And if no reason remains for maintaining the doctrine of the ray, then this doctrine is invalidated" (*Optics*, I, 1[58]).

What is *not* invalidated and is in fact preserved as an essential feature of the new theory is the concept of ray as an abstract line:

> "Moreover, all that mathematicians who hold the doctrine of the ray have used in their reasonings and demonstrations are imaginary lines which they call 'lines of the ray'. And we have shown that the eye cannot perceive any visible objects except through these lines alone. Thus the view of those who take the radial lines to be imaginary lines is correct, and we have shown that vision is not effected without them. But the view of those who think that something issues from the eye other than the imaginary lines is impossible and we have shown its impossibility by the fact that it is not warranted by anything that exists, nor is there a reason for it nor an argument that supports it. It is therefore evident from all that we have shown that the eye senses the light and colour that are in the surface of the visible object only through the form of that light and colour, which [form] extends from the object to the eye in the intermediate transparent body; and that the eye does not perceive any of the forms reaching it except through the straight lines which are imagined to extend between the visible object and the centre of the eye and which are perpendicular to all surfaces of the coats of the eye. And that is what we wished to prove" (*Optics*, I, 1[59-60]).

And Ibn al-Haytham finally adds:

> "[the matter] which we have shown, I mean the manner of vision, accords with the view of the learned among physicists and with the generally accepted view of mathematicians. And it is now clear from [what we have shown] that the two groups are right and that the two doctrines are correct, mutually compatible and not contradictory. But neither [doctrine] is complete without the other..." (*Optics*, I, 1[61]).

I have quoted Ibn al-Haytham's words at some length because they support my statement at the beginning of this paper, that the synthesis he conceived and expressed in the *Optics* was primarily a synthesis of two doctrines of vision, each of which he believed to be partially true and by itself incomplete. To understand his aim it would not therefore be appropriate to interpret his programme of combining the physical and the mathematical sciences in terms of a modern concept of "mathematical physics". Such a concept would have been far removed from his thought, even though some of his explanations (and indeed some of those of his predecessors) might be capable of being analyzed by means of it. (By the way, what would be the Greek, Arabic or Latin for the expression "mathematical physics"?) On the other hand, by keeping close to his own understanding of "physics" and "mathematics" we put ourselves in a better position to appreciate the nature of the problem he faced, the extent of his success in dealing with this problem and the inevitable

limitation of the solution he offered. For though his theory of light and vision was indeed revolutionary, it would be difficult to overlook the conservative influence of the elements it purposely incorporated. It offered no view of the nature of light other than that already maintained by natural philosophers, and it preserved the mathematical model of the visual-ray theory. True, it purified the latter theory by treating the rays as strictly mathematical lines on which the light extended, and it reversed the direction of the rays on the basis of empirical evidence in support of the view held by natural philosophers, and to that extent the synthesis achieved by Ibn al-Haytham was not a mere juxtaposition of already existing doctrines. But his theory remained imprisoned in the geometry of the visual cone.

One of the most fundamental contributions of Ibn al-Haytham's theory was undoubtedly the explicit formulation and application of the principle of point radiation. He was well aware of its implications and he clearly understood that it lay at the basis of the formation of pinhole images. It is therefore curious that he never attempted to use this principle to show how a real image, a *pictura,* can be constructed inside the eye from all the rays emanating individually from all points on the visible object. Instead, he employed his principle to justify the visual-cone model by arguing that only those rays that made up the cone were efficacious in producing vision, and as part of this argument he was forced to supply a psychological explanation of the perception of the non-visible form. Thus rather than introduce a new geometrical model, the principle was made to serve the old one, which would seem to suggest that the visual-cone model had loomed too large and too vividly in Ibn al-Haytham's mind. Of course, a psychological explanation had to be brought in at one stage or another in the process of vision, but in Ibn al-Haytham's theory such an explanation comes too soon, at the crystalline lens, and the theory consequently leaves out, either as unobserved or as irrelevant, the empirical fact of a real image formed within the eye.

VIII

Ibn al-Haytham's Lemmas for Solving "Alhazen's Problem"

I

"Alhazen's problem",* or "*problema Alhaseni* (or *Alhazeni*)", is the name given by seventeenth-century mathematicians to a problem which they encountered in the *Optics* of AL-ḤASAN IBN AL-HAYTHAM. The *Optics*, composed in the first half of the eleventh century, had been translated into Latin in the late twelfth or early thirteenth century,[1] and an edition of it by FRIEDRICH RISNER had been published at Basel in 1572.[2] CHRISTIAAN HUYGENS formulated the problem as

* A shorter version of this paper was read at the annual meeting of the History of Science Society which took place in New York in December 1979. I am grateful to A. ANBOUBA, J. L. BERGGREN, J. P. HOGENDIJK and E. S. KENNEDY for comments, suggestions and corrections on all or part of this paper. All errors and shortcomings that remain are of course my own. The attached translation of IBN AL-HAYTHAM'S lemmas is part of a project involving an edition and English translation of the Arabic text of IBN AL-HAYTHAM'S *Optics* (*Kitāb al-Manāẓir*). I wish to thank the U.S. National Science Foundation and the National Endowment for the Humanities, for their support of this research.

[1] Neither the name of the translator(s), nor the place or exact date of the translation has been ascertained. Of the twenty odd manuscripts that have been located in European libraries, the earliest are from the thirteenth century, and one of these (the Edinburgh Royal Observatory MS CR3.3 = MS 9-11-3(20)) is dated 1269 (see D. C. LINDBERG, *A Catalogue of Medieval and Renaissance Optical Manuscripts*, Toronto: The Pontifical Institute of Medieval Studies, 1975, pp. 17–19). The earliest mention of the Latin version of the *Optics* in the West occurs in a work by JORDANUS DE NEMORE who flourished in the period between 1220 and 1230 (see MARSHALL CLAGETT, *Archimedes in the Middle Ages*. Vol. I: The Arabo-Latin Tradition, Madison: University of Wisconsin Press, 1964, pp. 668–9 and 674).

[2] *Opticae thesaurus. Alhazeni Arabis libri septem, nunc primum editi. Eiusdem liber De crepusculis et nubium ascensionibus. Item Vitellonis Thuringopoloni libri X. Omni instaurati, figuris illustrati et aucti, adiectis etiam in Alhazenum commentariis, a Federico Risnero.* Basel, 1572. (Reprinted, New York: Johnson Reprint Corporation, with a valuable Introduction by D. C. LINDBERG.) '*Opticae thesaurus*' is clearly the collective title of the whole volume and should not be cited as the title of ALHAZEN'S 'seven books,'

that of finding the point of reflection on the surface of a spherical mirror, convex or concave, given the two points related to one another as eye and visible object.[3] He had found IBN AL-HAYTHAM's treatment of the problem "too long and wearisome" (*longa admodum ac tediosa*),[4] and, armed with the tools of modern algebra and analytic geometry, he set out to produce a solution of his own—a task which he finally fulfilled to his own satisfaction in 1672, having proposed an earlier solution in 1669.

"Long and wearisome" though IBN AL-HAYTHAM's treatment may have been, it certainly represented one of the high achievements of Arabic geometry, and its importance for the history of mathematics in Europe down to the seventeenth century is easily recognizable. HUYGENS' brief and elegant solution was itself based on the same idea which IBN AL-HAYTHAM had used six hundred years earlier—the intersection of a circle and a hyperbola.

This paper is concerned with "Alhazen's problem" as it appears in IBN AL-HAYTHAM'S *Optics*. The problem of finding the reflection-point occurs in this book as part of a long series of investigations of specular images which occupy the whole of Book V, and these investigations in turn presuppose a theory of optical reflection which is expounded in Book IV. Much of the character of IBN

as is often done. The seven books were together known in the Middle Ages as *Perspectiva* or *De aspectibus*, the titles sometimes shown in the extant manuscripts. It may be interesting to note that when the emir (or admiral) EUGENE OF SICILY translated PTOLEMY's *Optics* from the Arabic into Latin in the twelfth century, he chose as the title the original Greek '*Optica*' rather than any Latin rendering of the Arabic '*al-manāzir*' (see *L'Optique de Claude Ptolémée dans la version latine d'après l'arabe de l'émir Eugène de Sicile*, édition critique et exégétique par ALBERT LEJEUNE, Louvain: Bibliothèque de l'Université, 1956). EUGENE, whose native tongue was Greek, had access to the Greek text of EUCLID's *Optica* which, like the works of PTOLEMY and IBN AL-HAYTHAM, was called in Arabic *Kitāb al-Manāzir*. On EUGENE see C. H. HASKINS, *Studies in the History of Medieval Science*, New York: Frederick Ungar Publishing Co., 2nd ed., republished 1960, pp. 171 ff.

[3] See *Oeuvres complètes de Christiaan Huygens*, vol. XX (Musique et Mathématique Musique. Mathématiques de 1666 à 1695), La Haye, 1940, pp. 207, 265–71, 272–81, 328, 329, and 330–33; see especially p. 265. In 1669 HUYGENS expressed the problem in optical terms: "Dato speculo sphaerico convexo aut cavo, datisque puncto visus et puncto rei visae, invenire in superficie speculi punctum reflexionis" (*ibid.*, p. 265). In 1672 the formulation became purely mathematical: "Dato circulo cujus centrum *A* radius *AD*, et punctis duobus *B*, *C*. Invenio punctum *H* in circumferentia circuli dati, unde ductae *HB*, *HC* faciant ad circumferentiam angulos aequales" (*ibid.*, p. 328; also vol. VII, pp. 187–9). See note 4 below.

[4] *Ibid.*, p. 330. ISAAC BARROW was another mathematician in the seventeenth century who was annoyed by the excessive length of IBN AL-HAYTHAM's solution. In Lecture IX of his *Lectiones XVIII cantabrigiae in scholis publicis habitae* (first published at London in 1669), he described IBN AL-HAYTHAM's demonstrations as "horribly prolix" (see p. 74). Neither HUYGENS nor BARROW was, however, concerned to explain the character (objectionable or otherwise) of IBN AL-HAYTHAM's method of solution. Their approach was that of mathematicians, not of historians of mathematics. See the relevant remarks by SABETAI UNGURU in his edition and English translation of *Witelonis Perspectivae liber Primus* (Studia Copernicana XV), Wrocław, *etc.*: Ossolineum (The Polish Academy of Sciences Press), 1977, pp. 209–12.

AL-HAYTHAM's treatment of reflection-points can only be appreciated if understood with reference to this wider context. It should also be mentioned that IBN AL-HAYTHAM's researches extended to cylindrical and conical as well as spherical mirrors. IBN AL-HAYTHAM was therefore aiming to solve a wider and more complex set of problems than "Alhazen's problem" in HUYGENS' limited sense. Here, however, I am only concerned to give an account of that aspect of IBN AL-HAYTHAM's treatment which can be directly related to HUYGENS' formulation, and to present a full translation of the six lemmas which IBN AL-HAYTHAM proposed for solving the problem in all its generality. The clarifications which I hope to make are intended to be part of a more comprehensive study.

The limited problem with which we shall be concerned is, therefore, that of finding the point of reflection on the surface of a spherical mirror. Let us begin with IBN AL-HAYTHAM's solution as applied to the case of a convex mirror.

A and *B* (in Fig. 1.1) are, respectively, the given locations of the eye and the visible point. *G* is the centre of the mirror with a radius *GD*, given in magnitude. The plane of the circle is that containing lines *AG*, *BG*; and it is proposed to find on the circumference of the circle a point *D*, such that *AD* and *DB* will make equal angles with the tangent at *D*.

IBN AL-HAYTHAM takes at random a line *MN* (Fig. 1.2), which he divides in a point *F*, such that

$$\frac{MF}{FN}=\frac{BG}{GA}.$$

From point *O* at the middle of *MN* he draws the perpendicular *OC*, on which he takes a point *C*, such that

$$\sphericalangle OCN=\frac{1}{2}\cdot AGB.$$

Then, and this is the crucial step, through *F*, he draws line *QFS*, cutting *NC* in *Q* and the extension of *CO* in *S*, so that

$$\frac{SQ}{QN}=\frac{BG}{GD}.$$

Now IBN AL-HAYTHAM shows, *before coming to this proposition*, that two such lines can be drawn through *F*, producing two unequal angles at *N*. He takes the case of the larger of the two angles and further assumes that angle *SNQ* is obtuse. (I have reversed the order of presentation to spare the reader some of the suspense, but I shall return to this crucial construction.)

Having made this assumption, the construction of Figure 1.1 proceeds as follows:

Draw *GD* at an angle *BGD* equal to *SQN*: this gives the position of *D* which is now to be shown to be the point of reflection of the light from *B* to *A*.

IBN AL-HAYTHAM continues as follows: He produces *GD* to *E* and draws line *ZDT* tangent to the circle at *D*.

He then draws *DK* at an angle *GDK* equal to angle *QNF* (Fig. 1.1), and *BR* perpendicular to the extension of *DK*. (He can do the latter because angle *GKD* is acute.)

He further extends *DR* to *I*, so that *IR* is equal to *RD*, and joins *BI*.

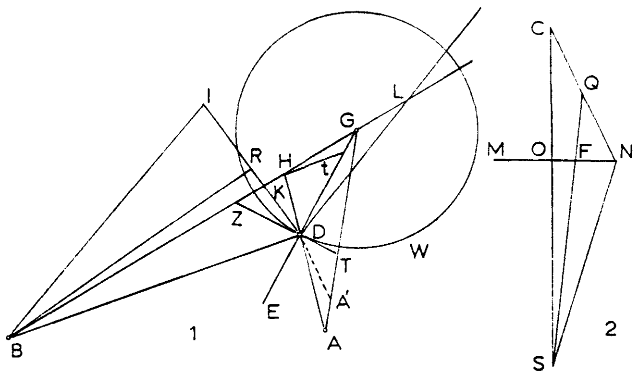

Fig. 1

Finally, he draws DL parallel to BI, constructs angle LDH equal to AGB, and draws Ht parallel to BD.

He proves that the extension of HD cuts GA at precisely point A, and finally deduces the equality of the angles made by AD and BD with DE, the normal to the tangent at D.

Figure 1 is not shown in the extant manuscripts of Book V of IBN AL-HAYTHAM's *Optics*. It is here constructed from the edited text of the *Optics*.[5] The inferred figure is essentially similar to the corresponding figures in KAMĀL AL-DĪN's commentary[6] and RISNER's edition of the medieval Latin translation, but is not identical with them.

I have deliberately added only one feature—the discontinuous line DA' as a *hypothetical* rectilinear extension of line HD. This merely simplifies the language of the proof without altering it in any other way.

Let us, then, say that HD produced cuts GA at point A'.

To prove that A' coincides with A, and, therefore, that HDA is a straight line, IBN AL-HAYTHAM has to show that GA' is equal to GA.

This he does by first considering triangles DHL and GHA', which are similar *by construction*, and this gives him:

$$\frac{DH}{DL} = \frac{HG}{GA'}.$$

Then he shows, again by consideration of similar triangles, that

$$\frac{DH}{DL} = \frac{HG}{GA}.$$

From which it follows that

$$GA' = GA.$$

[5] See below, n. 16.

[6] The "Commentary" by KAMĀL AL-DĪN AL-FĀRISĪ on IBN AL-HAYTHAM's *Kitāb al-Manāzir*, known as *Tanqīḥ al-Manāzir*, is believed to have been completed around A.D. 1300. KAMĀL AL-DĪN died in A.D. 1320. The *Tanqīḥ* has been published in an unsatisfactory edition in two volumes at Hyderabad, Dn. in 1928–1930.

(His proof involves taking HD as a mean proportional between BD and DL, *i.e.*

$$(1) \qquad \frac{BD}{DL} = \frac{BD}{HD} \cdot \frac{HD}{DL} \left(= \frac{BG}{GA} \right),$$

and HG as a mean proportional between BG and GA, *i.e.*

$$(2) \qquad \frac{BG}{GA} = \frac{BG}{HG} \cdot \frac{HG}{GA}.$$

Since

$$\frac{BD}{HD\,(= Ht)} = \frac{BG}{HG},$$

it follows, by substitution in (1), that

$$(3) \qquad \frac{DH}{DL} = \frac{HG}{GA}.)$$

Let us now return to the construction of the key figure on the right. IBN AL-HAYTHAM's chief contribution to the solution of this problem (and of the larger problem of finding the reflection-point or points on the surface of mirrors of other shapes) consists in the formulation and proof of six propositions or, as he properly calls them, lemmas (*muqaddamāt*)[7] which form the basis of his proofs. Except for elementary cases, some of which had been treated by PTOLEMY,[8] all constructions of reflection-points are presented by him as applications of these lemmas. In modern accounts of IBN AL-HAYTHAM's theory of optical reflection these lemmas are either ignored, cursorily dealt with, or re-formulated in modern terms. In what follows I shall try to keep as close as possible to IBN AL-HAYTHAM's procedure, my aim being largely to guide the reader through IBN AL-HAYTHAM's text.

Figure 2 is not from the *Optics*; it is a modern representation of two of IBN AL-HAYTHAM's lemmas, the first and the second. I have chosen to start with this figure because, being modern, it is quickly understandable, and it has the advantage (from the point of view of historical analysis) of being close to IBN AL-HAYTHAM's own figures. It is here reproduced, with some changes, from the important study published by M. NAẒĪF in 1943.[9]

[7] I write *muqaddamāt* (in the passive) and not *muqaddimāt*. A *muqaddama* is that part of a proof which is *put forward.*

[8] See ALBERT LEJEUNE, *Recherches sur la catoptrique grecque*, Brussels: Académie Royale de Belgique, 1957, pp. 71 ff.

[9] M. NAẒĪF, *al-Ḥasan ibn al-Haytham, buḥūthuhu wa kushūfuhu al-baṣariyya*, 2 vols., Cairo: Fouad I University, 1942–1943. This contains the best and most detailed study of IBN AL-HAYTHAM's treatment of the reflection-point(s) problem in any language; see vol. II, pp. 487–589. The two best historical accounts of "Alhazen's problem" in a European language are P. BODE, "Die Alhazensche Spiegelaufgabe in ihrer historischen Entwicklung ...", in *Jahresbericht des Physikalischen Vereins zu Frankfurt am Main*, for 1891–1892 (1893), pp. 63–107; and J. A. LOHNE, "Alhazens Spiegelproblem", in *Nordisk matematisk tidskrit*, 18 (1970), pp. 5–35 (with bibliography). For the transmission of IBN AL-HAYTHAM's problem to the Latin Middle Ages (in so far as it relates to conic

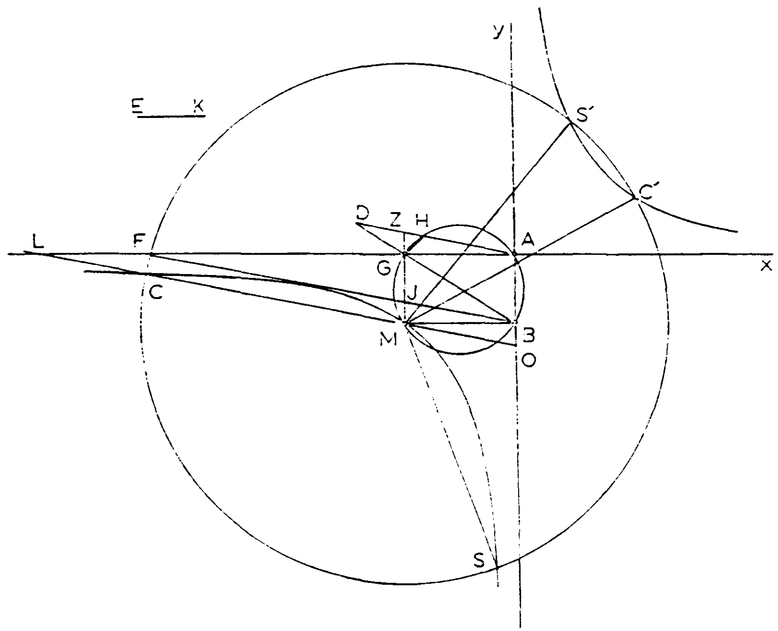

Fig. 2

We are given a point A on the circumference of a circle with diameter BG; and we are required to draw a line that cuts the circumference at a point, like H, and the diameter or its extension at another point, like D, such that DH equals a given line KE.[10]

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sections), see MARSHALL CLAGETT, *Archimedes in the Middle Ages*, vol. IV (A supplement on the medieval Latin traditions of conic sections, 1150–1566), Philadelphia: The American Philosophical Society, 1980, Chapter 1, pp. 3–31.

[10] Or, to phrase the problem differently, it is required to place between the diameter BG (or BG produced) and the circumference of the circle ABG a line equal to KE and verging towards the given point A. This is a particular case of the type of problem known to the Greeks as *neusis* (verging). PAPPUS, in his *Mathematical Collection*, presents several cases of the problem including that in which it is required to place a straight line of a given length between two straight lines given in position and verging towards a given point—a construction which, he tells us, the Greeks had ultimately solved by the use of conic sections. He himself shows a solution by means of the intersection of a hyperbola and a circle. The Greeks used the *neusis* as an intermediate step in the solution of the problem of trisecting an acute rectilineal angle. Their procedure appears to have become known to the Baghdad mathematicians of the ninth century, though not through direct translation of PAPPUS' text. J. P. HOGENDIJK sheds light on the transmission of this Greek method into Arabic, in "How trisections of the angle were transmitted from Greek to Islamic Geometry", *Historia Mathematica*, **8** (1981), pp. 417–38.

It may be noted further that Prop. 8 in the *Liber assumptorum* (attributed to ARCHIMEDES but found only in Arabic) assumes (without proof) a *neusis* construction in which a line segment of given length is to be placed between the circumference of a circle and the

Join AG, AB and produce them on both sides to form the rectangular axes x and y with A as origin.

Draw GM parallel to AB, and let it cut the circumference of the circle ABG in M.

Through M draw the hyperbola whose asymptotes are the two axes.

Then find the line MC whose product with KE is equal to the square of the diameter BG, *i.e.*

$$MC \cdot KE = \overline{BG}^2,$$

or

$$MC = \frac{\overline{BG}^2}{KE}.$$

The circle about M, with radius MC, will, in general, cut the two branches of the hyperbola in four points—let these be C, S, C', S'.

Join the lines MC, MS, MC', MS'.

Then each of the lines drawn from A parallel to these four lines will be the required line.

For example, line AHD, drawn parallel to MC cuts the circumference at H and the extension of the diameter BG at D, such that $DH = KE$.

In Figure 3 all four parallel lines are shown:

AH_2D_2, parallel to MS, cuts the circumference in H_2

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extension of the circle's diameter, such that the line segment verges towards a given point on the circle's circumference. Similar cases of *neusis* construction occur in ARCHIMEDES' work *On Spirals*, again without proofs. See T. L. HEATH, *The Works of Archimedes*, New York: Dover Publications, Inc. (reprint of 1912 edition), undated, Introduction, ch. V, pp. c–cxxii; *A History of Greek Mathematics*, vol. I (Oxford: The Clarendon Press), pp. 235–41; *A Manual of Greek Mathematics*, New York: Dover Publications, Inc. (reprint of the Oxford edition of 1931), pp. 147–52.

ABŪ SAHL AL-QŪHĪ, who flourished at Baghdad some fifty years before IBN AL-HAYTHAM died (see *Dictionary of Scientific Biography*, XI (1975), pp. 239–41), in a letter to ABŪ ISḤĀQ AL-ṢĀBI' (MS Ayasofya 4832, pp. 133^b–140^a, especially 138^a–139^a) assumes the solution of the following verging problem: to draw from a given point outside a given angle a line that cuts the sides of the angle, such that the intercept between these sides equals a given line. Instead of providing a proof AL-QŪHĪ simply says "We have shown how to do this in many places and it may often happen (*rubba-mā yattafiqu*) that we do not need [for this purpose] to resort to conic sections" (p. 138^b). (J. L. BERGGREN drew my attention to this passage.) It is known that IBN AL-HATHAM was acquainted with at least some of AL-QŪHĪ's works (see, for example, R. RASHED, "La construction de l'heptagone régulier par Ibn al-Haytham," *Journal for the History of Arabic Science*, **3** (1979), p. 341 (French), p. 228 (Arabic)). But the whole question of IBN AL-HAYTHAM's sources remains largely unexplored. That he was well versed in the methods of Greek higher mathematics is clear from several of his writings (including the *Optics*) and from the fact that he felt able to attempt a reconstruction of the lost book VIII of APOLLONIUS' *Conics*. This reconstruction, extant in a unique MS in Turkey (Manisa, Genel 1706, 1^b–25^b; see F. SEZGIN, *Geschichte des arabiscen Schrifttums*, V (Leiden: E. J. Brill, 1974), p. 140), and published in facsimile by NAZIM TERZIOĞLU as *Das achte Buch zu dem "Conica" des Appollonios von Perge*, rekonstruiert von Ibn al-Hayṯam, Istanbul, 1974, is being studied by J. HOGENDIJK of the University of Utrecht.

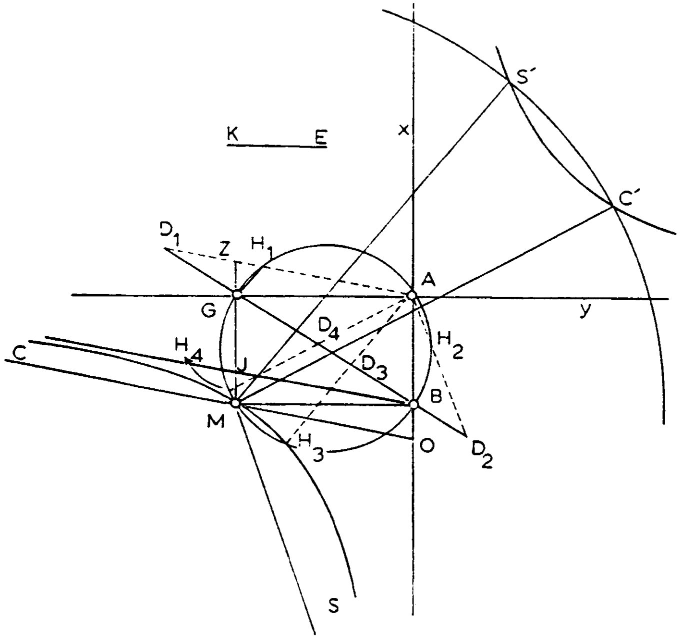

Fig. 3

and GB produced in D_2;

AD_3H_3, parallel to MS', cuts the circumference in H_3

and the diameter in D_3; and

AD_4H_4, parallel to MC', cuts the circumference in H_4 and

the diameter in D_4.

As in the case of AH_1D_1, the portion of each one of these lines between the circumference and the diameter is equal to the given line KE. That is H_2D_2, H_3D_3, H_4D_4 are each equal to KE.

The construction in Figure 3 therefore yields a general solution of our problem. But before we turn to Ibn al-Haytham's lemmas it should be noted that while the circle with radius MC will always cut the branch of the hyperbola through M in two points, three possibilities exist with regard to the other branch:

(a) the circle may cut it in two points, as in the figure (and this makes it possible to draw *two* lines satisfying the stated condition),

or

(b) the circle may touch that branch at one point (and this allows the construction of one line satisfying the stated condition),

or

(c) the circle may fall short of it altogether (and in this last case the required line cannot be constructed).

All this simply follows from the fact that the radius of the cutting circle, MC, is equal to $\overline{BG}^2/KE$ and therefore depends on KE.

With this picture in mind, Ibn al-Haytham's own procedure should now be easy to follow. As in all of his proofs, the problem is divided into particular cases which are taken up one by one. Figure 4 represents what I shall call case (*a*) in

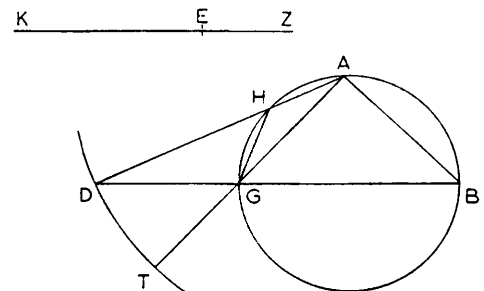

Fig. 4 = Lemma I

the first of the six lemmas: the given point A on the circumference of the circle having the diameter BG lies at the middle of the semi-circle BAG; and we are to draw a line, as AHD, cutting the circumference in H and the extension of BG (in this direction) in D, such that HD is equal to the given line KE.

KE is produced to Z such that

$$KZ \cdot ZE = \overline{AG}^2 \quad (KZ > AG)$$

and AT, equal to KZ, is drawn through G.

The circle about A with radius AT will cut the extension of the diameter BG, say at D,

and line AD will cut arc AG in H.

The required line is HD—which follows from the observation that triangles AGD, AHG are similar.

Case (*b*) in Lemma I is more complicated; it admits of three sub-cases (Fig. 5). The required line may be tangent to the circle at the given point A (as in 1), or it may cut the circle at a second point H which may lie on arc AG (as in 2), or on arc BA (as in 3).

Ibn al-Haytham provides proofs for all these cases, all based on the construction on the left. It is this construction which should now be described.

TN is a line taken at random.

Having drawn GZ (say in case 1) parallel to BA, the following angles and lines are then constructed:

$$\sphericalangle TNL = \sphericalangle DGA,$$

$$\sphericalangle TNM = \sphericalangle DGZ,$$

line MT // line LN,

and

line TQ // line MN.

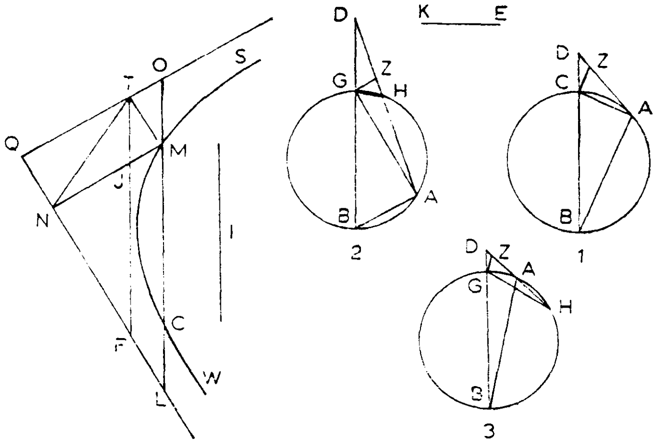

Fig. 5 = Lemma I

Referring to APOLLONIUS' *Conics*, Bk. II, Prop. 4, IBN AL-HAYTHAM then draws the branch of the hyperbola through M, with QT, QL as asymptotes (the similarity with Fig. 2 is apparent).

On the branch SMW, take a point C, such that

$$\frac{MC}{TN} = \frac{BG}{KE}.$$

Referring again to APOLLONIUS' *Conics*, Bk. II. Prop. 8, IBN AL-HAYTHAM states that the extension of MC on both sides will cut the asymptotes in points O and L, such that

$$OM = LC.$$

Draw TF parallel to OL, cutting NM in J.

Since surface $TMLF$ is a parallelogram, and so also is surface $TOMJ$, it follows that

$$MC = JF,$$

and therefore

$$\frac{JF}{TN} = \frac{BG}{KE}.$$

If AZ is now drawn at an angle

$$GAZ = NFT,$$

it will cut BG produced—say at D.

IBN AL-HAYTHAM shows, with reference to each of the three cases separately, that line AD will meet the cricumference at H and the extension of the diameter at D, such that $HD = KE$.

The difficulty with IBN AL-HAYTHAM'S approach, as compared with that of seventeenth-century mathematicians, becomes immediately apparent when we

note that Lemma I, consisting of four particular cases, is designed to take care of only one of the four lines in our reference Figure 3, namely line AD_1 which cuts the extension of the diameter BG on the side of G. IBN AL-HAYTHAM says nothing about line AD_2, cutting the extension of the diameter on the other side. But he provides a second lemma for the construction of lines AH_3, AH_4 which intersect the diameter itself. A brief look at this lemma will also be instructive.

In Figure 6, constructed from the text of Lemma II, A (in the right-hand figure) is the given point on the circumference of the circle with diameter BG; and we are to draw from A a line that cuts BG and the circumference in two points, such as E, D, so that DE is equal to the given line HZ.

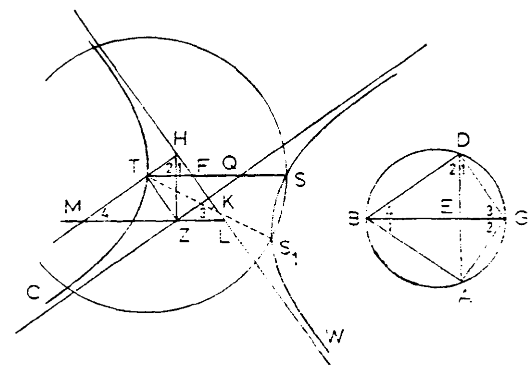

Fig. 6 = Lemma II

Having drawn AB, AG, IBN AL-HAYTHAM constructs angles $H1$ and $H2$ on either side of HZ, equal to angles $B1$ and $G2$ respectively. He completes the parallelogram $HKZT$, and draws through T the branch of the hyperbola with KH and KZ as asymptotes. Then, with T as centre and a radius equal to BG, he draws a circle that, according to his own explicit remarks, may or may not cut the opposite branch of the hyperbola. His text, however, is concerned with the case in which a meeting of the circle and that branch does take place, for example, at point S.

He joins TS, cutting the asymptotes at F and Q; and, through point Z, he draws LZM parallel to TS, and, like TS, cutting both asymptotes. LZM will cut the extension of HT, say in M. Finally, he draws GD at an angle with BG equal to MLH, and joins BD.

Considerations of the similar triangles indicated in the figure entail the equality of DE to the given line HZ.

The corresponding figures in RISNER and in KAMĀL AL-DĪN, inadequately and inexactly drawn, do not include the circle through S or the discontinuous line TS_1. This seems to reflect IBN AL-HAYTHAM'S remarks just referred to. He states that from T on one branch of the hyperbola, it may not be possible to draw more than one line that reaches the other branch. This, of course, would be the case when the circle touches that other branch at a point. He also notes that in some cases two such lines may be drawn (as in our Fig. 6), and, further, that for the

construction of the required line to be at all possible, it is necessary that BG, equal to the radius of the circle, must not, in his words, "be shorter than the shortest line that can be drawn from T to section SW".[11] As to the question of how this shortest line should be determined he refers the reader to Propositions 34 and 61 of Bk. V of the *Conics*—a correct reference which is omitted in RISNER.

So much for that part of IBN AL-HAYTHAM's proof. The next steps are not difficult to follow, but IBN AL-HAYTHAM's method of procedure remains the same. Lemmas III and VI are particular cases of one problem, and they establish their conclusions by reference to Lemmas I and II respectively.

Figures 7.1 and 7.2 are drawn from the text of Lemma III. In the triangle ABG, B is a right angle, and D a point given on BG (as in Fig. 7.1) or on its exten-

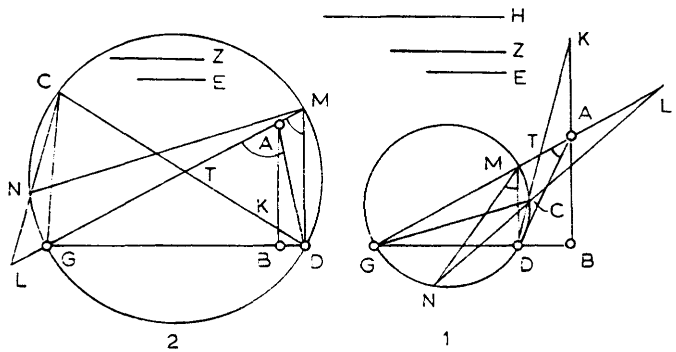

Fig. 7 = Lemma III

sion toward B (as in Fig. 7.2). It is required to draw from D a line that cuts the hypotenuse in a point, as T, and AB or its extension in another point, as K, such that

TK is to TG in a given ratio ($E:Z$).

From now on it will be easier to concentrate on Figure 7.1. Join AD; draw DM parallel to BA and describe the circle about the right-angled triangle MDG, which will have GM as diameter.

Construct angle DMN equal to angle DAG.

N will be on arc DG (Fig. 7.1), or on arc MG (Fig. 7.2).

Three more steps complete the figure. First, construct a line H, such that

$$\frac{AD}{H} = \frac{E}{Z} \text{(the given ratio)}.$$

Then, *applying Lemma I*, draw from N the line NCL, so that CL, the distance between the line's intersection with the circumference and the extension of diameter MG, is equal to H.

Now join DC and produce it in a straight line: it will cut LM, say in T. And join GC.

[11] See below, p. 318.

IBN AL-HAYTHAM shows that DT produced will cut BA produced (in Fig. 7.1) in a point K such that

$$\sphericalangle AKT = \sphericalangle TDM = \sphericalangle TGC.$$

Finally, from the similarity of triangles AKT and CGT, and also triangles LCT and ADT, it follows that

$$\frac{KT}{TG} = \frac{AT}{TC} = \frac{AD}{CL} = \frac{AD}{H} = \frac{E}{Z}, \quad \text{Q.E.F.}$$

The remaining case in this problem, represented by Lemma VI, relates to Figure 1.2, *i.e.* the auxiliary figure for the construction of the reflection-point on the surface of a spherical convex mirror.

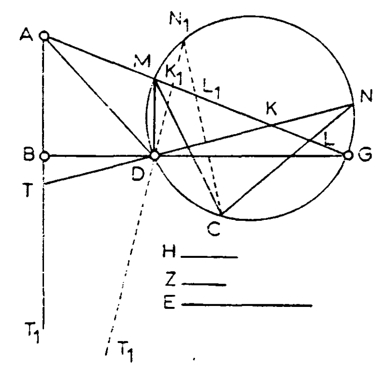

Fig. 8 = Lemma VI

Here (Fig. 8) from point D on side BG of the right-angled triangle ABG, we are to draw a line that cuts the hypotenuse in K and the extension of AB in T, such that

$$\frac{KT}{KG} = \frac{E}{Z} = \text{a given ratio}.$$

This IBN AL-HAYTHAM achieves on the basis of Lemma II which allows him to draw line CLN, cutting the diameter of the circle about MDG in L and the circumference in N, such that

$$LN = H,$$

where H is determined by

$$\frac{AD}{H} = \frac{E}{Z}, \text{the given ratio}.$$

We know, however, that it may be possible in this case to draw a second line, as CL_1N_1, which satisfies the stated condition, namely such that $L_1N_1 = H$. If that is the case, then, in addition to line $NKDT$, another line $N_1K_1DT_1$ can be drawn so that T_1K_1 is to K_1G as E is to Z. Again the figures in RISNER and

in KAMĀL AL-DĪN do not show the discontinuous lines in our figure. But IBN AL-HAYTHAM's text is explicit. This is what he says:

> "... it was shown earlier [i.e. in Lemma II] that there issue from point C two lines such that the segment of each of them that lies between the circle and the diameter [here segments LN and L_1N_1] will be equal to the given line [H]. Thus *if* two such lines are drawn from C, then there will issue from point D two lines in the given ratio; but the two angles produced at point G will be unequal ... [he means the angles made by TG or T_1G with AG]."[12]

This concluding comment is paraphrased in RISNER without the reference to the unequal angles at G.[13]

We come now to an important step in IBN AL-HAYTHAM's procedure, represented by Lemma IV.

In the plane of the circle with radius BG (Fig. 9.1), two points, say D and E, are given: and we are to find on the circumference of the circle a point A such that the tangent at A (AH in the figure) bisects the angle contained by AD and AE.

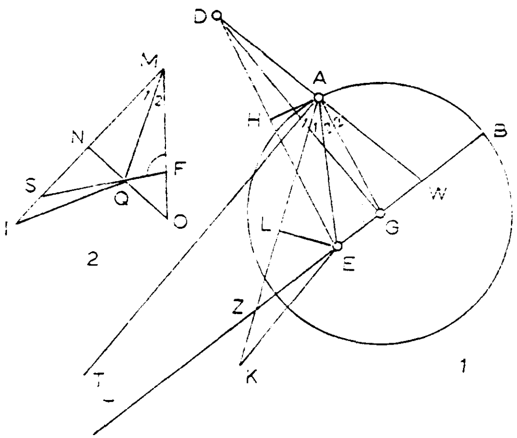

Fig. 9 = Lemma IV

Rather than summarize the proof, which *is* long, I shall be concerned to point out some features of it. The proof makes use of Figure 9.2 which is but case 2 of Lemma III (see Fig. 8.2), where from D on the extension of GB in the right triangle ABG, a line DKT is to be drawn, so that TK is to TG in a given ratio.

Similarly, to go back to Figure 9.2, SQF is drawn so that QF to FM is in a given ratio (—in this case, EG to GB in Fig. 9.1).

Now it is clear that the condition stated in this Lemma (that the tangent at A bisects angle DAE) is a particular case of a more general condition that can be stated by requiring that the tangent AH should make equal angles with AD and

[12] See below, p. 324. Emphasis added.

[13] *Opticae thesaurus. Alhazeni libri septem*, sec. 38, p. 150.

AE, without necessarily bisecting the angle contained by these two lines. Starting from this observation, NAẒĪF provides a generalized construction for Lemma IV that yields four points satisfying the more general condition.[14] This, in turn, yields a general solution of the problem of finding the reflection-point on the surface of a spherical concave mirror.

Figure 10 is an illustration of NAẒĪF's construction, where A and B are the positions of the eye and the visible object respectively, and P_1, P_2, P_3 and P_4 are reflection-points on the surface of the concave mirror with radius GM.

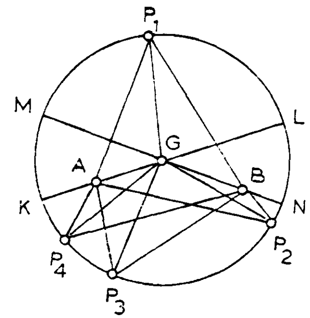

Fig. 10

NAẒĪF's construction is valid inasmuch as it is based on Lemmas III and VI which together comprize four possible cases. It does not, however, reflect IBN AL-HAYTHAM's intention, which (as NAẒĪF also points out)[15] is obviously to propose a particular construction (in which one of the two given points lies outside the circle) with a particular application in mind.

A similar observation applies to Lemma V. In Figure 11, E is a point given

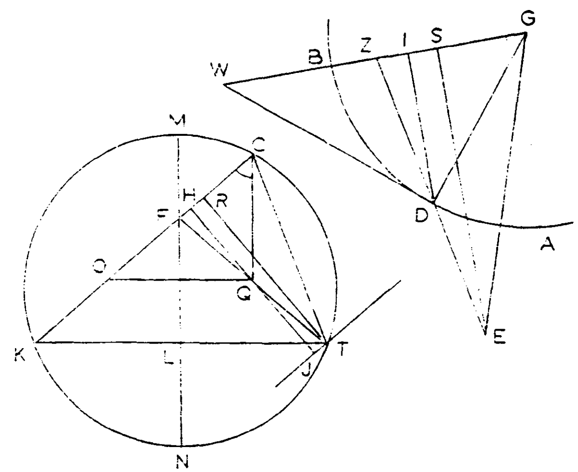

Fig. 11 = Lemma V

[14] NAẒĪF, *op. cit.*, vol. II, pp. 515–27.

[15] *Ibid.*, pp. 524–7.

outside the circle with radius BG; and it is required to draw from E a line that cuts the circumference in a point like D and the diameter in a point like Z, so that

$$DZ = ZG.$$

Having drawn the perpendicular ES, IBN AL-HAYTHAM takes a line $KT = ES$ on which he describes the segment of a circle that admits an angle equal to BGE. Then, having drawn the diameter MN through the middle of KT, he constructs line KFC, such that

$$FC = \frac{1}{2} BG.$$

This construction relies of course on Lemma II. But since the diameter MN is greater than the radius of the given circle BG, four lines can generally be drawn that satisfy the stated condition. However, IBN AL-HAYTHAM neither considers nor refers to any line other than KFC. Nor does he consider or refer to the case in which E lies inside the given circle.

So here again IBN AL-HAYTHAM is concerned with a particular case to be applied later to a particular construction.

This can be clearly illustrated by IBN AL-HAYTHAM's own construction for the reflection-point on the surface of a spherical convex mirror (Fig. 1). Here the conditions he lays down for drawing line SFQ (in particular, that angle SNQ must be obtuse) is equivalent to asserting that A and B (the two points related as object and eye) must be such that the line joining them neither cuts nor is tangent to the circle. If this condition does not obtain, no reflection from the convex side of the mirror will take place. (His investigation of this type of mirror is completed by a *reductio ad absurdum* proof that shows that no more than one reflection-point is possible.)

How, then, does IBN AL-HAYTHAM find the reflection-point (or points) on the surface of a spherical concave mirror? He enumerates the special cases and deals with them one by one. The two points related as object and eye may lie on the diameter of the mirror (or on its extension) at equal or unequal distances from the centre of the mirror. Or they may lie on different diameters, their distances from the centre being equal or unequal. IBN AL-HAYTHAM's piecemeal treatment of these cases, in which he applies his lemmas as required, makes for an even longer story than the one I have just summarized. But adding all these cases together we obtain a general solution of "Alhazen's problem" in HUYGENS' restricted sense. Long or not, this was an impressive achievement. But the historian's job is not completed before other investigations have been carried out. We still, for example, have to identify IBN AL-HAYTHAM's sources and find a detailed explanation for the character of his approach.

The preceding account had two limited aims: to give an accurate, though abbreviated, description of IBN AL-HAYTHAM'S procedure by providing exact figures that correspond to his own text, and to point out certain features of his proof that must be borne in mind in studying their character, their influence, and the reactions (and misunderstandings) they have given rise to. These two aims must be fully realized before we can put ourselves in a position to achieve an exact assessment of IBN AL-HAYTHAM's contribution, or make meaningful comparisons between his performance and that of later mathematicians.

II

Translation of Ibn al-Haytham's Lemmas[16]

[Lemma I: Figures 4 and 5]

Let circle ABG [Fig. 4], with diameter GB, be known [*ma*ʿ*lūma*]; let GB be produced on the side of G; let line KE be given [*mafrūḍ*] and let point A be given on the circumference of the circle. We wish to draw from A a line, as AHD, so that the part of it that lies between the diameter and the circle—such as HD—is equal to line KE.

Now arcs BA, AG are either equal to one another or not.

Let them be equal. We join lines BA, AG, and make the product of KZ and ZE equal to the square of AG. Line KZ will then be greater than line AG.

Draw AG and make AT equal to KZ;

with A as centre and with distance AT, draw an arc of a circle: it will always cut line GD—let it cut it at D.

Join AD: the line AD will be equal to line KZ.

AD will always cut arc AG, since the line drawn tangentially from A will be parallel to GB; for the line from point A joined to the circle's centre will be perpendicular to line GB, because of the equality of arcs AB, AG. Therefore line AD will cut arc AG—let it cut it at point H.

Join GH.

Angles AHG, ABG will together be equal to two right angles.

But angle ABG is equal to angle AGB;

therefore angle AHG is equal to angle AGD;

therefore triangle ADG is similar to triangle AGH.

It follows that the ratio of DA to AG is as the ratio of GA to AH, and, therefore, the product of DA and AH is equal to the square of AG.

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[16] The following translation is made from my (as yet unpublished) edition of the Arabic text in Book V of *Kitāb al-Manāẓir*. Book V survives in three MSS which are all preserved in Istanbul libraries: Fatih 3215, fols. 138^a–332^b, dated Jumādā II, 636/A.D. 1239; Ayasofya 2448, fols. 386^b–508^a, dated A.H. 869/A.D. 1464–1465; and Köprülü 952, fols 2^{a-b}, 74^a–81^b, 89^a–107^b, 134^a–135^b, dating probably from the 14th century A.D. All geometrical diagrams for Book V are missing from the Fatih and Ayasofya MSS. The Köprülü MS is incomplete but has the diagrams associated with the part of the text which it includes. I have made use of KAMĀL AL-DĪN's *Tanqīḥ* and of RISNER's edition of the medieval Latin version of *Kitāb al-Manāẓir*, both of which include the diagrams but not always accurately drawn.

In transliterating the Arabic I have used C for *ṣād*, J for *shīn* and t for *ṭā*'. All other transliterations are standard in recent literature.

But the product of KZ and ZE is equal to the square of AG;
therefore the product of DA and AH is equal to the product of KZ and ZE.
And DA is equal to KZ; therefore AH is equal to ZE.
It remains that line HD is equal to line KE.
And that is what we wished to do.

Now let arcs BA, AG be unequal [Fig. 5]. We join lines BA, AG, and draw GZ parallel to BA. Take a given line at random; let it be TN. Make angle TNL equal to angle DGA, and angle TNM equal to angle DGZ;
produce line LN on the side of N to Q, and draw line MT parallel to line NL;
further, draw line TQ parallel to NM, and produce QT on the side of T to O.
Then, through M, we draw the hyperbola of which lines OQ, QL are asymptotes (as has been shown in Proposition 4 in Book II of the *Conics* of Apollonius) — and let it be section SMW;
make the ratio of line I to line TN as the ratio of line BG to line KE;
draw in section SMW line MC equal to line I, and produce MC on both sides;
it will meet lines LQ, QO (as has been shown in Proposition 8 in Book II of the *Conics*)—and let it meet them in points L, O.
Then lines OM, LC will be equal (as has been shown also in Proposition 8 of the said Book).
Draw from point T line TF parallel to line OL, and let it cut line NM in point J.
Thus, surface $LMTF$ being a parallelogram, line LM will be equal to line FT.
But LM is equal to CO,
therefore CO is equal to TF;
and MO is equal to JT, because surface JO is a parallelogram,
it remains that FJ is equal to CM;
and CM is equal to I,
therefore line FJ is equal to line I;
and it follows that the ratio of line FJ to line TN is as the ratio of BG to KE.
On line GA and at point A draw angle GAZ equal to angle NFT.
This line, i.e. line AZ, will meet line GD, because the angles at points A, G are equal to the angles at points F, N—let it meet GD at D.
Now since angles AGD, ZGD are equal to angles FNT, JNT,
and angle GAD is equal to angle NFT,
triangles AGZ, ZGD, AGD are similar to triangles FNJ, JNT, FNT,
and, therefore, as ZA is to AG so is JF to FN,
and, as AG is to GD, so is FN to NT;
therefore as AZ is to GD so is FJ to NT.
But FJ is equal to I, and as I is to TN so is BG to KE,
therefore as AZ is to GD so is BG to KE.

And since line AD meets BD outside the circle on the side of G, line DA will either touch the circle at point A [Fig. 5.1], or it will cut arc AG [Fig. 5.2], or else cut arc AB [Fig. 5.3].
For, if arc AG is smaller than arc AB [Fig. 5.1], then the tangent drawn from A will meet the diameter BG on the side of G, and the line drawn from A parallel to diameter BG will cut arc AB; and, therefore, the lines which are drawn from A

and which meet GD above the tangent will cut the part of arc AB that is cut off by the parallel line. Further, the lines which are drawn from point A and which meet GD below the tangent will cut arc AG.

Now let arc AG be greater than arc AB [Fig. 5.2]; then every line drawn from A, meeting BG outside the circle on the side of G, will always cut [arc] AG.
For the tangent drawn from A will meet BG on the side of B,
and the line drawn from A parallel to the diameter BG will cut arc AG;
from which it follows (if arc AG is greater than arc AB) that all lines drawn from A so as to meet BG outside the circle on the side of G will cut arc AG.
Thus line AD will either touch the circle at A (as in the First Figure), or cut arc AG (as in the Second Figure), or else cut arc AB (as in the Third Figure).

[And, first,] let it be tangent [to the circle, as in Fig. 5.1].
Then angle GAD is equal to angle ABG,
and angle ZGD is equal to angle ABG,
therefore angle ZGD is equal to angle GAD.
Therefore the product of AD and DZ is equal to the square of GD;
and the product of BD and DG is equal to the square of AD (because AD is a tangent);
it remains that the product of DA and AZ is equal to the product of BG and GD.
Therefore as AZ is to GD, so is BG to DA;
but AZ to GD was shown to be as BG is to KE;
therefore as BG is to KE so is BG to DA;
and, therefore, line DA is equal to line KE.

Now let line AD cut arc AG, say at point H [Fig. 5.2].
Join GH.
Angle AHG will then together with angle ABG be equal to two right angles.
Therefore angle GHZ is equal to angle ABG;
and angle ZGD is equal to angle ABG; therefore angle GHZ is equal to angle ZGD;
therefore the product of HD and DZ is equal to the square of GD;
and the product of AD and DH is equal to the product of BD and DG;
it remains that the product of HD and AZ is equal to the product of BG and DG.
Therefore as AZ is to GD so is BG to HD;
but AZ to GD was [shown to be] as BG is to KE; therefore as BG is to HD so is BG to KE;
therefore line HD is equal to line KE.
Now let line AD cut arc AB, say at point H [Fig. 5.3].
Join HG.
Thus angle GHA is equal to angle GBA;
and angle ZGD is equal to angle GBA;
therefore angle GHD is equal to angle DGZ.
Therefore the product of HD and DZ is equal to the square of GD;

but the product of HD and AD is equal to the product of BD and DG;
it remains that the product of HD and AZ is equal to the product of BG and GD.
Therefore as AZ is to GD, so is BG to HD;
but AZ is to GD as BG is to KE;
therefore as BG is to HD so is BG to KE;
therefore line HD is equal to line KE.

We have thus shown in all cases how to draw from A a line that meets the diameter BG outside the circle on the side of G, so that the part of the line that lies between the circle and the diameter is equal to line KE
And that is what we wished to do.

[Lemma II: Figure 6]

Again, let [points] A, B, G be on the circumference of a circle; let BG be a diameter, and let line ZH be given; we wish to draw from A a line that cuts diameter BG and carries through to the circle, so that the part of it that lies between the circle and the diameter will be equal to line ZH.
Join lines AB, AG; and on line ZH and at point H construct angle ZHK equal to angle ABG, and angle ZHT equal to angle AGB;
from Z draw line ZT parallel to line KH, and ZK parallel to TH;
thus surface TK will be a parallelogram.
Draw through point T the hyperbola of which lines HK, KZ are asymptotes—let it be section TC, and let the opposite section be WS;
produce lines HK, ZK on the side of K to L, F, and with T as centre, and with a distance equal to diameter BG, describe a circle, and let this circle meet section WS at point S.
This circle will meet section WS if BG is not smaller than the shortest line that can be drawn from point T to section WS.
As to which is the shortest line that can be drawn from T to section WS, this has been shown in Propositions 34 and 61 in Book V of Apollonius' *Conics*.
Thus the circle described about T with distance BG, if it meets the section, will either touch it at one point or cut it in two points.
If it touches the circle, then only one line equal to BG can be drawn from point T to section WS.
But if the circle cuts the section in two points, then only two lines equal to BG can be drawn from point T to section WS.
Thus point S is either the point of tangency or one of the two points of intersection.
Join line TS; it will be equal to BG.
Line TS will thus cut lines HK, KQ—let it cut HK in point F, and KQ in point Q;
draw from Z a line parallel to TS, which will cut lines HK, HT, since line TS cuts these two lines—let that be line LZM;
thus ZM will be equal to TQ, because surface MQ is a parallelogram.

Now since CT, WS are opposite sections,
and TS cuts their asymptotes,
line TF will be equal to line QS (as is shown in Proposition 16[17] in Book II of the *Conics*).
And TF is equal to line ZL, because surface LT is a parallelogram,
therefore ZL is equal to QS;
and ZM is equal to TQ,
therefore LM is equal to TS;
and TS is equal to BG,
therefore LM is equal to BG.

We further construct on line BG, at point G, an angle BGD equal to angle MLH.
Angle MLH will be acute because angle LHM is right, being equal to ABG and AGB.
Line GD will therefore fall inside the circle—let it cut the circle at point D.
Join BD, AD, and let AD cut BG at point E.
Angle GDB will be a right angle, equal to LHM,
and angle BDE will be qeual to angle BGA which is equal to angle ZHM,
and angle GBD will be equal to angle LMH.
Thus triangle BGD will be similar to triangle LMH,
and triangle DEB will be similar to triangle HZM.
Therefore as GB is to BD, so is LM to MH;
and BD is to DE as MH is to HZ,
therefore as GB is to ED so is LM to ZH;
but LM is equal to BG,
therefore DE is equal to ZH.
We have thus drawn from point A line AED so that line ED is equal to line ZH.
And that is what we wished to do.

But if two lines equal to BG go from point T to section WS, then there will go from point Z to lines KH, HT two lines equal to line BG, producing between them and line HK two unequal angles.
Then if two angles equal to those angles are constructed on line BG at point G, two points will be produced on arc BG.
And if two lines are joined between them and point A, there will be cut off from each of these lines between arc BDG and diameter BG a line equal to ZH—this being shown by the demonstration we mentioned.

Further, if line BG is equal to the shortest line that can be drawn from point T

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[17] All three MSS have "11" instead of "16", the correct number of the proposition in Bk. II of the *Conics* both in HEIBERG's edition of the Greek text and in the Arabic copy in IBN AL-HAYTHAM's own hand (MS Ayasofya 2762). The wrong number "11" is written out in words in the Köprülü MS, and in the *abjad* notation in the Ayasofya and Fatih MSS.

to section WS, then only one line can be drawn from A to arc BDG so that the segment between the arc and line BG is equal to ZH.

If BG is greater than the shortest line, then there will go from A to arc BDG two lines in each of which the segment between the arc and the diameter will be equal to line ZH.

No more than two lines can be drawn from A to arc BDG so that the segment between the arc and the diameter will be equal to ZH. For the circle about centre T cannot cut section WS at more than two points, the centre of the circle being outside the section.

And, further, if BG is smaller than the shortest line, then a line cannot be drawn from A to arc BDG, so that the segment between the arc and the diameter is equal to ZH.

This construction is, therefore, either impossible, or it can be carried out once, or twice, but not more.

And that is what we wished to do.

[Lemma III: Figure 7]

Again, in triangle ABG let angle B be right; let D be given on line BG; and let the ratio of E to Z be known; we wish to draw from D a line like DTK so that
the ratio of TK to TG
is as the ratio of E to Z.

Join DA, and draw DM parallel to BA;
and on triangle DMG describe circle DMG; MG will be a diameter of the circle because MDG is a right angle.
Draw angle DMN equal to angle DAG;
MN will then cut angle DMG and, therefore, will cut arc DG (as in the First Figure),
or cut arc MG (as in the Second Figure);
let it cut [either] arc in point N.

Let the ratio of line AD to line H be as the ratio of E to Z;
and from N draw line NCL so that CL will be equal to H (as was shown earlier);
then join DC and produce it in a straight line—it will cut LM, say in point T;
and join GC.

Angle GCD will then be equal to angle GMD, and, therefore, equal to angle GAB,
therefore angle GCT is equal to angle TAK;
but angle CTG is equal to angle ATK;
therefore if line CT is produced in a straight line (as in the First Figure), it will meet line AK at an angle equal to angle TGC.
Produce CT and let it meet AK at K.
Then triangle AKT will be similar to triangle CGT (in both Figures):
therefore as AT is to TC, so is KT to TG.

Again, angle DCN is equal to angle DMN,
and angle DMN is equal to angle DAT,
therefore angle LCT is equal to angle DAT.
And triangle LCT is similar to triangle ADT,
therefore as AT is to TC, so is AD to LC.
And LC is equal to H,
therefore as AT is to TC, so is AD to H.
But AD is to H as E is to Z,
therefore as AT is to TC, so is E to Z.
And AT is to TC as KT is to TG,
therefore as KT is to TG, so is E to Z.
And that is what we wished to do.

[Lemma IV: Figure 9]

Again, let circle AB, with centre G, be given, and let D, E be two given points; we wish to draw from E, D, two lines like EA, DA, so that a line drawn tangentially to the circle, such as AH, will bisect angle EAD.
Join GD, GE, ED; and produce EG in a straight line to B.
Take any line at random, say MI, and divide it at S, so that
as IS is to SM, so is EG to GD;
then bisect line [IM] in N, and draw NO perpendicular to it;
make angle NMO equal to half of angle DGB,
and from S draw line SQF, so that
as QF is to FM, so is EG to GB;
and make angle EGA equal to angle SFM;
and join EA, QM;
then triangles EAG, QMF will be similar.

Make angle EAZ equal to angle QMS;
thus angle ZAG will be equal to angle SMO which is equal to half of angle DGB.
Produce AZ on the side of Z, and make the ratio of
AZ to ZK equal to the ratio of MS to SI, which is the same as the ratio of DG to GE.
Join EK, QI, and draw the perpendicular EL [to AK].
Thus the angles at points A, E, K, Z, L will be equal to the angles at points M, Q, I, S, N, and, therefore, the triangles will be similar.
Therefore AL will be equal to LK, and AE equal to EK,
and the ratio of KZ to ZA will be as the ratio of IS to SM, which is the same as EG is to GD.

Draw AT parallel to line EK.
Therefore angle TAZ will be equal to ZAE,
and as EA is to AT,

so will be *EZ* to *ZT*,
and *KZ* to *ZA*,
which is the same as *EG* is to *GD*.

Now make angle *GAW* equal to angle *GAE*.
Therefore angle *WAT* will be double of angle *GAZ*, which is equal to angle *FMN*, which is half of angle *DGB*;
therefore angle *WAT* will be equal to angle *DGW*;
therefore line *WA* will meet line *GD*—if line *AW* meets line *GB*,
and the triangle cut off by line *WA* produced will be similar to triangle *WAT*.
I say, then, that line *WA* will meet line *GD* at point *D*.
For, as *EG* is to *GD*, so is *EA* to *AT*;
and *EA* to *AT* is compounded of *EA* to *AW* and *WA* to *AT*;
therefore *EG* to *GD* is compounded of *EA* to *AW* and *WA* to *AT*.
And as *EA* to *AW*, so is *EG* to *GW*, because the angles at *A* are equal;
and as *WA* is to *AT*, so is *WG* to the line cut off by *WA* from line *GD*;
therefore the ratio of *EG* to *GD* is compounded of *EG* to *GW* and *GW* to the line cut off by *WA* from line *GD*.
But *EG* to *GD* is compounded of *EG* to *GW* and *GW* to *GD*,
therefore *GD* is the line cut off by *WA* and *GD*;
and thus line *WA* will go through to point *D*;
and, therefore, angle *TAD* will be equal to angle *EGD*.

Now make angle *GAH* right.
Then angle *ZAH* will be half of *EGD*, because angle *ZAG* is half of angle *DGW*.
Thus angle *ZAH* is half of angle *TAD*,
and angle *ZAE* is half of angle *TAE*,
therefore angle *EAH* is half of angle *EAD*.

But if line *AW* is parallel to line *GE*, then angle *EGA* will be equal to angle *GAE*;
therefore line *AE* will be equal to line *EG*.
But the angle next to angle *WAT* is equal to angle *TGD*,
and the angle at the intersection of *WA* with *GD* will be equal to angle *TGD*, because they are alternate angles,
therefore line *TA* will be equal to the line cut off by *WA* from line *GD*;
and line *EA* is equal to line *EG*;
therefore as *EA* is to *AT*, so is *EG* to the line cut off by *WA* from *GD*;
but *EA* is to *AT* as *EG* is to *GD*;
therefore the line cut off by *WA* from *GD* is the same as line *GD*;
therefore angle *TAD* will be equal to angle *EGD*.
And angle *ZAH* is half of angle *EGD*,
therefore angle *ZAH* is half of angle *TAD*;
but angle *ZAE* is half of angle *TAE*,
therefore angle *EAH* will in all cases be equal to half of angle *EAD*.
And that is what we wished to prove.

[Lemma V: Figure 11]

Again, let circle *AB* be given, with centre *G* and diameter [*sic*] *GB*, and let point *E* be given outside the circle, and we wish to draw from *E* a line, as *EDZ*, so that *DZ* will be equal to *ZG*.
Join *EG*, and from *E* draw *ES* perpendicular to line *GB*;
and make line *TK* equal to line *ES*;
on line *TK* describe the segment of a circle that admits angle *EGB*, and let it be segment *TMK*, and complete the circle;
bisect *TK* at *L*, and draw *LM* perpendicular to *TK* and carry it through to *N*;
MN will then be a diameter of the circle.
From point *K* draw line *KFC* so that line *CF* will be equal to half of line *GB*.
Join *TF*—it will be equal to *FK*.
Draw *CQ* parallel to *FN*, and *QO* parallel to *KL*;
angle *CQO* will then be a right angle, and *QF* will be equal to *FO*, because *TF* is equal to *FK*.
Then, since angle *CQO* is right and line *QF* is equal to line *FO*,
line *QF* will be equal to *FC*, and *FC* to *FO*.
Construct angle *BGD* equal to angle *KCQ*; join *ED* and carry it through to *Z*.
I say, then, that *DZ* is equal to *ZG*.

Demonstration:
From point *D* draw the perpendicular *DI*, and construct the right angle *GDW*:
line *DW* will then meet *GB*, because angle *DGZ* is acute because it is equal to angle *OCQ*—let them meet at *W*.
Join *TC*, and from *Q* draw the perpendicular *QH*,
draw *TJ* parallel to *CH*, and produce *HQ* to meet it, say at point *J*.
Draw the perpendicular *TR*: it will be equal to *JH*.

Then, since *CF* is half of *GB*, *CO* will be equal to *GD*;
and *TK* is equal to *ES*;
therefore as *TK* is to *CO*, so is *ES* to *GD*.
But as *GD* is to *DI*, so is *GW* to *WD*,
and as *GW* is to *WD*, so is *CO* to *OQ*,
therefore as *ES* is to *DI*, so is *TK* to *QO*, which is the same ratio as *TF* to *FQ*;
therefore as *ES* is to *DI*, so is *TF* to *FQ*.
And as *TF* is to *FQ*, so is *JH* to *HQ*;
and *JH* is equal to *TR*,
therefore as *ES* is to *DI*, so is *TR* to *QH*.
And as *GE* is to *ES*, so is *CT* to *TR*, because the two triangles [*GES* and *CTR*] are similar,
therefore as *EG* is to *DI*, so is *TC* to *QH*.
And *ID* is to *DG* as *HQ* is to *QC*,
therefore as *EG* is to *GD*, so is *TC* to *CQ*.
And angles *EGD*, *TCQ* are equal, and therefore the two triangles are similar,
therefore angles *GDZ*, *CQF* are equal.

And angles *DGZ*, *QCF* are equal,
therefore as *DZ* is to *ZG*, so is *QF* to *FC*.
And *QF* is equal to *FC*,
therefore *ZD* is equal to *ZG*.
And that is what we wished to prove.

[Lemma VI: Figure 8]

Again, let the right-angled triangle *ABG* have the angle *B* right; let *AB* be produced on the side of *B*, and let point *D* be given on *BG*; and, further, let *E* to *Z* be a given ratio; and we wish to draw from *D* a line,
such as line *TDK*, so that
as *TK* is to *KG*, so is *E* to *Z*.

Join *AD*,
and let *AD* to *H* be as *E* is to *Z*;
draw *DM* parallel to *BA*, so that angle *MDG* will be right;
on the triangle *MDG* describe a circle with diameter *MG*;
construct angle *DMC* equal to angle *DAG*;
from point *C* draw *CLN*, so that line *LN* will be equal to line *H*;
join *DKN* and carry it through on the side of *D* to *T*;
and join *GN*.

Therefore angle *DNG* will be equal to angle *DMG* which is equal to angle *BAG*.
But angle *NKG* is equal to angle *AKT*,
therefore line *KT* will meet line *AB*, say at point *T*,
and, therefore, triangles *ATK*, *NGK* will be similar;
therefore as *TK* is to *KG*, so is *AK* to *KN*.

And angle *DNC* is equal to angle *DMC* which is equal to angle *DAG*,
therefore triangles *AKD*, *NKL* are similar;
therefore as *AK* is to *KN*, so is *AD* to *NL*, which is the same as *E* is to *Z*;
therefore as *TK* is to *KG*, so is *E* to *Z*.
And that is what we wished to prove.

And it was shown earlier that there issue from point *C* two lines such that the segment of each of them that lies between the circle and the diameter will be equal to the given line. Thus if two such lines are drawn from *C*, then there will issue from point *D* two lines in the given ratio; but the two angles produced at point *G* will be unequal, I mean angle *TGK* and the angle corresponding to it.

IX

Psychology versus mathematics: Ptolemy and Alhazen on the moon illusion

That celestial magnitudes appear larger at the horizon than at higher altitudes is a commonly known phenomenon that has been recorded and investigated since antiquity. Because the phenomenon is particularly noticeable in the case of the moon, it has sometimes been referred to in recent times as the "moon illusion," a designation also reflecting the accepted understanding of the apparent enlargement as a psychological effect. But that is not how the phenomenon was always understood. A traditionally held belief in antiquity found expression in a brief passage in Ptolemy's *Almagest*, which compared the enlargement to the apparent magnification of objects immersed in water, thus proposing an explanation in physical rather than psychological terms. And it was this explanation that enjoyed wide acceptance in late antiquity and in the Islamic Middle Ages up to the end of the thirteenth century, which testified both to the great authority of the *Almagest* and to a remarkable lack of understanding of the mathematical theory of optical refraction. Already in the first half of the eleventh century, however, the mathematician Alhazen (Ibn al-Haytham) had freed himself from the erroneous view in the *Almagest* and, setting off from a new level of understanding some of the elements of which he found in Ptolemy's *Optica*, he had offered in his *Book of Optics* (*Kitāb al-Manāẓir*) a psychological explanation in terms of what modern psychologists have called, with some exaggeration, the size–distance constancy principle. The principle itself is clearly stated in Ptolemy's *Optica* and, until recently, it had been more or less taken for granted that a passage in the same work also contains an application of it to the phenomenon in question. This assumption was, however, challenged in 1976 when two psychologists showed that it was due to a confusion introduced by Della Porta in 1593 as a result of misreading a passage in Roger Bacon's *Perspectiva*.[1]

The fact is that the enormous literature concerned with explaining the phenomenon itself is in glaring contrast to the extreme paucity, almost the nonexistence, of significant studies devoted to the historical development of its investigation, especially in the ancient and medieval periods. The present chapter is a step toward improving this situation. Concentrating on the works of Ptolemy and the closely related writings of Alhazen, I shall follow a method favored by Marshall Clagett, namely, the method of presenting and elucidating texts, which seems particularly appropriate in this case. In analyzing the relevant texts in those two authors I shall be mainly concerned to describe the transition from a mathematical to a psychological explanation of the moon illusion. A secondary aim of my analysis will be to dispel one or two historical illusions.

I. Ptolemy

We shall examine three relevant passages that occur in three different works of Ptolemy: *Almagest*, *Planetary Hypotheses*, and *Optica*. It will be convenient to refer to these passages as the *A*-passage, the *PH*-passage, and the *O*-passage, respectively. It is generally accepted that the *Almagest* was the earliest of the three works, and some believe that the *Planetary Hypotheses* was probably the last of Ptolemy's compositions.

The A-*passage*

TEXT

The text of the *A*-passage reads as follows in G. J. Toomer's English translation of the *Almagest*:

> To sum up, if one assumes any motion whatever, except spherical, for the heavenly bodies, it necessarily follows that their distances, measured from the earth upwards, must vary, wherever and however one supposes the earth itself to be situated. Hence the sizes and mutual distances of the stars must appear to vary for the same observers during the course of each revolution, since at one time they must be at a greater distance, at another at a lesser. Yet we see that no such variation occurs. For the apparent increase in their sizes at the horizons is caused, not by a decrease in their distances, but by the exhalations of moisture surrounding the earth being interposed between the place from which we observe and the heavenly bodies, just as objects placed in water appear bigger than they are, and the lower they sink, the bigger they appear.[2]

COMMENTARY

Later we shall see what Alhazen thought of this passage in several of his writings. Here we shall note that most commentators of the *Almagest*, from Theon of Alexandria (fourth century A.D.) to Otto Neugebauer, have understood it to involve a strict analogy with optical refraction. But since, in viewing the apparently enlarged heavenly body, the eye is located in the denser medium of moist air, whereas the opposite is the case when observing an object immersed in water, the analogy seems to be false. (Or, as Neugebauer has put it, the optical situation invoked by Ptolemy is "not relevant" and the analogy itself "incorrect.")[3] How, then, could Ptolemy have committed such an error while his *Optica* clearly asserts that objects in a rare medium appear smaller, not larger, than they are to an eye placed in a denser medium? Several conjectures have occurred to those who pondered over this puzzle: Ptolemy cannot be the author of the *Optica* (a conjecture no longer seriously entertained by historians); or, Ptolemy did indeed write the *Optica*, but only after the *Almagest*, and after he had gained a better understanding of refraction and of optical matters generally; or, the analogy in the *Almagest* is so brief as to suggest that Ptolemy was simply helping himself to a current explanation which he had not carefully examined and which, therefore, should not be taken seriously. The suggestion that Ptolemy was referring not to refraction but to aerial perspective has little to support it; and it does not seem to have occurred to his ancient and medieval commentators.

Alhazen, at one time a subscriber to a totally erroneous conception of refraction that he tried to apply to the phenomenon in question, continued to see merit in the *A*-passage even after he later realized its basic irrelevance, and even after he had come upon his own psychological explanation of the phenomenon as an optical illusion. It will be interesting, when we consider his views in the second part of this chapter, to have before us a notion of what is contained in the most articulate interpretation of the *A*-passage that has come down to us from antiquity – that of Theon of Alexandria.

Relying on an account of refraction which he found (as he tells us) in a work of Archimedes on *Catoptrics*, Theon, in his *Commentary on the Almagest*, formulates an argument that incorporates the following statements:[4]

1. Two unequal magnitudes such as *AB*, *GD* (Figure 1), seen from *E* by means of the rectilinear rays *EAG*, *EBD*, will appear to be of equal size because they are viewed through one and the same angle *GED*.
2. *AB*, when placed below the water surface *ZH*, and seen by means of refracted rays such as *ETA*, *EKB*, will appear larger than it is because it is now viewed through an angle *TEK* > *AEB*.

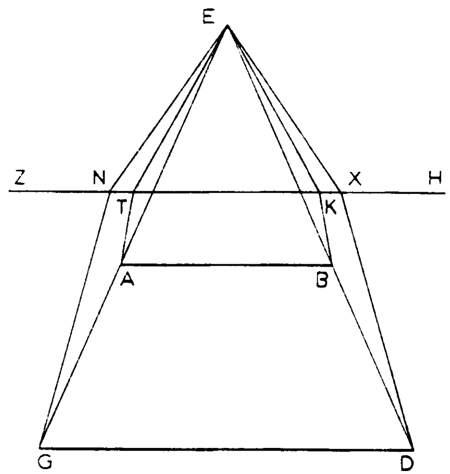


Figure 1

3. In the same manner, *GD* will look bigger when viewed through refracted rays *ENG*, *EXD* than when it is seen through the rectilinear rays *EG*, *ED*.
4. "Therefore the unequal objects *AB*, *GD*, which in pure air appear to be equal, will appear unequal when placed in water, and the lower one will look bigger, because they will be viewed through unequal angles."
5. Theon's construction of the enlarged images shows them in the same plane as the two magnitudes, that is, on the extensions of *AB* and *GD*.

The following comments may now be made:

First, statement (1) is untrue, and it implies that Theon either did not know or did not understand Ptolemy's *Optica* when he wrote his *Commentary on the Almagest*. The *Optica* clearly recognized the fact that apparent size is a function of the distance of the seen object from the eye as well as of the visual angle (a third variable also considered by Ptolemy is the inclination of the object's surface to the axis of the visual cone).[5]

Second, the explanation in (2) and (3) is correct in terms of a theory of monocular vision and a correct account of refraction stipulating that refraction into a denser medium takes place *toward* the normal to the surface of separation. However, (3) does not take us beyond (2), both being concerned with the same situation. Theon expressly attributes the explanation in (2) to Archimedes.

Third, the conclusion in (4) is irrelevant since *GD* is already *assumed* to be larger than *AB*. Theon's argument fails to show that one and the same magnitude appears to grow in size when it sinks deeper in water.

Fourth, (5) is another piece of evidence of Theon's ignorance of the *Optica*, in which work he would have noted that images of immersed objects appear closer to the water surface than the objects themselves.[6]

Fifth, Theon's argument simply consists in applying the irrelevant conclusion (4) to the phenomenon under discussion by observing that in horizontal viewing of a heavenly object, sight has to cover a longer distance through the moist air than in vertical viewing.

The PH-*passage*

Ptolemy wrote the *Planetary Hypotheses* in two books, of which only part 1 of Book I has survived in the original Greek. A ninth-century Arabic translation (revised by Thābit ibn Qurra, d. 901) of the complete Books I and II is extant in two manuscripts, one at Leiden University Library, the other at the British Library. In 1967, Bernard Goldstein published a facsimile of the BL MS together with variants from the Leiden MS, and his edition included a translation of the final part 2 of Book I and a commentary.[7] As is now well known, part 2, on planetary distances and sizes, is an account of "the Ptolemaic system of the world," the system representing the universe as a sequence of nested spheres which together occupy the entire celestial region between the sublunar sphere of fire and the sphere of the fixed stars.

Ptolemy first computes planetary distances from the earth on the basis of data derived from the *Almagest*: the geocentric extremal distances of the moon, the mean solar distance, and the ratios of extremal distances for every one of the planets. Following this, and using the value in the *Almagest* for the solar diameter, he computes the true diameters of the planets from an estimation of their apparent diameters and their distances.[8] Then, at the very end of this final part of Book I, and as if to answer a possible objection, Ptolemy adds what we have called the *PH*-passage. In the Appendix at the end of this chapter, I have provided an edition of the Arabic text of the *PH*-passage based on the two manuscripts mentioned and a quotation of the same passage in the *Nihāyat al-suʾl* of the Damascene astronomer Ibn al-Shāṭir (fourteenth century). Following is my translation of this edited text.

TRANSLATION OF THE *PH*-PASSAGE

{1} As for the reason why what appears to sight and is imagined in it regarding the size of the planets is not in proportion to their distances, we must recognize that this is the error that occurs to sight on account of the difference of perspective[s]. {2} This is manifest in all that appears and is

seen from large distances. {3} For just as the magnitude of the distances themselves cannot be recognized in what appears [in this case] to the eye, so also the difference between objects of unequal magnitudes cannot be known at those distances according to their proportion. {4} For by condensing and contracting that difference, sight reduces it to what is more familiar to it. {5} And, for this reason, every one of the stars is seen closer to us than it is in fact, because sight falls back to the distances to which it is accustomed and with which it is familiar. {6} Thus the case with the increases and decreases that size undergoes
* according to the increase and decreases in distances – which [increases and decreases] are less than the proportion [of the distances] – is like the case with the distances [themselves]. {7} For sight, as we have said, is unable to discern and perceive the amount of difference between all the kinds [of magnitude] we have mentioned.

NOTES ON THE TRANSLATION OF THE *PH*-PASSAGE

{1} 1. "sight" renders *naẓar* (first occurrence) and *baṣar* (second occurrence). 2. "the size of the planets"] The Arabic has "the size of their [or "its"] body": *ʿiẓami jirmihā*. Grammatically and contextually the reference could be to the planets generally, or to Venus, which is mentioned in the sentence immediately preceding the *PH*-passage (see the commentary following these notes). 3. "proportion" here renders *nisab* (ratios); in {3} *tanāsub* (proportion) is used. 4. "the difference of perspective(s)": *ikhtilāf al-manẓar* (or *al-manāẓir*)] A standard expression which refers generally to the variable appearance of objects in regard to magnitude and spatial relations. The Leiden MS and the quote in Ibn al-Shāṭir (in five of six manuscripts consulted) have the singular *al-manẓar*; the BL MS has the plural *al-manāẓir*.

{2} 1. "This is manifest"] *wa yatabayyanu dhālika*. I read *yatabayynu*, as in the quote in Ibn al-Shāṭir, rather than *nubayyinu*, as in the BL MS. To adopt the latter reading, meaning "we show this" or "we will show this," would be puzzling, since the sentence stops short of telling us where the showing is to take place. The word in the Leiden MS lacks the diacritical point(s) for the first letter, and can therefore be read *tabayyana* (it has been shown) as well as *nubayyinu*. *Yatabayyanu* is certainly more natural than any of these other possibilities.

{3} 1. "recognized" and "known" here translate one and the same Arabic verb, *ʿalima*. 2. "according to their proportion"] *ʿalā al-tanāsub allatī (sic) hiya ʿalayhi*. The pronoun *hiya* could refer to *al-ashyāʾ* (the objects) or to *al-abʿād* (the distances) – see the following commentary.

{4} 1. "condensing" and "contracting"] The Arabic words are *jamʿ* and *qabḍ*, respectively. The verb *jamaʿa* means to draw or bring together, or to collect. The second verb, *qabaḍa*, indicates the narrowing or shrinking effect of this collecting; hence my choice of the English words. 2. "that difference" replaces a singular masculine pronoun which, grammatically, can only refer back to the *tafāḍul* (difference or inequality), or the *tanāsub* (proportion), or to "what appears to the eye," all mentioned earlier in the same sentence.

{7} 1. "all the kinds [of magnitude] we have mentioned"] The two kinds mentioned are size and distance.

COMMENTARY

Neugebauer has interpreted this passage as indicating an abandonment of the explanation in the *Almagest* and as recognizing the phenomenon described in the *A*-passage as an optical illusion.[9] This is surprising since neither the context of the *PH*-passage nor the arguments in it would suggest such an interpretation. The *PH*-passage is clearly concerned with how the size of a celestial magnitude appears to vary with distance, not with how it appears to vary with the direction of sight while assuming the distance to be constant. Ptolemy is now thinking in psychological terms, but whether he now has a psychological explanation of the situation examined in the *Almagest* remains an open question.

Let us look at the statements in the *PH*-passage one by one. The passage begins by stating in {1}, as a matter of observation, that the size of a planet does not appear to vary in proportion to its distance from us, and, in {2}, that this is an error of sight which occurs whenever we look at very far objects. This means either that no reduction of size is observed when the distance of the far object increases, or that the reduction appears to be less than is required by the geometry of the visual cone. Subsequent statements in the passage imply that the latter is the case. The remark is then made in {3}, with reference to unequal objects at far distances from us, that we fail to perceive their inequality according to the proportion of their (true) sizes, just as we are not able to obtain a correct perception of the distances themselves. The Arabic is ambiguous (see {3} 2 of preceding "Notes on the Translation . . ."), allowing that the proportion intended might be that of the objects' distances, not their sizes; but then, on this alternative, we would be left with the question why the objects are supposed to be unequal. Thus while {1} is concerned with judging the size of what could be taken to be one and the same object at different distances, {3} seems to be concerned with estimating the apparent relative sizes of unequal objects at equal distances.

It is to be noted that while {3} asserts that we misjudge both distances and relative sizes of far objects, nothing is said about whether misjudging one of these magnitudes depends on misjudging the other. But the nature of the "error" involved in both cases is made clear in {4}: It consists in reducing the perceived magnitude, be it distance or size, to a more familiar and smaller scale; and this is stated explicitly in {5} with regard to distance.

Finally, {6} returns to considering the apparent variation of size with distance: Such variation does occur, we are told, but the apparent increase or decrease in size is less than the ratio of the distances because of sight's inability to estimate the amount of difference in this case – for the reason stated in {4}.

Apparently because the difference between extremal distances is (according to Ptolemy) appreciably greater for Venus and Mars than for any other planet, Ibn al-Shāṭir takes the *PH*-passage to be especially addressed to a question suggested by observation of these two planets. He wrote in a note at the end of Book I of his *Nihāya*: "You must know that the diameters of Venus and Mars are not seen in accordance with (ʿalā muqtaḍā) their distances. For their maximum distances being many times their minimum distances, their diameters should appear according to (*tabaʿ*) the distances: I mean that the diameter of each of them should appear to be many times greater at perigee than at apogee; but this is not so."[10] After quoting Ptolemy's passage, Ibn al-Shāṭir gives two reasons of his own: the diminution of the visible part of a sphere as it approaches the eye, and the disturbing effect of rays emanating directly from luminous bodies. (He assumes that all planets, except the moon, are self-luminous.)[11]

The O-*passage*

We come now to the passage in Ptolemy's *Optica* that has been generally understood to contain an explanation of the moon illusion in terms of the size–distance invariance principle. The *Optica* survives in a twelfth-century Latin translation from an earlier Arabic version of Books II to IV and part of Book V; neither the Greek original nor the Arabic translation is extant.

LATIN TEXT OF THE *O*-PASSAGE

***erit distantiis, eo quod debilitas sensum plus fit penes coniunctionem. {1} Vniuersaliter enim, cum uisibilis radius, quando cadit super res uidendas aliter quam inest ei de natura et consuetudine, minus sensit omnes diuersitates que in eis sunt, similiter etiam erit sensibilitas eius de distantiis quas comprehendit, minor. {2} Videtur autem hac de causa quod de rebus que sunt in celo et subtendunt equales

angulos inter radios uisibiles, ille que propinque sunt puncto qui super capud nostrum est, apparent minores; que uero sunt prope orizontem, uidentur diuerso modo et secundum consuetudinem. {3} Res autem sublimes uidentur parue extra consuetudinem et cum difficultate actionis.[12]

TRANSLATION OF THE *O*-PASSAGE

. . . {1} For, generally, just as the visual ray, when it strikes visible objects in [circumstances] other than what is natural and familiar to it, senses all their differences less, so also its sensation of the distances it perceives [in those circumstances] is less. {2} And this is seen to be the reason why, of the celestial objects that subtend equal angles between the visual rays, those near the point above our head look smaller, whereas those near the horizon are seen in a different manner and in accordance with what is customary. {3} But objects high above are seen as small because of the extraordinary circumstances and the difficulty [involved] in the act [of seeing].

NOTES ON THE TRANSLATION OF THE *O*-PASSAGE

{1} 1. "generally"] *uniuersaliter*. My guess is that the Latin renders a word like *bi-al-jumla* (Greek *holōs*). Ross and Ross translate: "it is a universal law that." The word occurs many times in Ptolemy's *Optica*.

2. "senses/*sensit*"] If the phrase *uisibilis radius* translates *shuʿāʿ al-baṣar* (the ray of sight), then the subject of *sensit* could be "sight," or the faculty of sight, rather than the "visual ray" itself, which would make a difference to an argument by Ross and Ross.[13]

3. "all their differences"] *omnes diuersitates que in eis sunt*. "Diuersitates" suggests the Arabic *ikhtilāfāt*, which it would not have been objectionable to translate as "inequalities," had it not been for the presence of "*omnes*." "*Omnes*" clearly implies that all kinds of visual differences between objects, including differences of size, are meant. Ross and Ross have "characteristics," which also preserves the general meaning. In the *PH*-passage, difference of size is expressed by *tafāḍul*.

4. "perceives"] *comprehendit*. The Latin word is used in this sense many times in Ptolemy's *Optica* and in the Latin version of Alhazen's *Optics*. Ross and Ross translate it here as "covers." The corresponding Arabic word is, most probably, *adraka*.

{2} 1. "in accordance with what is customary"] *secundum consuetudinem*. That is, according to what sight is accustomed to in familiar situations. The corresponding Arabic word may well be *iʿtāda* or *alifa*, both of which are used in the *PH*-passage.

COMMENTARY

The *O*-passage raises so many questions that a paraphrase of it is not possible without inserting a number of more or less crucial interpretations of the Latin text, in addition to those inevitably involved in translating it. One question already raised by Ross and Ross concerns the meaning of "less" (*minus, minor*) in sentence {1}. What is it that is here said to be reduced when objects are looked at in unusual circumstances: Is it the sensation itself or the seen object? Or, as Ross and Ross have put it, does Ptolemy's expression refer to "reduced ability to discriminate" or to "perception of something as smaller?"[14] (An Arabic word such as *aqall*, a probable equivalent of the Latin *minus*, would have been equally ambiguous. *Aṣghar*/smaller is less probable as it could hardly apply to *sensibilitas* = *ʾiḥsās*.) The application of *minor* to *sensibilitas* would seem to suggest that the former alternative is intended. But however we understand the words in {1} it is definitely asserted in {2} that an apparent *reduction of size* takes place, at least as a consequence.

It would seem to me that a reduction of distance is also intended. Ross and Ross cite results of experimental psychology to support the hypothesis that Ptolemy may well have associated apparently reduced size with *increased* distance.[15] But we have seen that Ptolemy, in the *PH*-passage, clearly asserts that the distances as well as the sizes of heavenly bodies do appear to be *smaller* than they really are, giving as a reason sight's tendency to compress *all* magnitudes perceived from afar. True, he makes this assertion there as a general statement, applicable in all situations. But the *PH*-passage shows at least that Ptolemy's beliefs may not have always been those of modern psychology. The real difficulty lies in making an inference with regard to the *O*-passage from a statement in another passage which may have been written later. And it remains true that neither in the *O*-passage nor in the *PH*-passage does Ptolemy assert a *dependence* of reduction of size upon a reduction of distance.

We are not told in {1} what an example of an unusual situation is, but such an example is provided by the contrast in {2} and {3} between viewing a heavenly object at a point near the zenith and viewing the same object near the horizon "in accordance with what is customary": Vision is said to be more difficult in the former, unusual case, with the result that the object looks smaller. Adding to this our conjecture that Ptolemy considered the distance, too, to appear smaller in the unusual situation, one might be tempted to offer the following interpretation of all these statements put together: (1) In the customary, horizontal viewing, a heavenly object appears to be farther away from us than when it is viewed with difficulty by looking upward; (2) objects that subtend the same angle at the eye appear to be smaller when they seem to be nearer; (3) therefore a zenith-star will appear to us smaller than a horizon-star. It may be argued that such an interpretation would be consistent with the *O*-passage as we have understood it, but the trouble with this interpretation is that neither (1) nor (2) can be found in that passage. (1) and (2) would have to be brought in and developed from other parts of the *Optica*. Would Ptolemy have only cited the difficulty of looking upward had he intended an explanation like the one just given?

II. Alhazen

Not surprisingly, it was the passage from the *Almagest*, the *A*-passage, that attracted the attention of Islamic mathematicians. In the Islamic world the *Optica* never achieved anything like the wide circulation enjoyed by the *Almagest*, and neither the *O*-passage nor the *PH*-passage was likely by itself to compel a comparison with the situation described in the *Almagest*. Almost all Arabic commentaries on the *Almagest* and all parallel discussions of its contents included remarks on the *A*-passage. At first these remarks were neither extensive nor illuminating, being for the most part limited to paraphrasing Ptolemy's text; but they grew in variety and sophistication after Alhazen's *Optics* became widely known through the "Revision" prepared by Kamāl al-Dīn at the beginning of the fourteenth century A.D. Alhazen's *Optics* thus marked a turning point in the Arabic discussions of the moon-illusion problem.

From his writings we gather that the problem claimed Alhazen's attention at various times that ranged over his entire scientific career.[16] Thus his full, mature explanation of the moon illusion is contained in Book VII of his *Optics*, a work composed relatively late in his career. His first comments on the *A*-passage are, however, to be found in a *Commentary on the Almagest*, undoubtedly an early composition which Alhazen wrote before he had access to Ptolemy's *Optica*. In a short treatise "On the Appearance of the Stars" (*Fī Ruʾyat al-kawākib*), written probably before the *Optics*, Alhazen made an attempt to reconcile the *A*-passage with the doctrine of refraction as expounded in Ptolemy's *Optica*. This treatise was thus written after Alhazen became acquainted with the *Optica*. He returned to the same subject in two treatises that dealt with doubts or difficulties (*shukūk*) in Ptolemy's works. One of these treatises has reached us in the form of a series of comments bearing the title "Solution of Difficulties in the *Almagest* Which a Certain Scholar Has Raised" (*Ḥall shukūk fī al-Majisṭī yushakkiku fīhā baʿḍu ahl al-ʿilm*). The other is his critique of Ptolemy known as *Dubitationes in Ptolemaeum* (*al-Shukūk ʿalā Baṭlamyūs*), very likely the last of all these works in order of composition and, as far as our problem is concerned, the least informative.[17] In what follows

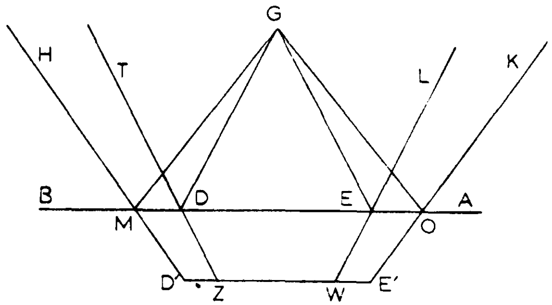


Figure 2

I shall deal first with Alazen's writings before the *Optics*, then with his views in the *Solution*, leaving his mature theory in the *Optics* to the end.

Alhazen's Commentary on the Almagest *and his* Treatise on the Appearance of the Stars

It will be instructive to look briefly at the primitive argument in the *Commentary on the Almagest*,[18] the sole purpose of which was to explain the apparent magnification of objects immersed in water and the increase of magnification with deeper immersion. The argument can be found already in an early-ninth-century compilation *On Optics and on Burning Mirrors* by a certain Aḥmad ibn ʿIsā, from which Alhazen may have obtained it.[19] The "explanation" begins with this curious statement:[20] "It has been shown in books of optics that rays of sight are reflected (*tanʿakisu*) from the surfaces of visible objects at equal angles and in straight lines, such as *EL*, *DT* [Figure 2], and that these lines enter into (*tanfudhu fī*) the transparent bodies and reach the object immersed in those bodies so that vision would occur by means of the reflected rays (*al-shuʿāʿāt al-munʿakisa*)."

Alhazen then argues as follows: The object *DE*, placed upon the water surface *AB* is seen by the eye at *G* through angle *DGE*. When sunk into the water to position *D'E'* the same object must now be seen through an angle *MGO* such that the extensions of the reflected rays *MH*, *OK* will reach *D'*, *E'*. "Thus it is evident that *DE* is seen on the water surface through the smaller angle *EGD*, and, when immersed in water, through the larger angle *OGM*; but that which is viewed through a larger angle appears larger, for we have shown previously that an object is seen according to the angle of vision; and for this reason, the deeper the object sinks the larger it will appear."

Such was the retarded understanding of refraction before Ptolemy's *Optica* became known.

With the treatise *On the Appearance of the Stars*[21] Alhazen reaches an entirely different level of understanding. He has now read and understood Ptolemy's theory of refraction in the *Optica* and he has become aware of new problems which, he feels, have not been fully dealt with by anyone, including Ptolemy. Most people think (he says at the beginning of the treatise) that the heavenly bodies are seen by rectilinear rays, while "experts in optics and mathematics" believe that the visual rays by means of which we perceive a star diverge upon striking the concave surface of the ether, thus causing the star to be seen as smaller than it is because the angle produced in the eye by the refracted rays will be smaller than that contained by the straight lines directly drawn to the extremities of the star's diameter. Both views are wrong, says Alhazen. The first ignores refraction altogether, and the second (he explains later in the same treatise) is true only in the special case in which the observed star is near the zenith.

> As for the excellent Ptolemy, he did not say in his *Optics* how sight perceives the bodies of the stars,[22] nor did he say how the ray [conceived as a solid volume] encompasses the star's body. He neither mentioned the angle through which the star is seen nor the mutual positions of the two rays that contain that angle and are refracted to the extremities of the star's diameter. [Ptolemy in the *Optics*] said [only] that a visible object is seen as smaller than its true magnitude when the eye is located in a medium denser than that in which the object is placed and when the two rays are refracted to the object's extremities away from the normal drawn from the eye to the separating surface. This is the case frequently described by writers on optics and repeated in their books.

To correct the generally received opinion Alhazen devotes the first part of his treatise to showing geometrically that (1) the apparent size of a star is diminished, regardless of altitude, as a result of being viewed by refraction; and (2) that the size is diminished less when the star is near the zenith than when it is near the horizon, because in the former case the amount of refraction is less and the difference between the angles of incidence and of refraction is less. The reduction in size should therefore in principle be more noticeable in horizontal viewing than in zenith viewing – which gives rise to the problem formulated by Alhazen as follows:

> A difficulty may occur to many mathematicians when they put Ptolemy's discourse in the *Optics*, where he shows that the magnitude of an object placed in a rarer medium than that in which the eye is located, appears to be smaller, and the greater the density of the latter medium the greater the

> diminution, and also says that the body in which the star is located is rarer than air, side by side with his discourse in the *Almagest* (*al-Taʿālīm*), where he says that the stars appear larger at the horizons because the vapor from the moisture surrounding the earth is interposed between the eye and the stars – for these two discourses are in appearance opposed to one another.

But only "in appearance," for, according to Alhazen, it is "possible" (*qad yumkinu*) that the interposition of thick vapor may cause the star to appear larger than it would when viewed in the absence of the vapor. He considers two cases (illustrated by Figures 3a and b) both yielding the same result. In both cases, let *ABE* be the plane of the azimuthal circle passing through the star *AT*. *AE* is the intersection of the circle with the horizontal plane through *E* where the observer is located. In Figure 3a let the vapor continuously fill the space between *E* and the concave surface of a rarer layer of air contained between *LN* and the ether surface *GD*. In Figure 3b a thin air lies on either side of the thick vapor contained between *OZ* and *LN*.

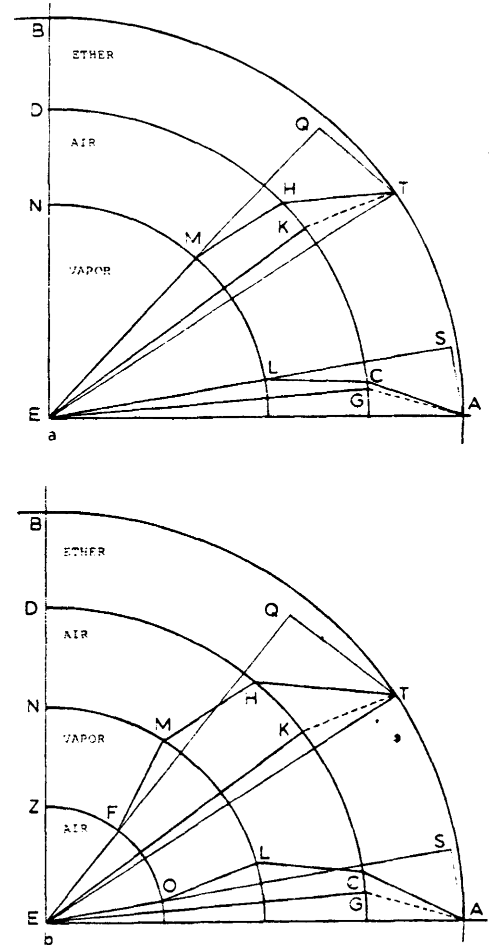
In both figures, let angle *GEK*, smaller than *AET*, be the angle of viewing *AT* in the absence of any dense moisture in the air between *E* and *GD* (I have added the broken lines *GA* and *KT*).

In the first case, in Figure 3a, the star *AT* will be seen by rays *EL* and *EM* that are refracted, first away from the normals at *L* and *M* into *LC* and *MH*, then again away from the normals at *C* and *H* into *CA* and *HT*, respectively.

In the second case, in Figure 3b, rays *EO* and *EF* are first refracted toward the normals at *O* and *F* into *OL* and *FM*, then refracted away from the normals twice at *L*, *M*, and at *C*, *H*, respectively.

After drawing the perpendiculars *TQ* and *AS* to *EM* and *EL* extended (in Figure 3a) or to *EF* and *EO* extended (in Figure 3b), Alhazen argues in both cases (a) and (b) that if $TQ{:}QO \geqslant AS{:}SE$, then *AT* will look bigger than it would if only one refraction took place at the ether surface. That is, he argues that angle *LEM* (or *OEF*) will be greater than angle *GEK*, the latter being the angle through which *AT* would be seen across a homogeneous atmosphere.

The argument cannot, of course, claim to constitute an explanation of the moon illusion as a constant phenomenon, nor was it intended to be such an explanation. It was merely meant to show that under certain conditions the apparent size of a celestial magnitude may be increased by the interposition of a dense moisture – an idea which, as we shall see, Alhazen retained and incorporated in the *Optics*. Since the treatise *On the Appearance of the Stars* makes no allusion to the *Optics* or to the psychological explanation offered in it, it seems reasonable to assume that it was written some time before the composition of that relatively late work, but this cannot be certain.



Figures 3a and b

Alhazen's treatise "Solution of Difficulties in the *Almagest* . . ."

Apart from the *Optics*, Alhazen's most interesting remarks on the moon illusion are contained in a treatise which, in the Istanbul MS Fatih 3439, folios 142b–150b, bears the title "Solution of Difficulties

in the *Almagest* Which a Certain Scholar Has Raised" (or: ". . . Which Some Scholars Have Raised"): *Ḥall shukūk fī al-Majisṭī yushakkiku fīhā baʿḍu ahl al-ʿilm*.[23] It consists of three distinct sections, the first of which (142b–145a) comprises five "difficulties" (*shukūk*) in Book I of the *Almagest*, and thus corresponds to title no. 38 in Ibn Abī Uṣaybiʿa's List III of Alhazen's writings.[24] Section II immediately follows under a new title, "Answers to Doubtful Questions (*masāʾil*) in the *Almagest*." These are nine questions which are not confined to Book I of the *Almagest*, and which occupy pages 145a–148b. Question nine ends with the statement "This is the last of your [*sic*] questions."[25] Section III (148b–150b) consists of four additional difficulties (*shukūk*) without an introductory title, but the fourth (concerning lunar parallax) is described as "one of the difficulties raised by Abū al-Qāsim ibn Maʿdān," a scholar otherwise unknown to me. The three sections may thus have been originally separate compositions, a conjecture confirmed by the fact that the same problem is treated in all of them. But all appear to have been written after the *Optics*, which is explicitly mentioned in sections I and III.

Only some of the difficulties discussed in this treatise can be reported here, and even these will have to be briefly summarized. The first of the five difficulties that make up section I concerns a problem suggested by the *A*-passage, which Alhazen paraphrases (or quotes?) as follows: "The apparent increase in the size of the stars when they are on the horizon is not perceived in them because of their decreased distance [from us] while in that position, but because a moist vapor that surrounds the earth rises between them and the eye, thus causing them to be seen thus; just as what is thrown in water appears larger, and the lower it sinks the larger it becomes."[26] It is important to be clear as to what the suggested problem is. It is not the problem of why the horizon-star is larger than the zenith-star. Nor is it the problem arising from comparing the apparently enlarged star which is viewed from a *dense* medium to the enlarged object viewed from a *rare* medium. (Alhazen had offered a solution to the first problem in his *Optics*, and he refers to the second in section III of the present treatise.) Rather it is the problem of simply explaining why an object placed in water appears to increase in size when it sinks deeper. In Alhazen's view the problem existed because a true statement in the *A*-passage about the magnification of immersed objects was not accurately explained in Ptolemy's *Optica*: the fault, he said, lay, not with the *Almagest*, but with the *Optica*.

Thus, referring to Book V of Ptolemy's *Optica*, and basing himself, as he says, on Ptolemy's "adopted method," Alhazen sets out to show how an immersed object looks larger than it is because the refracted rays will contain a larger angle than that contained by rectilinear rays.

When, however, the object is lowered, the angle between the visual rays, which now have to be refracted less in order to reach the extremities of the object in its farther position, must become smaller.

> Now if someone believes that the magnitude of an object is perceived according to the magnitude of the angle alone, then he will doubt Ptolemy's statement [in the *Almagest*] that the object increases in size as it sinks deeper. But the statement is subject to doubt only if the doubter relies exclusively on the angle. Such a reliance, however, would be a mistake; for Ptolemy has shown in the *Optics*, in the course of his discussion of size, that size is not perceived according to the angle alone, but according to the magnitude of the angle and of the [object's] distance and according to whether the object is inclined or frontally situated. Thus [Ptolemy] says in the second of his Propositions on size[27] that of two unequally distant objects and perceived through the same angle, the nearer never looks larger than the farther object, but is either seen as smaller [than the farther object] (when a sensible interval exists between them) or equal to it (when the difference between their distances [from the eye] is insensible). Thus if size were perceptible by means of the angle alone, then the two objects seen through the same angle, or through two equal angles, would always appear to be equal – which is not the case. Moreover, we have shown in our book on *Optics* that size is perceptible by [comparing] the angle to the distance and also in accordance with the angle alone.

In the course of citing a number of observations supporting the view just explained, Alhazen refers further to Ptolemy's "*Optics*, V, Prop. 17,"[28] after which he concludes:

> An object looks larger when it sinks deeper [in water] because its distance increases and because the distance of its image (*khayāl*) increases after it sinks and because the magnification entailed by the increase in distance is greater than the diminution entailed by the decrease in the angle [of vision]. And thus it evidently follows from the principles asserted by Ptolemy and from our own examples that an object must appear larger when it sinks deeper in water.

To represent the facts "more precisely" than in Ptolemy's *Optica*, Alhazen finally gives a proof of which the following is a simplified paraphrase.

Figure 4 is in the plane passing through the eye at *A* and the center of "the sphere of water" (i.e., center of the earth) at *I*, and intersecting the water-surface in arc *EZ*. *BG* and *WJ* are two positions of the same

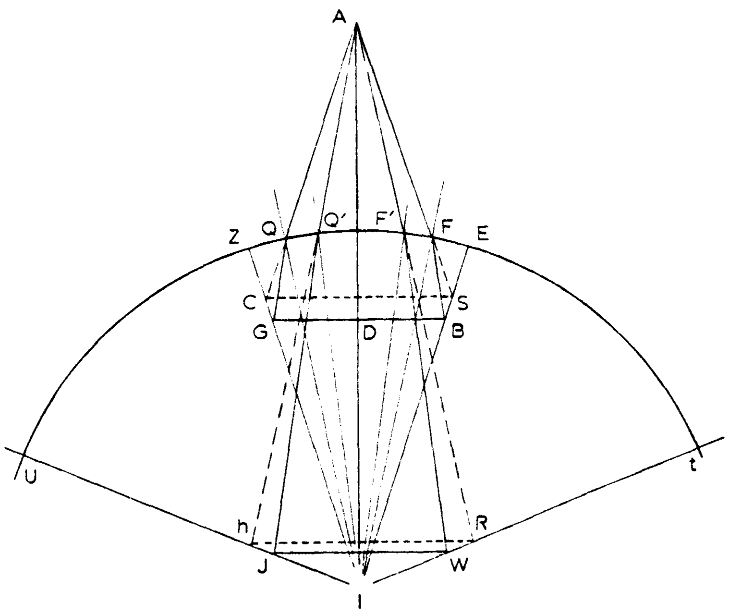


Figure 4

and similarly oriented object viewed from A – WJ being farther away from the eye than BG. The lines drawn from I are diameters of the sphere that cut the water-surface at right angles.

BG is first viewed by means of the refracted rays AFB and AQG. And since $\angle FAQ > \angle BAG$, BG will appear larger than it would if viewed by the rectilinear rays AB and AG.

At the lower position WJ, the object is viewed by means of refracted rays $AF'W$ and $AQ'J$, such that $\angle F'AQ' < \angle FAQ$. WJ should therefore look smaller than BG, if size were determined by the visual angle alone.

But, Alhazen maintains, the distance at which the object is seen is another factor which in fact outweighs the effect of the visual angle. To determine the locations of the images of BG and WJ he draws the normals to the refracting surface through the extremities of the two objects, and extends the incident rays to meet the normals at S, C and at R, h, respectively.

> Now Ptolemy has shown in the *Optics* that an object in water is seen at the location of the image; and if so, then it must be seen as having the magnitude of the image whether the angle of the image is greater or smaller, because the [object's] extremities are seen at the image's extremities. Therefore the magnitude BG will appear in water to be larger than it is because its image is larger than it. This, then, is the true explanation of the apparent magnification of objects seen in water.

A similar argument applies to WJ.

Two queries related to our problem make up the first question in section II.

Query 1: If refraction into a denser medium is a cause of the magnification of the horizon star, then where does this refraction take place if the eye is placed *inside* the dense medium? *Alhazen's answer:* It has to be assumed that the moisture rising on the horizon does not extend all the way to the observer's locality; so that a ray issuing from the observer's eye will first travel through rare air before it strikes the surface of the dense moisture. And it has to be assumed further that this surface must be plane, though it need not be continuous. That is, the refraction causing the magnification must be produced by parts of the moisture that are distributed on a plane surface, in the same way that the water droplets responsible for producing the rainbow are arranged in a regularly circular shape without this being the shape of the misty air itself.

Query 2: Where should the star's apparent size be closer to its real size – near the horizon or near the zenith? *Answer:* At the latter position, where the rays are refracted least.

There is no mention in section II of any psychological explanation, from which it does not follow that this section antedated the *Optics*. The answers given here are in fact offered as a *supplementary* explanation in the *Optics* (see the "Translation of Alhazen's text in *Optics*" below).

The first of the four doubts raised by Abū al-Qāsim ibn Maʿdān, and discussed in section III, assumes, like query 1 in section II, that the dense vapor continuously surrounds the entire surface of the earth. In support of his denial of this premiss Alhazen oberves that we are able to endure contemplating the sun for a while at rising and setting, but not when it is above our heads. This is "certain proof," he says, that the dense moisture at the horizon does not extend into the atmospheric areas directly above us. Alhazen thus maintains here that the magnifying effect of the moisture exists only for an observer located in the inhabited region of the northern hemisphere, and that such a moisture rises only from the water encircling that region. Concluding this section he writes:

This being made clear, I say that what I have stated so far are the answers to your questions.[29] However, other difficulties exist throughout the *Almagest*, but I merely answered the questions addressed to me. Among these difficulties that concern the magnification of the stars at the horizon is [the following]: the stars are in the heavens, and the heavens are rarer than the air; and if a visible object lies in the rarer medium while the eye lies in the denser medium, then the object must be seen as smaller than it is; and the greater the density of the medium close to the eye the smaller the object will appear; therefore the stars should appear smaller at the horizon than in the middle of the sky. I have explained this matter in my book on *Optics*, in the discussion on refraction, where I showed that the magnification of the stars at the horizon has a universal cause, other than the moisture, on account of which the stars and their mutual distances appear to be larger at the horizon than at the middle of the sky or at other altitudes higher than the horizon. I have also shown [there] that the moisture at the horizon adds to the increase in size that is due to the universal cause. I have not gone into this here because it is not one of the doubts you have raised.[30]

Alhazen's psychological explanation of the moon illusion

Alhazen offered his psychological explanation of the moon illusion at the end of the last chapter in Book VII of his *Optics* (*Kitāb al-Manāẓir*), a chapter devoted to the "errors of sight" due to refraction. The explanation follows a discussion of the role of refraction in viewing heavenly bodies. It is an original and sophisticated explanation which deserves a full commentary. Such a commentary cannot be given here, but, fortunately, Alhazen's text can stand on its own, being easily understandable without extra help.

The following translation is made from the Istanbul MS Fatih 3216 (copied at Baṣra in A.H.476/A.D.1084), folios 131b–138b, corresponding to page 280, line 54, to page 282, line 61, in F. Risner's edition of the medieval Latin translation of the *Optics*.[31] Additions and corrections made with the aid of this Latin text are enclosed in angle brackets. Expressions in parentheses are in the original Arabic text; my own additions are in square brackets. I have not noted omissions or departures in the Latin from the Arabic text.

I have used "magnitude" for *miqdār*, and "size" for both *ʿiẓam* and *miqdār*, two words used indifferently in the Arabic text. "Distance" and "interval" both stand for *buʿd*/*remotio*; "distance" always means

remoteness from the observer, unless otherwise specified. "To guess" or "conjecture" render *ḥadasa*/*perpendere, aestimare*. "Refraction angle" (*zāwiyat al-inʿiṭāf*) is the visual angle through which the object is viewed by refraction; "refracted angle" (*al-zāwiya al-munʿaṭifa*) is the angle through which the incident ray is refracted. I have translated *ʿilla* as "cause," "reason," and "explanation."

TRANSLATION OF ALHAZEN'S TEXT IN *OPTICS*, BOOK VII

We say that sight will perceive any star at the zenith to be smaller than in any region of the sky through which the star travels; that the farther the star is from the zenith the larger its magnitude will appear than it does at the zenith; that the star looks largest at the horizon; and that the same is true of the intervals between the stars. Now this is found to be so in fact, namely, that the stars, and their mutual distances, appear to be smaller at the middle of the sky than when they are far from it, and that the star (or interval between two stars) appears largest at the horizon. It remains for us to show the reason why this is so.

We say: It has been shown in Book II of this work, in our discussion of size, that sight perceives size from the magnitudes of the angles subtended at the center of the eye and from the magnitudes of the distances of the visible objects and from comparing the magnitudes of the angles to those of the distances. We have also shown there that sight neither perceives nor ascertains the magnitudes of the objects' distances unless these distances extend along near and contiguous bodies; that distances that do not so extend are not ascertainable in magnitude by sight; and that when sight cannot ascertain the magnitudes of the objects' distances, then it fails to ascertain the objects' sizes. We showed there, too, that when sight fails to ascertain the distance of an object, then it makes a guess in regard to the distance's magnitude by likening it to the distances of familiar objects at which it can perceive objects similar to that object in form and figure, then perceives the size of that object from the magnitude of the angle subtended by it at the eye-center as compared to the distance it has conjectured. But the distances of the stars do not extend along near bodies. Sight does not, therefore, perceive or ascertain their magnitudes, but merely conjectures their magnitudes by assimilating the stars' distances to the

distances of very remote earthly objects which it can perceive and whose magnitudes it conjectures.

Moreover, the body of the heavens is not seen as a sphere whose concavity faces the eye; nor is sight aware of the corporeality of the heavens, and only perceives of the heavens a certain blue color. As to the heavens' corporeality, extension in the three ⟨dimensions⟩, circularity and concavity – these sight has no way of perceiving. And when sight cannot identify something it likens it to one of the familiar objects that resemble it. Thus it perceives the sun and moon as flat, and perceives convex and concave bodies from an excessively great distance as flat, and also perceives arcs whose convexity or concavity faces the eye as straight. For when sight does not perceive [in the case of convex objects] the nearness of their middle points and the remoteness of their extremities, or (in the case of concave objects) the remoteness of their middles and the nearness of their extremities, it likens the convex and concave surfaces to flat surfaces and likens arcs to straight lines, because most familiar objects have flat surfaces and straight edges.

Nor, when the form of a star reaches the eye, is sight aware that it is a refracted form, or that it has been refracted from a concave surface, or that the body in which the star is is rarer than that in which the eye is. Sight rather perceives the form of a star as it perceives the forms that come to it in straight lines from objects located in the air. The forms of visible objects, when they encounter a body whose transparency differs from that of the body in which they were, do not undergo refraction for the sake of the [seeing] eye; nor will the eye be aware of their refraction or of the surface of the differently transparent body; rather, the refraction occurs in virtue of a natural property that is peculiar to the forms of light and color. The refracted forms of the stars thus reach the eye just as the forms of visible objects in the air reach it, and sight perceives them just as it does the forms of objects in the air.

Sight perceives the color of the sky and the extension of that color in length and breadth without perceiving the shape of the sky or identifying its figure by pure sensation. And when sight perceives a color as extended in length and breadth without perceiving its shape or identifying its figure, then it perceives it as flat, because it assimilates it to familiar flat surfaces that extend in length and breadth, such as those of walls and the ground. Similarly, sight perceives convex and concave surfaces from a large distance as flat, and also perceives the surface of the earth in wide areas as flat and is not aware of its convexity in the absence of hills and protrusions and depressions.

Sight, therefore, perceives the surface of the heavens as flat and perceives the stars in the same way as it perceives familiar objects scattered over wide areas of large dimensions, and assimilates the distances of the stars to those of familiar objects scattered over vast areas on the earth's surface the ends of which [areas] sight perceives as farther away than their middles and perceives those points that are close to the middle as less distant than those that are further removed from it. But if sight perceives under equal angles a number of objects scattered over a large area, while perceiving the magnitudes of the distances of those objects, then it will perceive the farther of those objects as larger. For it will perceive the size of the far object from comparing the angle subtended by that far object at the eye-center to a large distance, and will perceive the size of the nearer object from comparing the angle subtended by this near object (which is the same as the angle subtended by the far object) to a smaller distance.

Now this is found to be clearly the case, namely, that when two objects are viewed under the same angle (or under two equal angles), and their distances differ appreciably, then the farther object will look bigger. For let someone stand in front of a wide wall and raise his hand before one eye while closing the other, then look with one eye while holding his hand between it and the wall. His hand will screen a portion of the width of that wall. But since the hand and the [screened] width of the wall are seen at the same time, then they are seen through the same angle; and sight will at the same time perceive the [screened] width of the wall to be many times larger than the hand. And if the person moves his hand to one side so as to expose the hidden portion of the wall, and looks at the exposed portion and at his hand, he will perceive that portion to be many times larger than his hand. And he perceives his hand and the exposed portion by two equal angles. This experiment therefore shows that sight perceives size from comparing the angle to the magnitude of the distance.

Now sight perceives the surface of the heavens as flat and does not perceive its concavity, and it perceives the stars scattered over it. It therefore perceives separate and equal

stars as having unequal sizes, because it compares the angle subtended at the eye-center by the extreme star near the horizon to a large distance, while comparing the angle subtended by the star at or near the middle of the sky to a small distance. Thus it will perceive the star at or near the horizon to be larger than the star at the middle of the sky or near the zenith; and will perceive one and the same star (or interval between two stars) at different points in the sky to be of different magnitudes. It will thus perceive one and the same star to be larger at or near the horizon than at or near the middle of the sky, and will perceive the interval between two given stars to be larger at the horizon than at or near the middle of the sky, because it compares the angle subtended at the eye-center by the horizon-star to be a large distance and compares the angle subtended by the zenith-star to a small distance. And there is no great discrepancy between the two angles; rather, they are ⟨close⟩ though different. And the case is similar with intervals between the stars. But if the sense [of sight] compares two angles close in magnitude to two distances of greatly different magnitudes, then it will perceive the farther [object] to be larger.

The proof of the truth of this explanation is that the angles subtended at the eye-center by a given star from all regions in the sky ⟨when these angles are contained by refracted lines⟩ are equal ⟨to those through which the star is perceived⟩ by means of straight, unrefracted lines. For the eye being located at the center of the heavens, these angles will not be greatly reduced by refractions of the stars' forms. And if these reductions are not greatly different in magnitude, then the difference between the refraction angles through which the star (or interval between two stars) is perceived in different locations will not be great. And if so, then the size of the star (or interval between two stars) will not seem to differ greatly at different points in the sky on account of the difference between the angles. That the refraction angles are not greatly smaller than the angles contained by the straight lines has been shown in the experiment described in the chapter on refraction, where it was made clear that sight perceives the star by refraction. . . .[32] From which it is manifest that the refracted angles are too small to bring about a great difference between the angles through which sight perceives the star at different points in the sky. But there is a great difference in size between the star (or interval between two stars) at the

horizon and at the middle of the sky. Therefore the difference between the refraction angles cannot be the cause of that difference in size at the various points in the sky. And it has been shown that sight perceives the sizes of visible objects by ⟨comparing⟩ the angles to the distances. Therefore, if the difference between the angles is small, and the difference between the distances is great, and a visible object appears larger from the greater distance – if all that is so, then the reason why the stars (and intervals between them) are seen to be larger at the horizons than at or near the middle of the sky is the conjecture made by the sense [of sight] regarding their greater distance at the horizons than when they are at the middle of the sky.

What sight perceives regarding the difference in the size of the stars at different positions in the sky is one of the errors of sight. It is one of the constant and permanent errors because its cause is constant and permanent. The explanation of this is [as follows]: Sight perceives the surface of the heavens that faces the eye as flat, and thus fails to perceive its concavity and the equality of the distances [of points on it] from the eye. And it has been established in the mind that flat surfaces that extend in all directions about the eye are unequally distant from it, ⟨and that those directly above are closer to the eye than those to the right and left of it⟩. Now sight perceives those parts of the sky near the horizon to be farther away than parts near the middle of the sky; and there is no great discrepancy between the angles subtended at the eye-center by a given star from any region in the sky; and sight perceives the size of an object by comparing the angle subtended by the object at the eye-center to the distance of that object from the eye; therefore, it perceives the size of the star (or interval between two stars) at or near the horizon from comparing its angle to a large distance, and perceives the size of that star (or interval) at or near the middle of the sky from comparing its angle (which is equal or close to the former angle) to a small distance; ⟨and it perceives a great difference between the distance of the middle and that of the horizon⟩. We have thus stated the cause on account of which sight errs in regard to the difference in the size of the stars (or their mutual intervals); and it is a constant, permanent and unchanging cause.

That is also the cause of sight's perception of the stars as small on account of their remoteness. For the star, when remote, subtends a small angle at the center of the eye; and,

failing to ascertain the magnitude of the star's distance, the sentient [faculty] merely makes a conjecture in regard to this magnitude, comparing the distances of the stars to the distances of familiar but excessively far objects on the surface of the ground. Thus [the sense faculty] compares the angle produced by the star at the eye (which is a small angle) to a distance like the earthly distances, and in consequence of this comparison perceives the star as small. If sight had a true perception of the magnitude of the star's distance, it would perceive the star as large. Similarly, sight perceives all excessively far objects on the surface of the ground as small because it does not ascertain their distances. We have thoroughly explained this in Book III of this work.

And just as sight errs in regard to the magnitude of a star's distance because it fails to ascertain it and because it assimilates it to distances on the surface of the earth, so also it errs in regarding the distances of the star at different positions in the sky as unequal in magnitude, though these distances are equal, because, again, it assimilates them to certainly unequal distances to the right or left or in front of it on the surface of the ground. And just as its error in regard to the star's distance and size is permanent and constant, so also its error in the difference between the star's distances and size at various positions [in the sky] is permanent and constant. For the form of these distances does not vary in the eye from time to time but is always the same, and it is sight that assimilates it to the distances of familiar and excessively far objects on the surface of the earth.

The enlargement of heavenly objects at the horizon may frequently have another cause. This cause occurs when a thick vapor stands between the eye and the star positioned at or near the horizon, if the vapor is at or near the horizon and does not continue to the middle of the sky but rather forms a section of a sphere whose center is the center of the world because it surrounds the earth. If such a section terminates before [reaching] the middle of the sky, then the surface of it that faces the eye will be plane. But if the surface of the vapor facing the eye is plane, then the form[s] of the stars (and intervals between them) will be seen behind the vapor as larger than before the vapor occurred. Because the form of the star will [first] occur at the place on the heavens concavity from which it will be refracted to the eye.

Then, in the absence of the thick vapor, the form would extend from this place to the eye in straight lines. But, in the presence of the thick vapor at the horizon, this form will extend to the surface of the vapor that faces the [middle of] the sky, and thus occur in that surface. Sight will therefore perceive this form just as it would perceive objects placed in the vapor: that is, the form will extend through the thick vapor on straight lines then will be refracted at the surface of the vapor facing the eye, this refraction being away from the normal to the vapor's surface (which is a plane surface), since the air near the eye is rarer than the thick vapor. It follows from this that the form will appear to be larger than it would if it were seen rectilinearly. (This was shown in Proposition 1 of this chapter, namely, that when the eye is in the rarer medium and the visible object in the denser medium, and the surface of the denser medium is plane [then the object will look bigger than it is].) Thus the form that occurs in the surface of the vapor facing the middle of the sky is the visible object, and the medium in which this form is is the thick vapor, and the eye is in the rarer medium of air.

The principal cause on account of which the stars (and their mutual intervals) are seen at the horizons to be larger than at or near the middle of the sky is the one stated earlier. It is the inseparable and permanent cause. When, however, a thick vapor happens to rise at the horizons, it increases their magnification. But this is an accidental cause which always occurs [only] in certain regions of the earth and occasionally in others but is not permanent.

Appendix: Edition of the Arabic text of the *PH*-passage

The Arabic text of the *PH*-passage presents the translator with certain textual difficulties that make it necessary to declare how one proposes to read it before attempting to put it into another language. (See "Notes on the translation of the *PH*-passage" above.) For the reading proposed here, I have relied on two manuscripts of the Arabic text of Ptolemy's *Planetary Hypotheses*: British Library MS Add. 7473 (pp. 92a–b) and Leiden University Library MS Or. 1155 (p. 25b), both of which can be consulted in the facsimile and variants published by Bernard Goldstein and referred to earlier. (In the notes following the Arabic text of the *PH*-passage, I refer to these two manuscripts as *B* and *L*, respectively.) I have also used a quotation of the same passage in Ibn al-Shāṭir's *Nihāyat al-su'l* as preserved in six manuscripts: Bodleian Library, Marsh 139, pp. 36a–b; Hunt 547, 52b–53a; Marsh 290,

45b–46a; Marsh 501, 49a–b; Leiden University Library, Or. 194, 75a; Suleymaniye Library (Istanbul), 339, p. 138 (= 69b).*

> {1} Wa ammā al-sababu alladhī min ajlihi ṣāra mā yaẓharu li-l-naẓari wa yutakhayyalu ilayhi min ʿiẓami jirmihā laysa ʿalā nisabi abʿādihā fa-yanbaghī lanā an naʿlama annhu al-ghalaṭu alladhī yadkhulu ʿalā al-baṣari min qibali ikhtilāfi al-manāẓiri. {2} Wa yatabayyanu dhālika fī jamīʿi mā yaẓharu wa yurā ʿalā buʿdin kathīrin. {3} Fa-kamā anna al-abʿāda anfusahā lā takūnu kammiyyatuhā maʿlūmatan mimmā yaẓharu li-l-ʿayni fa-lā al-tafāḍulu fī-mā bayna al-ashyāʾi al-mukhtalifati al-miqdāri minhā yuʿlamu ʿalā al-tanāsubi allatī hiya ʿalayhi, {4} li-jamʿi al-baṣari wa qabḍihi iyyāhu [wa] tanqīṣihi lahu ilā mā huwa lahu ashaddu ilfan. {5} Wa li-dhālika narā kulla wāḥidin mina al-kawākibi qarīban minnā akthara min ḥāli ḥaqīqatihi li-inḥiṭaṭi al-baṣari ilā al-abʿādi allatī qad iʿtādahā wa alifahā. {6} Ka-dhālika al-ḥālu fī al-ziyādāti wa al-nuqṣānāti allatī taʿriḍu li-l-ʿiẓami bi-ḥasabi ziyādati al-abʿādi wa nuqṣānihā, fa-innahā takūnu aqalla mina al-nisbati, ka-al-ḥāli fī al-abʿādi, li-ʿajzi al-baṣari, ka-ma qulnā, ʿan tamyīzi wa idrāki aqdāri kammiyyati tafāḍuli kulli nawʿin mimmā dhakarnā.

{1} al-manāẓiri] al-manẓari *L* {2} Wa yatabayyanu] wa nubayyinu *B*, *L* – *B adds* ikhtilāf {3} mimmā] fa-mā *B* – fī-mā *L* / fa-lā] wa lā *B*, *L* / yuʿlamu] *first letter undotted in B* {4} li-jamʿi] jamʿ *B* / tanqīṣihi] *all letters undotted in B – 3rd, 4th and 5th letters dotted as* q, y *and* ḍ *in L* / lahu (*second occurrence*)] *omitted in B* (4} narā] *no dots for the first letter in B* / wa alifahā] *B adds*: fī-mā (?)bayyannā (as we have shown), (?)*or*: fī-mā baynanā (around us) {6} aqalla] anqaṣa *L*

Notes

1 See Helen E. Ross and George M. Ross, "Did Ptolemy Understand the Moon Illusion?," *Perception* 5 (1976):377–85.

2 *Ptolemy's Almagest*, translated and annotated by G. J. Toomer, New York: Springer-Verlag, 1984, p. 39.

3 O. Neugebauer, *A History of Ancient Mathematical Astronomy*, New York: Springer-Verlag, 1975, part II, p. 896.

4 For Theon's text, see A. Rome, ed., *Commentaires de Pappus et de Théon d'Alexandrie sur l'Almageste*. Vol. II: Théon d'Alexandrie, *Commentaires sur les livres 1 et 2 de l'Almageste* (Città del Vaticano: Biblioteca Apostolica Vaticana, 1936), pp. 346–51. An analysis of this text, including a French translation of the passage examined here, is in A. Rome, "Notes sur les passages des *Catoptriques* d'Archimède conservés par Théon

* I am grateful to George Saliba for photographs of the relevant pages in the last five manuscripts.

d'Alexandrie," *Annales de la Société Scientifique de Bruxelles*, ser. A, 52 (1932):30–41.

5 Albert Lejeune, ed., *L'Optique de Claude Ptolémée dans la version latine d'après l'arabe de l'émir Eugène de Sicile* (Louvain: Publications Universitaires, 1956), bk. II, paras. 52–63, pp. 38–46, esp. para. 56, p. 40. (The text of the *Optica* in this edition will hereafter be referred to as "Ptol. *Opt.*" See also Lejeune, *Euclide et Ptolémée: Deux stades de l'optique géométrique grecque* (Louvain: Publications Universitaires, 1948), pp. 95–9.

6 Ptol. *Opt.*, V, para. 6, pp. 225–6; also paras. 70–71, pp. 262–3.

7 Bernard Goldstein, *The Arabic Version of Ptolemy's Planetary Hypotheses*, Transactions of the American Philosophical Society, n.s., vol. 57, pt. 4, Philadelphia, 1967. A third copy of the *Hypotheses* has since been reported to exist in the Osmania University Library at Hyderabad, Dn. no. 306; see S. Sezgin, *Geschichte des arabischen Schrifttums*, VI (Leiden: E. J. Brill, 1978), pp. 94–95. I have been unable to consult this copy.

8 For accounts of Ptolemy's procedure see, in addition to Goldstein's translation and commentary, Olaf Pederson, *A Survey of the Almagest* (Acta Historica Scientiarum Naturalium et Medicinalium, 30), Odense, Denmark: Odense University Press, 1974, pp. 393–6; O. Neugebauer, *A History of Ancient Mathematical Astronomy*, II, pp. 919–22.

9 Neugebauer, *A History of Ancient Mathematical Astronomy*, II, p. 896. Having noted that Ptolemy's incorrect explanation in the *A*-passage must have antedated the *Optica*, Neugebauer continues: "Indeed, in the 'Planetary Hypotheses' this explanation is no longer upheld and the said phenomenon is recognized as an optical illusion, caused by wrongly estimating size in relation to nearby terrestrial objects, a topic further studied in his 'Optics.'" Neugebauer refers to Goldstein's translation of the *A*-passage. *

10 Bodleian Library MS March 139, p. 46a, lines 3–6.

11 Ibid., pp. 46a–b.

12 Ptol. *Opt.*, III, 59, pp. 115–16. (I have added the numerals in braces to facilitate references to the passage.) Lejeune supplies the following explanation for sentence {3}: "Parce que les distances verticales nous paraissent moindres que les distances horizontales et que nous interprétons les angles visuels en fonction de la distances supposée" (p. 116 n.53). But this piece of reasoning is not actually stated in the *O*-passage.

13 After raising the problem of interpreting "*minus sensit*" in sentence {1} (see first paragraph of "Commentary" on translation of *O*-passage), Ross and Ross go on to argue as follows: "But there is a more direct reason for doubting that Ptolemy intends any form of psychological explanation. It is clearly implied by the first sentence quoted that the images are already of different sizes by the time they are formed in the eye, since Ptolemy attributes the illusion to the visual ray itself, and not to the mind or brain in processing the sensory information it receives. So, if the ray is responsible, the change in size will already be present before any psychological factors can come into play" ("Did Ptolemy Understand the Moon Illusion?," p. 381). The argument loses force if the faculty of sight, and not what happens to the physical ray itself, is said to be responsible for the visual effect.

14 Ibid., p. 381.

15 Ibid.

16 See article "Ibn al-Haytham" in *Dictionary of Scientific Biography* VI (1972), pp. 189–210, esp. p. 205.

17 The Arabic text of the *Dubitationes* has been edited by A. I. Sabra and N. Shehaby as *al-Shukūk ʿlā Baṭlamyūs*, Cairo: Dār al-Kutub, 1971. An English translation of the section of this work that deals exclusively with Ptolemy's *Optica* is in A. I. Sabra, "Ibn al-Haytham's Criticisms of Ptolemy's *Optics*," *Journal of the History of Philosophy* 4 (1966):145–9. The reference to the *A*-passage occurs in the first section of the book, which is concerned with problems in the *Almagest*. Ibn al-Haytham's criticism here is the same as that quoted below from section III of his *Ḥall shukūk fi al-Majisṭī*.

18 This has survived in Istanbul MS Ahmet III 3329, copied in Jumādā II, 655 (A.D. 1257), 123 fols.

19 Aḥmad's work is extant in two Istanbul MSS: Laleli 2759_2, and Ragip Paṣa 934, both of which are listed in Max Krause, "Stambuler Handschriften islamischer Mathematiker," *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abt.B: Studien, Band 3 (1936):513–14. The relevant pages are 72a–74a in the first manuscript, and 54a–56a in the second.

20 The text of the whole argument in Ibn al-Haytham's *Commentary on the Almagest* is reproduced in the edition of the *Shukūk* cited in note 17 above, pp. 74–7.

21 The treatise *Fī Ruʾyat al-kawākib* is reproduced by Mullā Fatḥollāh Shirwānī (d. A.H. 891/A.D. 1486) in an optical appendix included in his commentary on al-Ṭūsī's *Tadhkira*, as I have noted in a photograph of this appendix from MS 493 in the Central Library of Tehran University, kindly provided by Dr. Ḥusayn Maʿṣūmī. I have also consulted a transcription of the same treatise made by Anton Heinen from a manuscript in a private collection at Lahore; see Heinen's "On some hitherto unknown manuscripts of works by Ibn al-Haytham," to appear in the Proceedings of the Second International Symposium on the History of Arabic Science, held in Aleppo, 1979.

22 Ibn al-Haytham was, of course, aware of Ptolemy's treatment of atmospheric refraction in Book V of the *Optica*. What he means here is that Ptolemy did not explain how refraction affects perception of the star's size.

23 I have not been able to consult two other manuscripts bearing the same title; see Sezgin, *Geschichte des arabischen Schrifttums* VI (Astronomie), p. 258 (no.13).

24 Cf. *Dictionary of Scientific Biography* VI, pp. 206–7.

25 "*wa huwa ākhiru mā dhakarahu min al-masāʾil.*" I take the third person singular form in *dhakarahu*, not literally but as the common polite form of address, thus assuming that sections II and III are pieces of correspondence with, possibly, the same person mentioned in section III (see first paragraph under "Alhazen's treatise . . . "; Abū al-Qāsim ibn Maʿdān).

26 Compare Ibn al-Haytham's quotation of the same passage in *al-Shukūk ʿalā baṭlamyūs*: "The sun is seen larger at the horizon than when it is in the middle of the sky only because a moist vapor that surrounds the earth occurs between it and the eye, thereby causing it to be seen thus – just as what is thrown in water appears larger, and the deeper it sinks the larger it becomes" (p. 5).

27 This is Proposition 7 in Book II in Lejeune's edition of Ptol. *Opt.*, p. 40:

> Veluti si fuerint due quantitates *ab*, *gd*, habentes eundem situm [i.e., orientation] et subtendentes eundem angulum qui est *e*. Cum ergo distantia *ab* non sit equalis distantie *gd*, sed propinquior ea, *ab* utique numquam apparebit maior quam *gd* secundum quod decet propter propinquitatem suam. Sed aut minor apparebit, quod fit cum distantia alterius ad altera habuerit sensibilem quantitatem: aut uidebitur equalis ei, quod fit cum quantitas diuersitatis distantie fuerit insensibilis.

Ibn al-Haytham's words are almost a verbatim reproduction of Ptolemy's text.

28 Corresponding to Proposition 98 in Lejeune's edition of Ptol. *Opt.*, V, 76, pp. 264–5.

29 *masāʾilihi* (his questions). For changing the form of this word from third to second person, see note 25, above.

30 *min shukūkihi* (of the doubts he has raised). For translating this in the second person, see note 25, above.

31 F. Risner, ed., *Opticae thesaurus. Alhazeni Arabis libri septem, etc.*, Basel, 1572.

32 The ellipsis stands for a sentence I cannot make out in Arabic or Latin. In the Latin translation, which is fairly literal, it reads as follows: "Et videt stellam fixam ex polo mundi, et remotio eius est ipso in una revolutione; nam haec diversitas invenitur parva; ex quo patet, etc." (Risner, *Opticae thesaurus*, p. 282, lines 5–6).

X

SENSATION AND INFERENCE IN ALHAZEN'S THEORY OF VISUAL PERCEPTION

I

Alhazen's theory of visual perception occupies a considerable part of his *Optics*, itself a large work in seven books averaging about forty-three thousand words each.[1] Of the eight chapters that constitute the Arabic text of the first book, five are taken up by a theory of vision that includes a discussion of the effect of light upon sight, a description of the structure of the eye, an explanation of the manner of vision, a statement of the uses of the various parts of the eye, and finally an account of the conditions of vision.[2] Book II, on the objects of vision and the manner of their apprehension, contains the core of the theory of perception. Book III is on the errors of direct vision and deals, among other things, with errors of "pure sensation," of recognition, and of perceptual inferences. The errors of vision through reflection from mirrors of various types are the subject of Book VI, and the last chapter of Book VII is concerned with errors due to refraction. Some of these considerations already figure, sometimes prominently, in Ptolemy's *Optics*, from which Alhazen's own investigations clearly started. But no such detailed, elaborate, and systematic treatment of the subject of perception has come down to us from any writer in antiquity or in the Middle Ages prior to the eleventh century.[3]

Nevertheless, Alhazen's views, and, to a slightly lesser extent, those of Ptolemy, have been almost totally neglected by historians,[4] and, it seems, for the same reason. Historians of philosophy who are concerned with the history of perception have usually regarded works on optics as scientific or mathematical and therefore falling outside their domain, whereas historians of science and mathematics have tended to ignore the psychological sections in such works as properly belonging to philosophy. Even historians of optics who have given attention to Alhazen's doctrine of vision appear for the most part to have assumed that it was feasible to elucidate the account presented in book I of the *Optics* without exploring the subsequent account on perception, as if the two could meaningfully be divorced from one another.

The questions that can be asked about Alhazen's theory of perception are many and of different kinds. Some are historical and concern its relation to preceding theories (for example, those of Ptolemy and Aristotle), or the extent and consequences of its influence on later thinkers in the Islamic world and in Europe. Others concern the nature of Alhazen's approach to the problem of perception, the character of his treatment, or the details of his explanations. In the following pages an attempt will be made to clarify certain key concepts in Alhazen's treatment of perception, with emphasis on the connection between the accounts of the first and second books of the *Optics*, and on the distinction between sensation and inference that is basic to the second book.

II

The account of vision (*ibṣār*: *visio*)[5] in book I is an account of how a faithful representation (the word is not Alhazen's) of the distribution of light and color on the surface of a visible object is conducted through the medium to the eye and thence to the brain. Strictly speaking, however, there is no vision *of the object* without the concurrence of certain mental operations that it is the aim of book II to expound. And though Alhazen speaks of the occurrence of the representation in the brain as a sensation (*ihsās*: *sensus*), this sensation, according to him, never takes place without being accompanied by some of those operations. His first account may therefore properly be described as a theory of the physical and physiological conditions of vision.[6]

This theory combines two concepts that it had become traditional to oppose to one another: the concept of form (*ṣūra*,

eidos, *forma*) associated with Aristotle and his followers, and the concept of ray on which mathematicians like Euclid and Ptolemy had based their geometrical explanations of vision.[7] Alhazen regarded this combination as a synthesis between a physical and a mathematical approach to the study of light and vision. He characterized the physical approach as one concerned with *what* things are—*what* light and transparency, for example, are—and the mathematical approach as one aiming to determine *how* things behave; for example, *how* light extends in transparent media or *how* it is reflected from polished surfaces or refracted when passing from one medium into another.[8] From the physicists or natural philosophers he obtained such statements as: light is an "essential form" of self-luminous bodies, and an "accidental form" in bodies that are illuminated from outside; transparency is an "essential form" in virtue of which light is transmitted from one point to another; and a ray is an "essential form" that extends rectilinearly in transparent bodies.[9] From the mathematicians, and from Ptolemy in particular, he inherited the experimental and mathematical character of his entire book.

The Arabic version of Ptolemy's *Optics* that was known to Alhazen, like the extant Latin translation made from it in the twelfth century, lacked the first part in which Ptolemy proposed a theory of luminous and visual radiations.[10] This accident had important consequences for the subsequent history of optics. It signaled the absence of a theoretical basis that students of the subject were invited to make up for as best they could. Alhazen's achievement can be viewed as an attempt to fulfill this task by subjecting the "physical" or peripatetic doctrine of vision to a geometrical treatment in terms of rays. In the *Optics* he identified the physicists' position as that according to which "vision is brought about by a form (*ṣūra*) that comes from the visible object to the eye and through which sight perceives the form (*ṣūra*) of the object"; whereas mathematicians agreed that "vision is brought about by a ray that goes from the eye to the visible object and by means of which sight perceives the object; that this ray extends on straight lines whose extremities meet at the center of the eye; and that each

ray through which a visible object is perceived has as a whole the shape of a cone whose vertex is the center of the eye and whose base is the surface of the object."[11] Obviously, these two doctrines could not simply be juxtaposed without modification. The resulting synthesis retained the mathematicians' rays, for example, but only as imaginary lines on which the forms extended.[12] And, for the first time, it provided an answer to the question of how a form representing the object passed without distortion through the eye considered as a dioptric structure. It is this question and the answer given to it by Alhazen that should now concern us.

The Greek atomists had explained vision as the impinging on the eye jelly of a stream of skins or idols that are continually being separated from the surfaces of visible objects. Now Alhazen's "forms" are not corporeal, but they are supposed by him to radiate in a manner that would have well suited an atomistic theory. For just as it would have been natural in such a theory to assume that atoms went off from all points of the object-surface in all directions (for why should they behave otherwise?), so Alhazen's forms radiated not from the object as a whole but from every point on it rectilinearly in every direction. And the fact that these forms were immaterial would make them less vulnerable to objections which the earlier atomists might perhaps have found more difficult to answer.

The forms that radiate in this manner are forms of the light and the color in the object, whether essential or accidental. Accidental or acquired light and color are *not reflected* from an opaque object, but, rather, having been "received" and "fixed" in the object, they behave like essentially inherent light and color, i.e., radiate from all points in all directions. Reflection proper is a case of sending back the impinging light and color, and it takes place in a given plane at a given angle. Colors, it must be emphasized, are no less real that light. They may even be capable of sending out their forms independently of light, but they are never visible without it. The forms of color are always found mingled with those of light.[13]

Alhazen's explanation of the manner of vision, in the limited sense indicated above, as an application of this principle of

the radiation of forms to a certain view of the construction and functioning of the eye. Starting from current anatomical knowledge originally derived from Galen, he conceived of a geometrical arrangement of the principal coats of the eye that served his own purpose. The uvea is a sphere that contains the albugineous, crystalline, and vitreous humors, in this order. It is placed close to the cornea and therefore eccentrically to the eyeball. The front surface of the crystalline is, however, concentric with the cornea, or rather with that part of the cornea which directly faces the pupil. The middle of the pupil, the center of the uvea, and the center of the eyeball all lie on one straight line that extends to the middle of the optic nerve. The center of the eye lies behind the interface that separates the crystalline and vitreous humors. This interface is either plane or spherical so that the line of centers is perpendicular to it. It follows that lines drawn from the eyecenter to the surface of the cornea (the so-called lines of the ray, or radial lines) are all perpendicular to that surface and to the crystalline-surface.[14]

Now, in accordance with the stated principle, the light and color (or their forms) that extend along divergent lines from a given point on an object placed opposite the eye, will spread over the eye-surface. As this is true for every point on the object, the light and color from all these points will concur at every point of the eye-surface. And since refraction will take place at all points of that surface, still more confusion of forms will occur on the crystalline-surface. To make sure that a form preserving the order of the parts of the object as a whole is laid out on the crystalline-surface, Alhazen assumes that only forms going through the eye along lines perpendicular to its surface are effective in the process of vision. Those lines are, of course, the radial lines drawn outward from the eye-center. The points where these lines intersect the crystalline-surface will have a one-to-one correspondence with all points on the object.[15] The sensation that, according to Alhazen's repeated statements, takes place first at the crystalline is a sensation of the ordered form as it penetrates the body of the crystalline along the radial lines.

III

The doctrine that the crystalline is the sensitive organ in the eye had been urged by Galen and his many followers;[16] and, as Alhazen stated, "That sight perceives the visible objects through the straight lines whose extremities meet at the center of the eye is accepted by all mathematicians, there being no disagreement among them about it. And these lines are what mathematicians call lines of the ray."[17] These ideas, however, now receive certain refinements as a result of their being taken into a new theoretical scheme. First, the reception of light and color through lines of the visual cone is attributed simply to the "nature" of sight.[18] Gradually, however, a distinction between two modes of reception emerges:

> . . . The crystalline is disposed both to receive and perceive [the forms]. Thus the forms traverse it on account of the receptive and also perceptive power in it and through which it is disposed to have perception. And since it is disposed to receive these forms through the radial lines, the forms traverse its body along those lines.[19]

Again—the distinction becoming clearer:

> As for the sensitive organ, i.e., the crystalline humor, it does not receive the forms of colors and lights in the way they are received by the air and the non-sensitive transparent bodies, but in a different manner from that in which the transparent bodies receive them. For this organ being disposed to sense those forms, it receives them *qua* sensitive in addition to its receiving them *qua* transparent.[20]

And, finally, in book II:

> The sensitive organ [i.e., crystalline] does not receive the forms in the same way as they are received by transparent bodies. For the sensitive organ receives these forms and senses them, and the forms go through it on account of its transparency and on account of the sensitive power that is in it. Therefore it receives these forms in the manner proper to sensation (*qabūla iḥsāsin*), whereas transparent bodies receive them only in the manner proper to transmission,

(*qabūla ta'diyatin*) without sensing them. And if the sensitive body's reception of these forms is not like their reception by non-sensitive transparent bodies, then the forms do not extend through the sensitive body along the lines required by transparent bodies, but rather along the extension of the parts of the sensitive body. Sight is thus characterized by receiving the forms along the radial lines alone, because it is a property of forms to extend in transparent bodies along all straight lines and therefore they come to the eye along all straight lines. But if sight received them along all lines on which they arrive, the forms would not [appear] to it ordered. And therefore sight has come to be characterized by receiving the forms through those [radial] lines alone, so that it would perceive the forms with the order they have on the surfaces of visible objects.[21]

That sensation takes place by means of lines perpendicular to the crystalline-surface is not, therefore, simply or primarily due to the superior strength of action along those lines,[22] but rather to a property of the crystalline itself, namely, its selective sensitivity. It is a function of the crystalline, as a sensitive body, to sort out the forms coming to it from different sides, pick up those forms that extend on the perpendiculars, and hand them down along the same privileged directions to the vitreous humor. The latter thus receives a total form whose elements correspond one-to-one with their origins in the visual field. The vitreous humor, in addition to its transparency and sensitivity, has the further property of conveying the total form as an integral whole to the optic nerve,[23] where different points of the form will sensitize different parts of the visual spirit. Since these parts are separately confined to different filaments along which they travel, a total form, undisturbed by the bending of the nerve, will eventually arrive at the brain. Already, at the common nerve, the form coming from one eye will have coincided with the form from the other symmetrically disposed eye, and it is this united form that the *ultimum sentiens* perceives.[24]

IV

Two questions may now be briefly considered: what, in the light of what we have seen, could be the meaning of Alhazen's statement that sensation first takes place in the crystalline, and

what should we make of his concept of form? Both questions are clearly important for an understanding of his theory of perception.

Alhazen speaks generally of the effect of light in the eye as something "of the nature of pain"; and it is the pain felt in the eyes when gazing at intense light that he cites as an empirical evidence in support of the intromission hypothesis.[25] At one place he appears to be saying further that the visual sensation *in* the crystalline does not itself differ generically from the sensation of pain, even if no pain is felt:

> The effect that light produces in the crystalline is of the nature of pain. But while some pains are such that they disturb the organ suffering them and upset the soul, others, being slight, are found to be bearable. . . . Pains of this description are not felt, and the subject suffering them does not judge them to be pains on account of their slightness. . . . Now the effects of lights in the eye are all of the same kind and only vary by more or less. That being so and the effect of strong lights being of the nature of pain, all luminous effects in the eye are of the nature of pain and only vary by more or less. But due to the slight effect in the eye of weak and moderate lights they are not perceived as pain. The crystalline's sensation of the effects of lights is therefore of the same nature as the sensation of pain.[26]

A few pages later, however, Alhazen makes his position clearer:

> It may be said [objected] that the forms occurring in the eye do not reach the common nerve, but rather it is the sensation taking place in the eye that extends to the common nerve in the same way that the sensation of pain and of tangible objects extend; and when this sensation reaches the common nerve the last sentient perceives the sensible object. . . .
>
> We reply that the sensation produced in the eye no doubt reaches the common nerve. But the sensation produced in the eye is not only a sensation of pain, but a sensation of an effect of the nature of pain, and a sensation of luminosity, and of color, and of the order of the parts of the object.[27] Now the sensation of colors and of the order of the object's parts is not of the nature of pain. We shall show later on [in bk. II] how the eye's sensation of each one of these things is produced. But the sensation of the form of the visible object as it is can only be produced by the sensation of

everything in this form. Further, if the sensation that takes place in the eye reaches the common nerve and it is from the sensation produced in the common nerve that the sensitive faculty perceives the form of the visible object, then the sensation occurring in the common nerve is a sensation of the light and the color and the order. Thus, in any case, the thing that comes from the eye to the common nerve and from which the last sentient perceives the form of the object is a thing from which the last sentient perceives the light and color in the visible object and the configuration of its parts. But the thing from which the last sentient perceives the light and color and order is a certain form. Thus from the form produced in the eye there comes, in any case, to the common nerve a certain ordered form. And from the ordered form occurring in this nerve the last sentient perceives the form of the object as it is in itself. Therefore, the sensation of the effect produced in the surface of the crystalline reaches the common nerve, and so does also the form of the light and color that occurs in the surface of the crystalline, and it gets there with the order it has on the crystalline-surface.[28]

No sensation, whether of pain or of form, is "accomplished" until it arrives at the last sentient, which resides in the front of the brain.[29] When the effect produced in the eyes by a bright light reaches the brain, it is perceived as pain that, indeed, is felt in both eyes. But the sensitive faculty does not become aware of the forms coming to it as located in the crystalline. In what sense, then, is the crystalline said to be the place where *sensation* of forms *first* occurs? There can, I think, be only one simple answer to this question: by selecting the forms that extend in certain special directions, the crystalline performs in fact the *first* necessary operation in the process of vision. Alhazen asserts: "It is only at the crystalline that the forms of visible objects are set in order by means of the radial lines, for it is at that organ that sensation begins."[30] This, of course, can be put the other way round: sensation begins at the crystalline *because* it is there, and only there, that the forms are properly arranged in respect to veridical perception; visual sensation is therefore said to begin at the crystalline because it depends, in the first place, on a property that exclusively belongs to the crystalline humor.

V

"Form" is an undefined term in the *Optics*. Two expressions in the long passage just quoted perhaps come nearest to a definition: form is that "thing that comes from the eye to the common nerve and from which the last sentient perceives the form of the object," "a thing," that is, "from which the last sentient perceives the light and color in the visible object and the configuration of its parts." Though it may be described as an optical array, a form in Alhazen's sense is not a picture depicted anywhere in the eye, and should not therefore be mistaken for the image produced in a pin-hole (or lens) camera, or the impression made by a material *eidōlon*.[31] As a representation of the object, it is perceptible only after it has been singled out from a multitude of confused rays on the crystalline-surface and transmitted to the brain; and it is perceptible only to the faculty of sense. From a historical point of view one may say that the crystalline's selective power performs a function corresponding to, but only corresponding to, that which Kepler later ascribed to the crystalline by regarding it as a lens that casts a distinct picture on the retina. It organizes the visual matter before this matter is presented to the sense faculty.

More insight can be gained into Alhazen's concept of form by considering what he has to say about "ascertained forms." It may be profitable to look now at his account of this special category, though it occurs in the last chapter of book II, after the general theory of perception has been given.[32] Sight, it has been said, perceives a visible object by receiving its form. This form is composed of the particular properties that make up the visible appearance of the object, such as its shape, size, color, and so on. A particular property, say color, thus comes to the sensitive faculty as part of a composite form that combines a multitude of properties. Now some properties, such as shape or color, may appear to the sense of sight as soon as the eye looks at the object. That is, they are visible at a glance. Others, however, such as the letters of small script, only become apparent after the object has been contemplated and scrutinized.

There are thus two modes of perception, the one immediate (*idrāk bi 'l-badīha, comprehensio superficialis*),[33] the other contemplative (*idrāk bi 'l-ta'ammul, comprehensio per intuitionem*). Alhazen then introduces the concept of "ascertained form" (*ṣūra muḥaqqaqa, forma certificata*). A true form (*forma vera*) of an object, he says, is one that manifests all visible properties of the object. How can we ascertain that such a form has been received? Only by contemplation, he answers. For it is only by contemplation that we may apprehend the fine features of objects. And even if such subtle features were totally absent from an object, so that a quick glance at it would give us *all* its visible properties, we could not be certain of this without scrutinizing the object. A true form may, therefore, be present to the sensitive faculty by virtue of an immediate perception; but contemplative perception, an operation involving the inspection of all parts of the object, is a condition for obtaining an ascertained form.

Contemplation is an operation effected both by the eye and the faculty of judgment (*virtus distinctiva*),[34] which, as we shall see, is involved in all normal acts of perception. The eye, by successively orienting itself to various parts of the object, causes the forms of those parts to be received along the axis of the visual cone or along lines close to it; and vision along such lines is clearer than vision along other lines. The faculty of judgment then discerns (*tumayyiz, distinguet*) the colors of various parts, their similarity or difference, their relations to one another, and so on, until, in the end, the disposition of the whole object composed of all those parts becomes clear to it.[35]

Alhazen explains the manner in which the form of a visible object is ascertained as follows. When the eye looks at an object, the sense perceives the total form of the object as a whole in some vague way, while clearly perceiving that part of the object where it is intersected by the visual axis. By moving the eye over the whole surface of the object, the sense gains a succession of perceptions of the object's total form, each focusing on a different part of it. The ascertained form of the whole object is the outcome of this succession of perceptions. As a result of the faculty of judgment's comparing and discerning of the various

details contained in each one of these perceptions, there is finally formed "in the imagination" (*al-takhayyul, imaginatio*) a total configuration truly representative of the visible object.[36] Alhazen finally remarks, however, that to "ascertain fully" the form of an object is to ascertain it to the limit (*ghāya*) possible for sense perception. Ascertainment is a relative concept—relative, that is, to the faculty of sense.[37]

All this means that to obtain a form approximating the visible features of an object is a highly complex affair that involves other operations besides those explained in book I of the *Optics*. The forms sent out to the visual faculty every time the eye glances at the object, though unconfused and organized, are but the raw material from which the "true form" will be built up, insofar as it is attainable, by the faculty of judgment. Even as layers in the process of building up the increasingly truer form, the successively grasped total forms do not remain unchanged in the final product; some of their features (e.g., their peripheral fuzziness) must be thrown out in order to be replaced by others more truly representative of the object.

But let us now go back to the beginning of book II where these complicated ideas are introduced.

VI

All objects of vision (*al-ma'ānī al-mubṣara, intentiones visibiles*), says Alhazen, are properties of physical bodies.[38] Not all properties are perceived in the same manner. Two of these, color and luminosity, are perceived by "pure sensation." Perception of all others involves acts of comparison, discernment, and inference, all of which are performed by the faculty of judgment. Consider the similarity (or difference) of two objects, taken by Alhazen as a paradigm case and presented by him in a rather striking manner. This can only be perceived through perception of the similarity (or difference) between their forms. But the similarity (or difference) of two forms is not identical with either or both of them. And nothing is received from the objects other than their respective forms. In particular, they do not send out a third form from which their similarity (or differ-

ence) can be perceived. Perception of similarity (or difference) can therefore be achieved only by comparing (*qiyās*, *comparatio*)[39] the forms and grasping that property which they have in common (or in respect of which they differ). "That being the case, the sense of sight's perception of the similarity and difference of forms is not by pure sensation, but rather through comparison of the forms that it perceives by pure sensation."[40]

The case of perceiving two similar colors, say two greens of which one is brighter than the other, is particularly instructive, involving as it does an object of pure sensation. The sense (*al-ḥāss*, *visus*) will perceive their similarity in being green and also their difference in respect of brightness. "Now to distinguish between the two greens is not the same as the sensation of green, for the latter is due to the eye's becoming green through the agency of the green; and the eye has become green through the agency of two greens; and as a result of becoming green through the agency of both greens the sense perceives them to be of the same kind. Thus its perception that one green is brighter than the other, and that they are of the same kind, is a judgment (*tamyīz*, *distinctio*) of the coloring that takes place in the eye, and not the sensation of the coloring itself."[41] By analogy we may say that the eye's sensation of light is due to the eye's being illuminated through the agency of light and, again, that the sense of sight perceives the similarity and difference of lights through comparison and discernment.

The preceding account, couched in terms very close to those of Alhazen, is misleading in two respects. It gives the impression, or rather states, that forms, as well as light and color as such, are perceived by pure sensation; and it ascribes the acts of comparison and discrimination to "the sense of sight." That neither of these conceptions is intended quite as it stands is made clear by later statements. First, we read:

> Not everything perceived by the sense of sight is perceived by pure sensation, but rather many visible properties are perceived by discernment and inference in addition to the sensation of the visible object's form, and not by pure sensation alone. *Now sight does not possess the power of discernment* (*quwwat al-tamyīz*), but rather it is the *faculty of judgment* (*al-quwwa al-mumayyiza*) *that discriminates those properties*. But the discrimination performed by the faculty of judgment cannot take place without the mediation of the sense of sight.[42]

This clears up one point: the sense of sight is said to be capable of certain acts beyond pure sensation because it is the medium through which these acts are performed by the faculty of judgment. But we still have the assertion in which it is seemingly implied that the object's form is perceived by pure sensation. The following passage should dispel any doubt regarding this point too. In it Alhazen is primarily concerned to distinguish the role of recognition, an act involving memory and comparison, from that of pure sensation, but in the course of making this distinction he specifically states that perception of forms as forms is a function of the faculty of judgment.

> Now recognition is not pure sensation. For the sense of sight perceives the forms of visible objects from the forms that come to it from the colors and lights of those objects. And its perception of lights *qua* lights and colors *qua* colors *is* by pure sensation. But those features in the form which, or the like of which, it previously perceived, and which, or the like of which, it remembers having perceived, are at once perceived by recognition from significant traits (*amārāt*) in that form. *The faculty of judgment then discerns* (*tumayyiz*) *that form, thus perceiving from it all of its properties, such as order*, outline (*takhṭīṭ*), *similarity, difference and all properties of the form whose perception is not effected by mere sensation or recognition.* Therefore, among the things that are perceptible by the sense of sight, some are perceived by pure sensation, others by recognition, and others still by a discerning and an inference that exceeds the inference of recognition.[43]

The presence of form in the eye is a coloring and illumination produced by the colors and lights coming from points on the object. A form, however, is not just light and color (light *qua* light and color *qua* color), but a pattern whose outline and order of parts are discerned only by the faculty of judgment. If, as Alhazen says, pure sensation is only of light as such and of color as such, then it follows that there exists no state of consciousness that can be described as pure sensation. We are aware of the sense of coloring and illumination only as part of the discerned

form whose perception *qua* form must involve judgment by the discerning faculty. Thus the concept of form, first introduced in book I of the *Optics* as a necessary condition for veracious vision, is seen to be ultimately absorbed into a psychological concept elaborated by the theory of perception given in book II.

VII

To recapitulate. A form is an optical array disengaged by the crystalline humor and presented through the optic nerve to the faculty of sense. The visual material of which this form is composed, the light and color in it, are registered as light and color sensations. But the perception of the received configuration as an ordered disposition of light and color is the work of a mental faculty over and above mere sensation. There is a process that turns the disentangled visual material into a perception of form and, ultimately, into a perception of an object lying out there with all its visible properties—shape, size, position, and so on. Seeing an object is not the result of a mere imprinting on the mind (brain) of a form emanating from the object. It is an inference from the material received from the object as sensation.

A special category of inferential perception is what Alhazen calls perception by recognition (*idrāk bi 'l-ma'rifa*, *comprehensio per cognitionem*). Through it sight, or rather the cognitive faculty, recognizes an individual to be a member of a certain species or as the same individual it previously had acquaintance with. Memory (*dhikr*, *rememoratio*) is an essential element of recognition—a proof that the latter is not a result of merely registering the form given in sense perception. In the case of recognizing an individual as such, the conclusion "this is my friend *x*" is arrived at by means of comparing the presently received form of *x* with the previously received but presently memorized form or forms of *x*. In the case of recognition of a species ("this is a horse"), the conclusion is derived from a comparison of the form present in sense-perception with the previously received and presently memorized forms of individuals

belonging to the same species.[44] Together with this account in terms of separate individual forms, Alhazen introduces a concept of "universal form" (*ṣūra kulliyya*, *forma universalis*) that is remarkable for its thoroughly empirical character, no matter what one might think of its degree of philosophical sophistication.

A universal form is established "in the imagination" as a result of repeated perceptions of individuals that belong to the same species. Such individuals have visible properties of which some, e.g., color or shape, are the same for all of them. A universal form consists of the totality of *particular* properties that individuals of the same species have in common. Every time an individual is presented in sense perception, the sensitive faculty, *qua sensitive*, perceives the universal form which exists in that individual. To recognize what a thing is (*mā'iyya*, *quidditas*), therefore, is nothing more than to recognize the coincidence of the presently perceived universal form in the individual with the universal form already present "in the imagination." Perception of such a coincidence or similarity is something that the faculty of judgment automatically seeks to achieve. When an object is seen, the faculty of judgment immediately undertakes to search for a similar form among those stored in the imagination. No recognition will take place if no such form is found.[45] What is remarkable about this explanation is that, unlike those of all major Islamic philosophers of the peripatetic school, such as al-Kindī, al-Fārābī and Avicenna, it nowhere appeals to an *intellectus agens* as a source of *sui generis* universal forms. It should be emphasized that, according to Alhazen, the universal form in an individual object is merely a collection of some of the particular properties making up the concrete form of the object, and it is conveyed to the cognitive faculty along with the object's total sensible form. No sensible object, whatever the sense faculty, is perceived to be what it is (*mā'iyya*) except by recognition.[46]

Recognition is distinguished from other inferential perceptions by the fact that it does not require inspection (*istiqrā'*, *inductio*) of all features of the recognized form or object, and this explains why it takes place in an exceptionally short time.[47] A familiar

word on a piece of paper, for example, is recognized, not by scrutinizing the order and shape of every letter, but, perhaps, by simultaneously noticing the first and the last letter in it, or by "perceiving the configuration (*tashakkul*) of the totality of the form" representing the written word.[48] Such features, which, as a result of being grasped and compared with a memorized form of a whole object give rise to recognition of the object, are called by Alhazen "significant traits" or "signs" (*amārāt, signa*), and he accordingly calls "perception by sign" (*comprehensio per signum*) such a mode of apprehension. It is an example of how inferences may take place within us without our being aware of them. His aim is to argue that if we are not conscious of inferential perceptions as inferences, that is only because repetition and habit have turned them into perceptions by recognition or sign, which take place in an extremely short time:

> . . . The perception of many of the objects of vision that are perceived by discernment and inference (*bi 'l-tamyīz wa 'l-qiyās, per rationem et distinctionem*) takes place in an extremely short interval of time, and in many cases it is not manifest that their perception occurs through discernment and inference because of the speed of the inference through which those objects are perceived. . . . For the shape or size of a body, or the transparency of a transparent body, and such like properties of visible objects, are in most cases perceived extremely quickly, and not immediately, since they are perceived by inference and discernment. . . . The quickness of the perception of these properties by inference is only due to the manifestness of their premisses and to the fact that the faculty of judgment has been accustomed to discern those properties.[49]

Again:

> And similarly with all objects of vision that are perceived by inference: when the faculty of sight repeatedly perceives them, its perception of them turns into [perception] by recognition without resuming the inference by which it [formerly] perceived their identity.[50]

Still using the term *idrāk* (here consistently rendered as "perception"), Alhazen extends his account to include "perception" of syllogistic conclusions and uses this example to make a distinction between performing an inference and being aware of how it is done. Upon hearing the statement "this thing can write," a man of "sound judgment" will "without an appreciable interval of time" conclude that "this thing is a man." His inference will be effected by means of a universal premise that is "established in the soul" (undoubtedly as a result of previous experience), "manifest" to the faculty of judgment, and "present to the memory." But "the faculty of judgment does not syllogize by ordering and combining and repeating the premises as in the verbal ordering of syllogism." " . . . For that faculty perceives the conclusion without the need for words or for repeating and ordering of premises, or the need for repeating and ordering words."[51] Alhazen continues: "The order of words that make up the syllogism is but a description (*ṣifa*) of the manner in which the faculty of judgment perceives the conclusion, but the faculty of judgment's perception of the conclusion needs neither a description (*na't*) of that manner nor the order of the manner of perception."[52] That is to say, the faculty of judgment need not be aware of the forms of inferences as these forms are displayed in *our* descriptions of them, nor does it need the means (words) *we* employ to perform those inferences. It need not be a "little man" within. "Perception" of a visible property, or of a conclusion in an inference about objects, should therefore be distinguished from "perception" of the manner in which it is achieved. A higher-level inference is required to identify those inferences that the faculty of judgment performs without discerning their manner of production.[53]

The visible properties are "many," but they generally fall under twenty-two categories:[54] light (*ḍaw', lux*), color (*lawn, color*), distance (*bu'd, remotio*), position (*waḍ, situs*), solidity (*tajassum, corporeitas*), shape (*shakl, figura*), magnitude (*'iẓam, magnitudo*), discreteness (*tafarruq, discretio & separatio*), continuity (*ittiṣāl, continuum*), number (*'adad, numerus*), motion (*ḥaraka, motus*), rest (*sukūn, quies*), roughness (*khushūna, asperitas*), smoothness (*malāsa, levitas*), transparency (*shafīf, diaphanitas*), opacity (*kathāfa, spissitudo*), shadow (*ẓill, umbra*), darkness (*ẓulma, obscuritas*), beauty (*ḥasan, pulchritudo*), ugli-

ness (*qabīḥ*, *turpitudo*), and the similarity (*tashābuh*, *consimilitudo*) and dissimilarity (*ikhtilāf*, *diversitas*) between any of these properties or any of the forms composed from them. Alhazen calls these "particular" objects of vision to distinguish them from the more complex properties that fall under them: such as order, which comes under position; curvature, which comes under shape; equality and inequality, which come under similarity and dissimilarity; laughter and weeping, which come under shape and movement; and so on. He is convinced that there exist no visible properties that cannot be reduced to the "particular properties" (*intentiones particulares*) either individually or in combination. Alhazen's major contribution to the history of perception lies in the impressively detailed and often original explanations of how each of these particular properties is apprehended. To discuss his explanations here without being able to give them their due share of attention would be a sign of failure to appreciate his achievement and a disservice to the history of our subject. He devoted to them two thirds of the entire second book of the *Optics*. One thing, however, that can and should be pointed out here is that these explanations are all guided by one logical conclusion of the distinction outlined above. If "light as such" and "color as such" are the only objects of pure sensation; further, if the apprehension of all the other properties is an act of a discerning faculty, which accompanies the optical stimulation reaching the brain from the eye; and, further, if this act is in the first place an inference that may not appear as such because it is performed automatically and often very quickly; then to explain the manner of perception of all those inferential properties will be to formulate descriptions of the inferences involved in apprehending each of them. Alhazen was quite clear and indeed persistent about this. He saw his task as one of providing models of inference whose conclusions are judgments about the nature of a color, the distance, size, or shape of a visible object, or the beauty of a human face.[55] This was an ambitious program that cannot fail to impress readers of the *Optics* by its consistent application. While describing inferences at the basis of our perception of objects as distant from us (disappearance of the object when the eyes are closed

or turned away from it, and so on), Alhazen finds occasion to answer an objection raised by the visual ray theorists against the intromission hypothesis. The answer is worth quoting here because it reveals the extent to which his general view of vision was involved in his psychological theory:

> Because the visible object is perceived in its own place, the upholders of the doctrine of the ray came to believe that vision occurs by means of a ray issuing from the eye and ending at the object, and that vision is achieved by the end points of the ray. They argued against natural scientists, saying: if vision takes place by a form that comes from the object to the eye, and if the form exists inside the eye, then why is the object perceived in its own place outside the eye while its form exists in the eye? But these people forgot that vision is not accomplished by pure sensation alone, but is rather accomplished by means of discernment and prior recognition, and that without these no vision can be effected by sight, nor would sight perceive what the visible object is at the moment of seeing it.[56]

1. The Arabic text of Alhazen's *Optics* (*Kitāb al-Manāẓir*) has not been published. The Latin translation made by an unknown person in the late twelfth or early thirteenth century, and known in the Middle Ages as *Perspectiva* or *De aspectibus*, was published by F. Risner in the volume bearing the collective title *Opticae thesaurus. Alhazeni Arabis libri septem, nunc primum editi, eiusdem liber de crepusculis et nubium ascensionibus, item Vittellonis Thuringo-Poloni Libri X, omnes instaurati, figuris illustrati et aucti, adjectis etiam in Alhazenum commentariis a Federico Risnero* (Basel, 1572; repr., New York, 1974). Reference will be made to the Arabic MSS and to Risner's edition. In all instances my English translation will be from the Arabic, but Risner's text will sometimes be quoted for comparison.

2. These are chapters 4-8, which, alone, make up the entire bk. I in the medieval Latin version. Chapters 1-3 (preface; properties of sight; properties of light and manner of its radiation) have not been found in any of the Latin manuscripts of the *Optics*.

3. Alhazen died ca. 1040 in Cairo, where he spent the latter part of his life. For biographical and bibliographical information, see the article "Ibn al-Haytham" in *Dictionary of Scientific Biography*, ed. C. C. Gillispie (New York, 1972), 6:189-210.

4. The most detailed study of Alhazen's psychology of vision is in Arabic: M. Naẓīf, *al-Ḥasan ibn al-Haytham, His Researches and Discoveries in Optics*, 2 vols., (Cairo, 1942-43); see vol. 1, chaps. 2-3, pp. 240-338. There is an account in German: H. Bauer, *Die Psychologie Alhazens auf Grund von Alhazens Optik dargestellt*, in the series *Beiträge zur Geschichte der Philosophie des Mittelalters*, vol. X, no. 5 (Münster in Westfalen, 1911). G. F. Vescovini studies Alhazen's influence on fourteenth-century empiricist theories of cognition in *Studi sulla prospettiva medievale* (Turin, 1965). The best account of Ptolemy's theory of perception is in A. Lejeune, *Euclide et Ptolémée, deux stades de l'optique géométrique grecque* (Louvain, 1948).

5. Alhazen's *baṣar*, like the Greek *opsis* and the Latin *visus*, means both eye and sight or sense or faculty of sight. He has a special word for the activity of seeing or vision, namely *ibṣār*, corresponding to the Greek *horasis* and the Latin *visio*.

6. The limited scope of the explanation of vision in bk. I is indicated by Alhazen toward the end of that book as follows: "This is the manner of vision generally. For that which sight perceives of the visible object by mere sensation is only the light and color in that object. As for the other things that sight perceives of the visible object, such as shape, position, magnitude, movement, and the like, these sight does not perceive by mere sensation, but through inference and signs (*bi-qiyās wa-amārāt*: *per rationem et signa*). We will later explain this thoroughly in the second Book when we enumerate the things perceived by sight" (*Optics*, I.6; Fatih MS 3212, fol. 105ᵃ). See Risner, p. 15, lines 11-15.

7. Thus al-Kindī in the ninth century argued against any explanation of vision in terms of "form," and in favor of a theory exclusively formulated in terms of rays; while Avicenna in Alhazen's own time took precisely the opposite view (see Lindberg's chapter, this volume). In medieval Islam not only the mathematicians but practically all the early *mutakallimūn* or dialectical theologians adopted the visual-ray theory. It is more than likely that the writings on *kalām*, of which large sections were devoted to discussion of "physical" questions, constituted the immediate source of Avicenna's detailed knowledge of the arguments in support of that theory.

8. In the *Discourse on Light* (*Maqāla fi 'l-ḍaw'*), a short work composed after the *Optics*, Alhazen wrote: "Discussion of the nature (*māhiyya*) of light belongs to (*min*) the natural sciences (*al-'ulūm al-ṭabī'iyya*), and the discussion of the manner (*kayfiyya*) of the radiation (*ishrāq*) of light depends on (*muḥtāj*) the mathematical sciences (*al-'ulūm al-ta'līmiyya*) on account of the lines on which the lights extend. Again, discussion of the nature of the ray belongs to the natural sciences, and the discussion of its shape (*shakl* and *hay'a*) belongs to the mathematical sciences. And similarly with regard to the transparent bodies through which the lights pass: the discussion of the nature of their transparency belongs to the natural sciences, and the discussion of how (*kayfiyya*) light extends through them belongs to the mathematical sciences. Therefore, the discussion of light and of the ray and of transparency must be composed of (*yajibu an yakūna murakkaban*) the natural and the mathematical sciences" (*Majmū' Rasā'il Ibn al-Haytham*, Hyderabad, 1357 A.H., no. 2, p. 2). There is a French translation of the *Discourse*: R. Rashed, "Le 'Discours de la lumière d'Ibn al-Haytham," *Revue d'histoire des sciences et de leurs applications* 21 (1968):198-224. In the *Optics* Alhazen speaks of the synthesis in terms that directly refer to the question of vision: "Our inquiry combines the natural and the mathematical sciences. It is dependent on the natural sciences because vision is one of the senses and these belong to the natural things. It is dependent on the mathematical sciences because sight perceives shape, position, magnitude, movement and rest, in addition to its being especially concerned with straight lines. Since it is the mathematical sciences that investigate these things, the inquiry into our subject truly combines the natural and the mathematical sciences" (bk. I, ch. 1, Istanbul MS Fatih 3212, fols. 2^{a-b}).

9. These statements occur in the *Discourse*, cited in the preceding note.

10. One of Alhazen's earlier works on optics was a summary of Euclid and Ptolemy in which he reconstructed the contents of the first part, which, he said, was missing from Ptolemy's *Optics*; see the article in *Dictionary of Scientific Biography* referred to in note 3 above, p. 190, col. B. That work is now lost. A modern attempt at reconstruction is in A. Lejeune's *Euclide et Ptolémée*. Ptolemy's Greek text has not survived. A critical edition of the Latin version is A. Lejeune, *L'Optique de Claude Ptolémée dans la version latine d'après l'arabe de l'émir Eugène de Sicile* (Louvain, 1956).

11. MS Fatih 3212, fols. 2^a-3^b.

12. *Optics*, I.6; MS Fatih 3212, fol. 104^a: "Moreover, all that mathematicians who hold the doctrine of the ray use in their reasonings and demonstrations are imaginary lines, which they call lines of the ray. And we have shown that the eye does not perceive any of the visible objects except through these lines. Thus the opinion of those who take the radial lines to be imaginary is correct, and we have shown that vision is not effected without them. But the opinion of those who think that something issues from the eye other than the imaginary lines is impossible, and we have shown its impossibility by the fact that it is not warranted by anything that exists, nor is there a reason for it nor an argument that supports it." See Risner, p. 15, lines 1-5, where these statements are somewhat compressed. See also D. C. Lindberg, "Alhazen's Theory of Vision and Its Reception in the West," *Isis* 58 (1968):325-27.

13. *Optics*, I.6; MS Fatih 3212, fol. 82^a: "Moreover, the form of the color is always mixed with the form of the light and not distinct from it, for sight perceives light always mingled with color. It is therefore most appropriate that the eye's sensation of the color of the visible object and of the light that is in it should only occur through the form that is mixed of that light and color, and that comes to the eye from the surface of the object." See Risner, p. 7, sec. 14, lines 11-13.

14. The structure of the eye is the subject of *Optics*, bk. I, chap. 5; MS Fatih 3212, fols. 72^a-81^b; Risner, bk. I, chap. 4, pp. 3-7. The following chapter expounds the manner of vision.

15. The picture is complicated in book VII where Alhazen tries to explain how objects lying outside the visual cone are seen. This explanation in terms of refraction has been left out of the above account because the questions that concern us here are independent of it.

16. See Galen, *On the Usefulness of the Parts of the Body*, trans. M. T. May (Ithaca, N.Y., 1968), vol. 10, chap. 3, pp. 469-74.

17. *Optics*, I.6; MS Fatih 3212, fol. 98^a; Risner, p. 13, lines 16-18.

18. *Optics*, I.6; MS Fatih 3212, fol. 97^b: ". . . The nature of sight is to receive what comes to it of the light of visible objects, and . . . its nature is further characterized by receiving only those forms that come to it through certain lines . . . , namely, the straight lines whose extremities meet only at the center of the eye, these lines being alone characterized as diameters of the eye and perpendicular to the surface of the sensitive body [i.e., the crystalline humor]. Thus perception occurs through the forms coming from the visible objects, and these lines are, as it were, an instrument of sight by means of which the visible objects appear to it distinct and the parts of each visible object ordered." See Risner, p 12, sec. 20, line 28-p. 13, line 5. For "surface of the sensitive body" (*saṭḥ al-jism al-ḥāss*), the Latin has "*superficiem visus sentientis*."

19. *Optics*, I.6: MS Fatih 3212, fols. 106^b-107^a; Risner, p. 15, sec. 25, lines 7-9: "Et etiam glacialis est praeparatus ad recipiendum istas formas, et ad sentiendum ipsas. Formae ergo pertranseunt in eo propter virtutem sensibilem recipientem."

20. *Optics*, I.6: MS Fatih 3212, fol. 117^a; Risner, p. 17, sec. 30, lines 1-4: "Membrum vero sentiens, scilicet glacialis, non recipit formam lucis et coloris, sicut recipit aer, et alia diaphana non sentientia, sed secundum modum diversum ab illo modo. Quoniam istud membrum est praeparatum ad recipiendum istam formam: recipit ergo istam, quatenus est sentiens, et quatenus est diaphanum."

21. *Optics*, II.2: MS Fatih 3213, fol. 7^{a-b}; Risner, p. 26, sec. 4, lines 1-9: "Et receptio formarum in membro sentiente non est, sicut receptio formarum in corporibus diaphanis; quoniam membrum sentiens recipit istas formas, et sentit eas, et pertranseunt in eo propter suam diaphanitatem et virtutem sensibilem, quae est in eo. Recipit ergo istas formas secundum receptionem sensus. Corpora autem diaphana non recipiunt istas formas, nisi receptione, quae recipiunt ad reddendum, et non sentiunt ipsas. Et cum receptio corporis sentientis ab istis formis non sit sicut receptio cor-

porum diaphanorum, non sentientium; extensio formarum in corpore sentiente non debet esse secundum verticationes, quas corpora diaphana exigunt. Visus ergo non est appropriatus receptioni formarum ex verticationibus linearum radialium tantum, nisi quia proprietas formarum est, ut extendantur in corporibus diaphanis super omnes verticationes rectas." The remainder of the quoted passage is lacking in Risner's text.

22. *Optics*, I.6: MS Fatih 3212, fol. 90^b; Risner, p. 10, lines 27-30: "But the effect of the lights that come along the perpendicular is stronger than the effect of those that come along inclined lines. Therefore, it is most appropriate that the crystalline should perceive, through each point on it, the form that comes to this point along the perpendicular alone, without perceiving through the same point that which comes to it along refracted lines."

23. *Optics*, II.2; MS Fatih 3213, fol. 8^a: "In addition to sensing these forms, the posterior part [of the crystalline], namely, the vitreous, and the receptive power that is in it, has the property of only preserving their arrangement." Risner, p. 26, sec. 4, lines 12-13: "Posterior autem pars quae est humor vitreus, et virtus recipiens, quae est in illo corpore, non est appropriata cum suo sensu istarum formarum, nisi ad custodiendum eorum [*sic*] ordinationem tantum."

24. See Risner, pp. 26-27, secs. 5-6.

25. See *Optics*, bk. I, chap. 1, sec. 1, in Risner's edition.

26. *Optics*, I.6; MS Fatih 3212, fols. 107^a-108^a. See Risner, p. 15, sec. 26, line 1-p. 16, line 4.

27. Strictly, as the theory of bk. II makes clear—see below, perception of the order of the parts of the object is not perception "by pure sensation," which can only be of "light as such" and of "color as such." The question that concerns Alhazen here, however, is not "what is sensation?", but "what are the conditions for veridical perception?" He is concerned to argue that a "form" must in any case be presented to the last sentient. Some features of the form, including order, are only discerned by the faculty of judgment.

28. *Optics*, I.6; MS Fatih 3212, fols 112^b-113^b. This whole passage, including a part not quoted here, is reduced to nine lines in Risner's text: see p. 16, sec. 27, line 45-p. 17, line 3.

29. Risner, p. 16, sec. 27, lines 31-32: "sensus non completur, nisi per illud [ultimum] sentiens tantum, non per oculum tantum."

30. *Optics*, II.2; MS Fatih 3213, fol. 6^b; Risner, pp. 25-26: "Lineae ergo radiales non iuvant ad ordinationem formarum rerum visibilium, nisi apud glacialem tantum, quoniam apud membrum istud principium est sensus."

31. Kepler, in *Ad Vitellionem paralipomena* (Frankfurt, 1604), p. 193, makes the distinction, which should be borne in mind here, between image as a theoretical entity and as a picture: "Definitio. Cum hactenus Imago fuerit Ens rationale, iam figurae rerum vere in papyro existentes, seu alio pariete, picturae dicantur."

32. See Risner, pp. 67 ff.

33. *Optics*, II.4; MS Fatih 3213, fol. 132^a; Risner, p. 67, sec. 64, esp. lines 20-22. In the Latin version the more commonly used expression for *idrāk bi 'l-badīha* is *comprehensio per aspectum*.

34. Alhazen's *al-quwwa al-mumayyiza* (faculty of judgment), which he often calls simply *al-tamyīz* (discernment), corresponds to some extent to Aristotle's *dynamis kritikē*. The cognate verb *mayyaza*, also frequently used in the *Optics*, means to differentiate, distinguish, discriminate, discern. The Latin version employs *virtus distinctiva*, *distinctio*, and *distinguere*. For these terms in Ptolemy see A. Lejeune, *L'Optique de Claude Ptolémée*, Index.

35. *Optics*, II.4; MS Fatih 3213, fol. 133^b: "The faculty of judgment discerns all

the forms that come to it: thus it discerns the colors of [their] parts, and their difference if they are different, and the order of the parts in relation to one another, and their details, and the structure (*hay'a*) of each of them, and all [other] features (*ma'ānī*) that appear as a result of contemplating the object, and the structure of the whole object as composed of those parts and features." Risner, p. 67, lines 14-17: "Et virtus distinctiva distinguet omnes formas venientes ad ipsam, et distinguet colores partium, et divarsitatem colorum, et ordinationem partium inter se. Et generaliter distinguet omnes intentiones rei visae, quae apparent per intuitum et formam totius rei visae compositam ex illis intentionibus."

36. *Optics*, II.4; MS Fatih 321, fol. 136^a; Risner, p. 69, sec. 66, lines 1-3: "Et etiam dicamus, quod quando visus comprehenderit aliquam rem visam, et fuerit certificata forma eius apud sentientem, forma illius rei visae remanet in anima, et figuratur in imaginatione. . . . "

37. *Optics*, II.4; MS Fatih 3213, fol. 152^{a-b}: "This ascertainment is relative to the sense. For 'ascertained' and 'perfectly ascertained' (*ghāyat al-taḥqīq*) here mean the limit (*ghāya*) of what the sense (can) perceive. In addition to all that, the sight's perception of visible objects takes place in accordance with the sight's strength, for sights vary in respect of strength and weakness." Risner, p. 75, lines 8-12: "Et ista certificatio, quae est respectu sensu, est intentio certificata, et est dicere finem certificationis in istis locis, finem illius, quod potest comprehendi a sensu. Et cum omnibus istis comprehensio visibilium a visu est secundum fortitudinem visus; quoniam sensus visus oculorum diversatur secundum vigorem et debilitatem." As in several other places the Latin is clearly inadequate.

38. The Arabic *ma'nā* (pl. *ma'ānī*) means sense, notion, concept, and so on. In medieval and particularly philosophical literature, it was frequently used in the general sense of "thing," "matter," "affair," and so on, and referred to objects that lacked a special name. As employed by Alhazen, the word has nothing to do with *intentio* as a directing of the mind.

39. In Alhazen's *Optics*, the word *qiyās* is used to mean comparison, analogy, analogical argument, inference, syllogism. All of these were established usages in his time. The Latin version renders *qiyās* variously as *comparatio*, *ratio*, *syllogismus*. It has *ratiocinatio* for *istidlāl* (inference), which is sometimes used by Alhazen.

40. *Optics*, II.3; MS Fatih 3213, fol. 20^{a-b}; Risner, p. 30, sec. 10, lines 20-21: "Et cum ita sit, comprehensio ergo sensus visus a consimilitudine formarum, et diversitate illarum, non est per solum sensum, sed per comparationem formarum inter se." The Latin omits the clause "which it perceives by pure sensation."

41. *Optics*, II.3; MS Fatih 3213, fol. 21^a; Risner, p. 30, sec. 10, lines 25-30: "Sed distinctio inter duas viriditates non est ipse sensus viriditatis, quoniam sensus viriditatis est ex viridificatione visus ab utraque viriditate, et comprehendet, quod sunt unius generis [sic]. Comprehensio ergo visus, quod altera viriditas est fortior altera, et quod duae sunt unius generis, est distinctio colorationis, quae est in visu, non ipse sensus coloris."

42. *Optics*, II.3; MS Fatih 3213, fol. 22^{a-b} (italics added); Risner, p. 31, lines 2-4: "Non ergo omne, quod comprehenditur a visu, comprehenditur solo sensu, sed multae intentiones visibiles comprehenduntur per rationem et distinctionem cum sensu formae visae. Visus autem non habet virtutem distinguendi, sed virtus distinctiva distinguit istas res; attamen distinctio virtutis distinctivae in istis rebus visibilibus non est, nisi mediante visu."

43. *Optics*, II.3; MS Fatih 3213, fols 24^b-25^a (italics added). The whole passage is rendered in Risner's text by two sentences: "Cognitio autem non est solo sensu. Intentiones ergo quae comprehenduntur a sensu visu quaedam comprehenduntur solo sensu, quaedam per cognitionem, quaedam per rationem et distinctionem" (p. 31, sec.

11, lines 31-33).

Another explicit statement occurs later, on fol. 37^{b}: "The faculty of judgment perceives most of the particular properties in the visible object by discerning the properties in that form [of the object], namely, the order of its parts, the shape of its periphery, the configuration of those parts, the difference between them in respect of color, position and order . . . [and so on]." Risner, p. 35, lines 6-10: "Et virtus distinctiva comprehendit plures intentiones particulares, quae sunt in re visa, ex distinctione intentionum, quae sunt in illa forma ab ea, scilicet ex ordinatione partium formae, et ex figuratione illius, quod continet formam, et ex figuratione partium eius, et diversitate colorum, et situum et ordinationum, quae sunt in partibus illius formae, et ex consimilitudine et diversitate earum."

44. *Optics*, II.3; MS Fatih 3213, fols 22^{b}-23^{a}; Risner, p. 31, line 5-sec. 11, line 16.

45. *Optics*, II.4; MS Fatih 3213, fols. 138^{b}-140^{a}; Risner, p. 69, sec. 67-p. 70, sec. 68.

46. *Optics*, II.3; MS Fatih 3213, fol. 24^{b}; Risner, p. 31, sec. 11, lines 29-30: "Et non comprehendetur quidditas alicuis rei visae, neque alicuis rei sensibilis alio sensu, nisi per cognitionem."

47. *Optics*, II.3; MS Fatih 3213, fol. 23^{b}; Risner, p. 31, sec. 11, lines 11-16.

48. *Optics*, II.3; MS Fatih 3213, fol. 24^{a}; Risner, p. 31, lines 16-18. The clause in quotation marks is missing from Risner's text. The Arabic is "*aw min idrākihi li-tashakkul jumlat al-ṣūra.*"

49. *Optics*, II.3; MS Fatih 3213, fol 25^{a-b}; Risner, p. 31, sec. 12, lines 1-8.

50. *Optics*, II.3; MS Fatih 3213, fol. 27^{b}. Risner's text starts "Et similiter sunt omnes intentiones, quae comprehenduntur per rationem, . . . " but omits the rest of the passage: see p. 32, lines 18-19.

51. *Optics*, II.3; MS Fatih 3213, fol. 26^{a-b}; Risner, p. 32, lines 4-8; "Quod virtus distinctiva non arguit per compositionem et ordinationem propositionis, sicut componitur argumentatio per vocabula. . . . Quoniam virtus distinctiva comprehendit conclusionem sine indigentia in verbis, et sine indigentia ordinationis propositionum, et ordinationis verborum."

52. *Optics*, II.3; MS Fatih 3213, fol. 26^{b}; Risner, p. 32, lines 8-11: "Quoniam ordinatio verborum argumenti non est, nisi modus qualitatis comprehensionis virtutis distinctivae a conclusione. Sed comprehensio virtutis distinctivae ad conclusionem non indiget modo qualitatis, nec ordine qualitatis comprehensionis."

53. *Optics*, II.3; MS Fatih 3213, fols. 30^{a}-31^{a}; Risner, p. 32, sec. 13, line 1 p. 33, line 13: "Et etiam multoties non apparet qualitas comprehensionis intententionum visibilium, quae comprehenduntur ratione (i.e., *qiyās*, inference) et cognitione (i.e., *ma'rifa*, recognition), quoniam comprehensio earum non fit valde velocior, et quia comprehensio qualitatis comprehensionis non est nisi per secundum argumentum post primum argumentum, per quod fuit visio. Virtus autem distinctiva non utitur isto secundo argumento, in tempore, in quo comprehendit aliquam intententionem visibilem, neque distinguit qualiter comprehendit illam intentionem. . . . Comprehensio ergo qualitatis comprehensionis, et quae comprehensio eiusmodi comprehensionis est, non est, nisi per argumentum et distinctionem non velocem. Et propter hoc non apparet multoties qualitas comprehensionis rerum visibilium, quae comprehenduntur ratione apud comprehensionem."

54. *Optics*, II.3; MS Fatih 3213, fol. 34^{a-b}; Risner, p. 34, sec. 15. As Alhazen mentioned elsewhere, Ptolemy had counted seven, see A. I. Sabra, "Ibn al-Haytham's criticisms of Ptolemy's *Optics*," *Journal of the History of Philosophy* 4 (1966): 46. Ptolemy wrote: "Dicimus ergo quod uisus cognoscit corpus, magnitudinem, colorem, figuram, situm, motum, et quietem" (Lejeune, *L'Optique de Claude Ptolémée*, p. 12). But since his list does not include light, it is doubtful that he meant it as a complete enumeration.

55. *Optics*, II.3; MS Fatih 3213, fol. 34^{a-b}; Risner, p. 34, sec. 15, lines 1-3: "Et cum declarata sint omnia ista, incipiemus modo ad declarandum qualitates comprehensionis cuiuslibet intentionum particularium, quae comprehenduntur per visum, et qualitates argumentorum (*kayfiyyat al-maqāyīs*, manner of inferences), per quae acquirit virtus distinctiva intentiones comprehensas sensu visus."

56. *Optics*, II.3; MS Fatih 3213, fol. 49^{a-b}; Risner, p. 38, line 1-p. 39, line 4: "Et ex comprehensione rei visae in suo loco, opinati sunt ponentes radios, quod visio esset per radios exeuntes a visu, et pervenientes ad rem visam, et quod visio esset per extremitatem radii, et ratiocinati sunt contra physicos, dicentes. Cum visio fuerit per formam venientem a re visa ad visum, et illa forma pervenit ad interius visus, quare comprehenditur res visa in suo loco, qui est extra visum, et forma eius iam parvenit ad interius visus. Et non sciverunt isti, quod visio non completur solo sensu tantum, et quod visio non completur, nisi per cognitionem et distinctionem antecedentem, et si cognitio et distinctio antecedens non esset, non compleretur in visu visio."

XI

FORM IN IBN AL-HAYTHAM'S THEORY OF VISION

I

It was no innovation for someone in the eleventh century to promulgate the doctrine that vision was mediated by the reception of visible forms into the eyes of the perceiver. Among natural philosophers, the doctrine had been continuously maintained by commentators and followers of Aristotle from Alexander of Aphrodisias to Abū 'Alī ibn Sīnā (d. 1037). But when Ibn al-Haytham, writing as a mathematician, decided to appropriate the Aristotelian doctrine as a fundamental proposition in his *Optics* (*Kitāb al-Manāẓir*),[1] he was breaking away from a long-estab-

* Department of the History of Science, Harvard University, Cambridge, MA 02138, USA. Paper read before the Society for Medieval and Renaissance Philosophy, at the annual meeting of the American Philosophical Association, December 28, 1986.

[1] The *Optics*, in seven Books or *maqālāt*, was rendered into Latin by an unknown translator (or translators) at the end of the twelfth or, perhaps more likely, in the early thirteenth century. The earliest surviving manuscripts of this translation are from the thirteenth century; and one of these, MS CR3.3 at the Royal Observatory, Edinburgh, is dated 1269 (see D. C. Lindberg, *A Catalogue of Medieval and Renaissance Manuscripts*, Toronto, 1975, pp. 17–19). The Latin text, circulating in the Middle Ages as *De aspectibus* or *Perspectiva*, was first published from two as yet unidentified manuscripts by Friedrich Risner in a collective volume bearing the title, *Opticae thesaurus* ..., Basel, 1572; reprinted, with a historical Introduction by D. C. Lindberg, New York, 1972 – hereafter referred to as: Risner, followed by page and line numbers (e. g. Risner 13:30). The 1572 volume included Witelo's *Perspectiva* and a treatise on dawn and twilight (*De crepusculis*) which was wrongly attributed to Ibn al-Haytham (cf. A. I. Sabra, "The authorship of the *Liber de crepusculis*", *Isis*, 58 (1967), pp. 77–85 and 560). The present writer has published a critical edition of the Arabic text and an English translation of Bks I-II-III, *On Direct Vision*, as: *Kitāb al-Manāẓir* ..., Kuwait, 1983; *The Optics of Ibn al-Haytham. Books I-III: On Direct Vision*, in two parts, Transla-

lished tradition that went back to Ptolemy and Euclid, whose writings on vision had passed into Arabic versions about the same time as did those of Aristotle and the Greek commentators. That was a crucial decision of important consequences for the later study of visual perception, not only in Islamic science and philosophy, but also, and in fact to a much larger extent, in the writings of European philosophers from the thirteenth to the early seventeenth century. Ibn al-Haytham's decision marked, in effect, the emergence of a new approach which coincided neither with that of the earlier natural philosophers nor with that of the mathematicians – an approach which resulted, not in juxtaposing elements of earlier doctrines, but in rethinking those elements in such a way as to suggest new problems and new methods of solution. As was probably inevitable, many of these problems were serious problems in the psychology of visual perception which had not received systematic treatment from earlier writers on optics.[2]

Ibn al-Haytham himself characterized his approach in the *Optics* as a 'synthesis' (*tarkīb*) between the natural-philosophical account of vision in terms of forms, and the mathematical mode of treatment, which he learnt from Euclid and Ptolemy, in terms of lines and angles.[3] What this meant was that he proposed to subject form to geometrical analysis, something which no Aristotelian before him had thought of doing, and which, naturally, no mathematician operating with the visual-ray hypothesis felt the need for. A first result of this synthesis was an explanation of how a form that represents the visible object enters the eye – an explanation in which the physics of light and colour and the anatomy of the eye, as well as geometry, were brought into play in

tion and Commentary, London, 1989. The Arabic text and English translation will be cited by Book, chapter and paragraph numbers (e. g. II, 3 [24]); these numbers are identical in the Arabic and English versions.

[2] Ptolemy went some way in recognizing a role for psychology in the explanation of the visual act, and it was from Ptolemy that Ibn al-Haytham's investigations started. For comparisons of the doctrines and approaches of these two authors, see English translation of *The Optics of Ibn al-Haytham*, Introduction and Commentary.

[3] For various expressions of the idea of synthesis in Ibn al-Haytham, see *The Optics of Ibn al-Haytham. Bks I-III*, Commentary, note on I, 1[2].

a distinctive manner. Since both the physics and the physiology were presented in terms of the ubiquitous 'form', we shall try first to see what this word meant in the context of Ibn al-Haytham's physics.[4]

In the *Optics* of Ibn al-Haytham there is no special exposition of his doctrine of light and colour and, in particular, of their ontological status. The book is not a philosophical dissertation on what light and colour are, but a mathematical and experimental study of their properties, especially, but not exclusively, in so far as these relate to the problem of vision. Nevertheless, an underlying doctrine is clearly implied, and a clear account of this doctrine can be easily assembled from statements in Chapter 3 of Book I and from a short treatise, the *Discourse on Light* (*Qawl*, or *Maqāla*, *fī al-Ḍaw'*), which Ibn al-Haytham wrote sometime after the *Optics*.[5] The picture which can be obtained

[4] In ordinary, non-technical Arabic, the word *ṣūra*, here translated as 'form' and in the Latin version as *forma*, means likeness, image, picture, illustration, as well as form, shape, figure, appearance. In Arabic translations from the Greek it rendered a number of words which included *eidos*, *eidōlon*, *eikōn*, *idea*, *morphē*, *typos* (see S. M. Afnan, *A Philosophical Lexicon in Persian and Arabic*, Beirut, 1969; H. Daiber, *Aetius Arabus*: *Die Vorsokratiker in arabischer Überlieferung*, Wiesbaden, 1980 – see arabisch-griechisches Glossar). Most often in Arabic philosophical writings, *ṣūra* corresponded to *eidos* in the Aristotelian sense of form, kind, or nature. *Eidos*, of course, also carried the more ordinary senses of form (or figure), shape, and appearance. Ibn al-Haytham, as will be seen, employs *ṣūra* in a variety of ways some of which reflect its ordinary as well as its technical/Aristotelian usage, while others are direct consequences of his own theory. He has a special word for specular image, *khayāl*, which he characterizes as a *ṣūra* seen in a mirror (*Optics*, I, 1 [7]).

[5] The differing titles of the two works already indicate a difference of emphasis: the later book was about *light*, the earlier primarily about *manāẓir*, a word which corresponded to the Greek *optika* and which sometimes rendered the cognate *opseis* (visual rays) (see the article on *Manāẓir* in *The Encyclopaedia of Islam*, New edition).

It should be noted that Chapters 1–3 in Bk I are missing from the Latin translation, both in Risner's edition and in all extant manuscripts. And, to my knowledge, there is no evidence that these chapters were ever available to Latin readers. No one, for example, has pointed out passages or terms or doctrines in medieval Latin writers that could have derived only from those chapters. It remains possible, however, that a complete Latin manuscript once existed from which the same chapters were removed together with the name of the translator.

from the relevant passages in these two works is coherent and unambiguous, and it can be described briefly and precisely. Light is a form (*ṣūra/forma*) in virtue of which material bodies shine forth into the surrounding medium. Either it naturally inheres in the body, in which case it is considered an "essential" form (*ṣūra jawhariyya*); or it is temporarily "fixed" in the body's surface, and in this case it is said to be an "accidental" form (*ṣūra 'araḍiyya*). The light that shines from naturally inherent or essential light (*ḍaw' dhātī*) is called "primary" (*awwal*); that which shines from accidental light (*ḍaw' 'araḍī*) is called "secondary" (*thānin*). Both self-luminous objects, such as the sun or a flaming torch, and illuminated ones, such as the surface of the ground in daylight or in moonlight, are said to be luminous or shining (*muḍī'a/luminosa*) because light emanates from both kinds of objects, although in one case the emanating light is "primary", and in the other, "secondary". Colour is an objective quality of bodies, distinct from light; and, like light, it either inheres in the coloured body or is temporarily cast upon it by another coloured body. Illuminated colours have the capacity to "shine" (*ashraqa*) or "extend" themselves (*imtadda*) into the surrounding medium, but whether they can do so in the absence of light is left an open question. We might add that it is also an undecidable question, since perception of colour is asserted to be impossible without the accompanying light. Bodies are divided with respect to their behaviour in relation to light into four classes: (1) essentially luminous, or self-luminous, bodies (*ajsām muḍī'a min dhātihā*) are those in which light naturally inheres; they are not coloured but possess "something like colour" (*shay' yajrī majrā al-lawn*); (2) opaque objects (*ajsām kathīfa*) cause the light shining upon them to be fixed (*thabata*) in their surfaces as accidental light from which secondary light radiates, and this secondary light is always accompanied by radiation of the object's colour; (3) a spe-

For one of several editions of the *Discourse on Light*, see J. Baarmann, "Abhandlung über das Licht von Ibn al-Haiṭam", *Zeitschrift der Deutschen Morgenländischen Gesellschaft*, 36 (1882), pp. 195–237; reviewed by E. Wiedemann in *ZDMG*, 38 (1884), pp. 145–48. Another edition, to which reference will be made here, appeared as *Risāla* 2 in: Ibn al-Haytham, *Majmū' al-Rasā'il*, Hyderabad, Dn., A. H. 1357. A French translation is included in R. Rashed, 'Le "Discours de la lumière" d'Ibn al-Haytham', *Revue d'histoire des sciences et de leurs applications*, 21 (1968), pp. 198–224.

cial class of opaque bodies called mirrors have smooth surfaces that cause the incident light to be turned back or reflected in a determinate plane and direction; (4) transparent bodies, like air or water, allow the light to pass through them, but being always endowed with a certain degree of density (*ghilaẓ*) they also cause some of the light to be fixed in them, and they always resist (*mānaʿa*) the passage of light more or less.[6]

This brief outline of Ibn al-Haytham's doctrine of light and colour is enough to acquaint the reader with his use of 'form' as a term denoting a corporeal property which either naturally inheres in or temporarily supervenes upon a physical object. Form in this sense is not to be confused with visible shape or figure or appearance; it simply refers to the light and colour themselves as physical properties of the luminous and coloured object.[7] In what

[6] In the *Discourse on Light* Ibn al-Haytham asserts "as a universal proposition" held by "the learned among physicists" that "... every property (*maʿnā*) that exists in a natural body and is one of the properties that constitute the body's essence (*māʾiyya*) is called a substantial [or essential] form (*ṣūra jawhariyya*); for the substance (*jawhar*) of any body only consists of the totality of those properties in that body which are inseparable from it as long as its substance remains unchanged. Now light in every self-luminous body is one of the properties that constitute the essence (*māʾiyya*) of such a body. Therefore light in every self-luminous body is a substantial form in that body. And the accidental light (*al-ḍawʾ al-ʿaraḍī*) that appears on the opaque bodies on which [light] shines from other [bodies] is an accidental form [in the opaque bodies]" (Hyderabad edition, p. 2, lines 10–18). In a similar manner, Ibn al-Haytham is led to assert that transparency, being one of the properties that constitute the essence of a transparent body, is a "substantial form" in such a body (ibid., p. 6, lines 10–12). See also p. 3, lines 21–23; p. 4, lines 6–9; p. 5, lines 5–15; p. 7, lines 6–9; p. 9, line 18-p. 10, line 12; p. 10, lines 4–12.

In the *Optics*, for "accidental" and "essential" lights, see: I, 3 [29], II 3 [48, 52]; I, 3 [69, 88, 95, 97, 110], II, 3 [52]; for "primary" and "secondary" lights, see: I, 3 [21]; I, 3 [88–98]; for "primary" and "secondary" forms of colour and of light, see: I, 6 [99, 100, 101, 103, 104, 107; 105], 8 [9]; for the colour of selfluminous bodies, see: I, 2 [12], 3 [113]; for the objectivity of colour, see: I, 3 [132–139]; for the extension of colour, see: I, 3 [138–141]; and for the radiation of colour, see I, 3 [113–121].

In Risner's text, the expressions *lux accidentalis* or *lumen accidentale* occur at 35:21 and 35:59; while *lux essentialis* occurs at 35:56. *Forma prima* can be found at 18:30, 31, 50; 19:57; and *forma secunda* at 18:29, 30, 50, 57; 19:17, 24, 26, 35; 23:41.

[7] In *Optics*, I, 3[1–112], the explanation of the manner of radiation of light is presented without employing the word 'form'. It is "light" itself that is spoken of here as radiating from shining objects or, rather, from points on the shining objects – as, for example, in the following passage: "the light shining from a self-luminous body into the transparent air ... radiates from every part of the luminous body facing that air, ... and it issues from every point on the luminous body in every straight line that can be imagined to extend in the air from that point" (I, 3[21]; cf. also I, 3[110]). At I, 3[20], we already meet 'form' in the (Aristotelian) sense of nature, in an argument aiming to establish that even the smallest parts of a physical object must behave (with respect to their optical properties) in the same way as the object as a whole, as long as parts retain the "nature" or "form" of the whole object: "This property [= the radiation of light from every part of a self-luminous body] being manifest in the case of the larger parts of self-luminous bodies, their smaller parts – even when extremely small and as long as they preserve their form – must also be luminous; light will radiate from these parts in the same manner as it does from the larger ones ... For this property is natural to self-luminous bodies and inseparable from their essence. Now small and large parts have the same nature as long as they preserve their form. Therefore, the property that belongs to their nature must exist in each part (whether small or large) provided that that part maintains its nature and form. ..." (See also I, 3[98] where it is again asserted that the smaller parts will always share the same qualities of the larger ones as long as they "preserve the form of their species".) It is when Ibn al-Haytham starts his investigation of the radiation of colour that he begins to conduct his discussion in terms of "forms" of colour and of light: "we find that many of the colours in opaque bodies that shine with accidental light accompany the lights that radiate from those bodies – the form of the colour being always found together with the form of the light. And similarly with bodies that shine with their own light: their lights are found to be similar to their forms which are of the same sort as colours. For the form of the sun's light that is of the same sort as colour is similar to the form of the sun. Similarly, the form of the light of fire is similar to the form of the fire." Light and colour, then, exist as forms in the shining and coloured bodies from which forms of light and colour emanate in all directions; see also I, 3[132–136, 138, 140–143].

follows I shall refer to form-as-property by the abbreviation 'f_p'. As has been noted, the Latin translation of the *Optics* always renders *ṣūra* by *forma*, never by *species*: when the latter term is used, it corresponds to the Arabic *nawʿ*, which represented the Greek *eidos* in the sense of natural class, kind or sort of objects. The expressions *lux essentialis*, *lumen accidentale*, *forma prima* and *forma secunda* (whether of light or of colour) are also found in the Latin version, corresponding to *ḍawʾ dhātī*, *ḍawʾ ʿaraḍī*, *ṣūra ūlā*, and *ṣūra thāniya*, respectively.[8]

[8] The distinction between essential and accidental lights calls to mind Ibn

II

Now the view which Ibn al-Haytham ascribed to the 'physicists', and which he wanted to take into his own theory, was the view that a form, not just of light and colour, but *of the object*, came from the object to the eye where it initiated the process of vision.[9] More than a century and a half before Ibn al-Haytham, the philosopher al-Kindī, in a work now extant only in a Latin translation known as *De aspectibus*, put forward what he believed to be a definitive argument against this view.[10] The argu-

Sīnā's well known tripartite division of light into *ḍaw'/lux*, *nūr/lumen* and *shu'ā'/radius* (See *Avicenna's De Anima, being the psychological part of Kitāb al-Shifā'*, ed. F. Rahman, Oxford, 2nd ed., 1960, Maqāla III, Faṣl 1, esp. pp. 91–92; S. Van Riet, ed., *Avicenna Latinus*: *Liber De anima I-II-III*, Louvain-Leiden, 1972, pp. 169–73 and Lexiques, under the terms cited above.) In general, Avicenna's *lux* corresponds to Ibn al-Haytham's essential light; *radius* often denotes what Ibn al-Haytham would call "primary light"; while *lumen* sometimes denotes Ibn al-Haytham's "primary light", sometimes his "accidental light". Another Avicennian category is *barīq/splendor*, the glittering light from shining smooth surfaces. It is not advisable, however, to merge the classifications in these two authors with one another, despite the possibility of their sharing a common origin. Nor does Ibn al-Haytham have a terminological distinction between *lux* and *lumen*, contrary to the Latin translation of the *Optics*; he employs *ḍaw'* for essential, accidental, primary and secondary lights. This accords with the doctrine, which Ibn al-Haytham seeks to establish experimentally, that primary and secondary lights (or forms) behave in identical manner: namely, they issue from all points on the shining surface in all rectilinear directions, whether the light *in* the surface is essential or accidental.

[9] *Optics*, I, 1[3]: "The learned among [physicists] settled upon the opinion that vision is effected by a form which comes from the visible object to the eye and from which sight perceives the form of the object."

[10] Vision, argues al-Kindī, cannot but take place in one of the following manners: (1) A form (*forma*) of the visible object reaches the eye where it makes an impression; (2) A visual power (*virtus*) goes out of the eye and extends itself to the object; (3) These two things occur at the same time; (4) The object's form impresses the air which in turn makes an imprint in the eye. (Al-Kindī's text is as follows: "Dico igitur impossibile esse, quin [1] oculus sua recipiat sensibilia peruenientibus et currentibus suorum sensibilium formis ad eum, quemadmodum plures antiquorum extimauerunt, et sigillentur in eo, aut [2] ab eo procedat uirtus ad sua sensibilia, cum qua ea recipiat, aut [3] haec duo sint simul, aut [4] eorum formae sint sigillantae in aere et im-

ment is brief, even somewhat abrupt, being presented as an immediate consequence of regarding the received form as a replica

pressae, et aer sigillet eas et imprimat in oculo, quas oculus comprehendit uirtute sua receptibili eius, quod aer in eo impressit lumine mediante" *De aspectibus*, Prop. 7, p. 9, in A. Björnbo and S. Vogl, eds, *Alkindi, Tideus und Pseudo-Euclid*: *Drei optische Werke*, Abhandlungen zur Geschichte der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen, Heft XXVI.3, Leipzig/Berlin, 1912.)

There is nothing in [1] that differentiates it from the view usually attributed to Peripatetic philosophers by Islamic medieval writers, and I take it to be associated in al-Kindī's mind with "Aristotelian" doctrine. [2] describes a view shared by Euclid and Ptolemy, whose theories diverged, however, with regard to the manner in which the visual power proceeded from the eye – Euclid thinking in terms of discrete radial lines, Ptolemy in terms of a continuous cone of radiation. [3] is a descendant of the Platonic view according to which vision resulted from the mingling of the light issuing from the eye with the external light (not the object's form, as here). [4] looks very much like the Democritean theory as described in some detail by Theophrastus (see G. M. Stratton, ed. and trans., *Theophrastus and the Greek Physiological Psychology before Aristotle*, London/New York, 1917, pp. 108–11: "Vision [Democritus] explains by the reflection [*emphasei*] ⟨in the eye⟩, of which he gives a unique account. For the reflection [*emphasin*] does not arise immediately in the pupil. On the contrary, the air between the eye and the object of sight is compressed by the object and the visual organ, and thus becomes imprinted [*typousthai*]; since there is always an effluence of some kind from everything. Thereupon this imprinted air, because it is solid and is of a hue contrasting ⟨with the pupil⟩, is reflected in the eyes, which are moist. A dense substance does not receive ⟨this reflection⟩, what is moist gives it admission". Words between square brackets added; see also Aetius in Daiber, *Aetius Arabus*, p. 202, lines 11–12.).

Lindberg reverses the authorship of [1] and [4], identifying [1] with the atomists and [4] with Aristotle, but without giving a special reason; see his *Theories of Vision*, p. 22. According to al-Kindī, [1], [3] and [4] all involve the assertion that a form of the object eventually makes an impression in the eye through which the eye perceives the object; and it is this assertion which he refutes by the argument paraphrased above. Here is the argument as it has reached us in the Latin translation: "Si ergo sensibilium formae procederent, donec ad oculum peruenirent, et sagillarentur in eo, aut duae causae essent simul, quaeque uidelicet earum ad suam comparem curreret, aut sensibilia suas formas imprimerent in aere et sagillarent, oporteret, ut circuli, qui cum aspiciente in una consistunt superficie, procederent et currerent ad aspicientem, et uiderentur quemadmodum sunt secundum esse. Sed non apparet ita. Immo cum circuli et aspiciens in una consistunt superficie, circuli nullo modo uidentur. Non ergo restat, ninsi ut ab aspiciente ad res, quae aspiciuntur, procedat uirtus, qua eas recipiat" (*De aspectibus*, ed. cit., p. 9, lines 21–29).

or representation of the object's own form. For something to be a formal representation of an object, al-Kindī appears to be arguing, it must accurately reproduce the object's own form. For example: if the object is round, then roundness is a feature of the object's own form; and the form *representing this* must convey roundness. When, however, we view flat round objects placed edgewise before our eyes, we do not see them as round objects, that is, in accord with what they are in themselves (*quemadmodum sunt secundum suum esse*), but as straight, thick lines; therefore it cannot be the case that we see objects as a result of receiving their forms. For al-Kindī, the argument demonstrates the advantage of the geometrical model employed by adherents of the visual-ray theory, like himself: With the help of this model, and by drawing straight lines from a point in the eye to points on the opposed surface of the object, one was better able to account for the phenomenon described.

Al-Kindī's argument was directed against all explanations in terms of forms, whether these were the immaterial *eidē* of Aristotle or the material *eidōla* of the atomists, but seems to be more directly applicable to the former. It was, in any case, the Aristotelian version which Ibn al-Haytham wanted to incorporate into his synthesis; and whether or not he was acquainted with al-Kindī's work, his theory can be said to be the first intromission theory of vision that attempted to offer a solution to al-Kindī's problem in terms of the geometrical language favoured by al-Kindī.

The solution, which I shall now outline, made use of two new ideas, one of which belonged to the physics of light and colour, the other to the physiology of vision. The first introduced what Vasco Ronchi has called analyzing the visible object into punctiform elements.[11] and it consisted in regarding a shining surface (whether self-luminous or illuminated) as an aggregate of shining points each of which behaved independently of the others. Ibn al-Haytham explicitly states the principle that from the light and colour at each and every point on the shining coloured surface, light and colour rectilinearly radiate in every direction. Or, as he also phrases it, from the light and colour at every point on the

[11] Vasco Ronchi, *Histoire de la lumière*, trans. J. Taton, Paris, 1956, p. 38.

surface, a form of that light and colour rectilinearly emanates in every direction.[12] But since, as we have seen, the light or colour exists in the object as a form/property f_p, then the emanating form, being some sort of likeness or reproduction of f_p, may be called an image of f_p. We are thus led to introduce the following definition of form-as-image of a shining point on a coloured surface: Let f_p be the form/property of a given point P on the shining surface of a coloured object; then, a form/image f_i of f_p is said to exist at any other point I if and only if I lies on the unobstructed path of radiation from P. In most cases (the only cases known to Ibn al-Haytham), the path of radiation from P is that of direct, reflected or refracted radiation from P. Since f_p is an optical property of point P (namely, light or colour) we may call f_i an optical image of f_p. But note that f_i is also an optical property f_p of point I, being what Ibn al-Haytham would call an accidental form of point I.

The principle just stated yields one consequence of great importance for the theory of vision. It is that optical images f_i's of all points on a shining surface, say the sunlit façade of a building, will exist at every single point in front of that surface, provided only that unobstructed straight lines can be drawn from that single point to every point on the surface.

Writers on vision, before and after Ibn al-Haytham, sometimes seemed to express the fact embodied in this consequence in a different manner, by regarding the accumulation of f_i's at a single point, not as a confluence or confusion of images of points on the object, but as an image *of the total object* considered as a spacial configuration of *distinct* points. We read in Plotinus, *Ennead* IV.5, in the course of an argument against the view that we see through the agency of a medium affected by the visible object:

[12] See, for example, *Optics*, I, 3 [141–143]. The notion of forms of individual points of light and colour is central to the explanation of the manner of vision in Bk I, ch. 6 of the *Optics*. Ibn al-Haytham begins his explanation thus: "We say . . . that when the eye faces a visible object, there comes from each point on the surface of the object to the whole surface of the eye the form of the colour and the light that exist in that point. And from each point on every visible object facing the eye there also comes in that moment to the whole surface of the eye the form of the colour and the light that are in that point" (I, 6 [12]; see also I, 6 [13–55 *passim*]).

> For if the intermediary air was affected, the affection would presumably have to be a bodily one; but this means there would have to be an impression, as in wax. Then a part of the seen object would have to be stamped on each part of the air; so that the part of the air in contact with the eye would receive a part of the seen object just as large as the part which the pupil of the eye would receive according to its own size. But as it is, the whole object is seen, and all those who are in the air see it, from the front and sideways, from far and near, and from the back, as long as their line of sight is not blocked; *so that each part of the air contains the whole seen object,* the face for instance [ὥστε ἕκαστον μόριον τοῦ ἀέρος ὅλον οἷον τὸ πρόσωπον τὸ ὁρώμενον ἔχειν]; but this is not a bodily affection, but is brought about by higher necessities of the soul belonging to a single living being in sympathy with itself.[13]

Thus, from the fact that an object can be seen from any point in the air surrounding the object, Plotinus felt entitled to speak of *every part* of the air as containing *the whole object* seen. And, indeed, the experience of seeing, which is here invoked as evidence for what is being argued for, does normally consist in having a distinct image of the whole visible aspect of the object. But such an image cannot be obtained by drawing lines from points on the object to the facing point occupied by the eye. Images of all points on a shining object will exist everywhere in the facing atmosphere, but these images will all be confused.

Towards the end of the fifteenth century, Leonardo da Vinci wrote (1492) in more explicit language:

> Every body in light and shade fills the surrounding air with infinite images [*similitudini*] of itself; and these, by infinite pyramids diffused in the air, represent [*rappresentano*] this body throughout space and in every part.[14]

According to Leonardo, an image, a likeness of my face for instance, exists now at every point on this page; and he knows this to be a fact on the grounds that he can construct a pyramid or cone of rays with my face as base and the point on the page as apex. But I do not see a likeness of my face anywhere on the page. And the reason for that is that each of the images generated by means of those pyramids is but a confused aggregate of images of points on my face. Leonardo knew, however, of a simple way to segregate the individual point-images and order them so as to form a visible likeness of the object from which they proceed. I am referring to his use of the *camera obscura*. To quote Leonardo again from the same manuscript:

> It can clearly be shown that all bodies are, by their images [*similitudine*], all in all the surrounding atmosphere, and all in each part as to substance [*per corpo*], form [*figura*], and colour; this is seen by the images [*spetie*] of many and various bodies which are reproduced in one single perforation through which they transmit the objects by lines which intersect and cause reversed pyramids, from the objects, so that they are upside down on the dark plane where they are first reflected.[15]

The principle of the device referred to here was known to Ibn al-Haytham,[16] but he made no attempt to apply it to the problem of vision. Instead of treating the eye as a pin-hole camera (or, as Kepler was to do later, as a lens-camera) he precipitously appealed to physiology and, almost at once, to psychology. He thus missed the chance to obtain a real likeness of the visual field inside the eye. This had implications for his concept of form, as we shall see presently. But although his failure to obtain a real image in the eye entailed a faulty account of the physiology of vision, it had no significant effect on his psychological theory of visual perception.

III

We come now to Ibn al-Haytham's physiological hypothesis, the second of the two ideas mentioned above. He assumes the surface of the eye (i. e. cornea) and the front surface of the crys-

[13] Translation by A. H. Armstrong in the Loeb edition of *Plotinus*, vol. IV, Cambridge, MA and London, 1984, p. 293.

[14] MS B. N. 2038, 6b, in J. P. Richter, ed. and trans., *The Literary Works of Leonardo da Vinci*, 2 vols., 3rd ed., New York, 1970, I, p. 136, no. 63.

[15] Ibid., I, p. 136, no. 61.

[16] *Optics*, I, 6 [85–88]; Risner 17:24–17:36. See also E. Wiedemann, "Über die *Camera obscura* bei Ibn al Haiṭam", in *Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen*, 46 (1941), pp. 155–69, reprinted in *idem*, *Aufsätze zur arabischen Wissenschaftsgeschichte*, 2 vols., Hildesheim/New York, 1970, vol. I, pp. 87–101; M. Naẓīf, *al-Ḥasan ibn al-Haytham ...*, Cairo, 1942–43, pp. 180–204; and the article on Ibn al-Haytham in *Dictionary of Scientific Biography*, ed. C. Gillispie, vol. VI, New York, 1972, pp. 195–96.

talline humour to be spherical and concentric, their common centre being the same as that of the eyeball. The body of the crystalline is denser than the air outside, and it also differs in density from the so-called vitreous humour lying immediately behind the crystalline and in touch with it. The eyeballs are set in the cavities of the optic nerves along which the visual spirit, as bearer of visual impressions, performs a shuttle service between the eyes and the brain.

Now consider the forms or images f_i's that accumulate at any point on the surface of the eye, having arrived there from all points on the visible object. Some of these point-forms will be refracted into the eye and, upon reaching the forward surface of the crystalline humour, refracted again into this body. Other point-forms, namely those that arrive across the atmosphere along lines drawn from the common centre and are, therefore, perpendicular to both surfaces of the cornea and crystalline, will pass through these two surfaces without suffering any change of direction.

To obtain what he considered a *distinct form* (or image) *of the whole visible surface* inside the crystalline, Ibn al-Haytham postulates that only the latter group of f_i's can be effective in the process of vision. That is to say, he assumes that only impressions produced in the crystalline by the perpendicular point-forms will ultimately be presented to the brain, these perpendicular forms being the only ones that have a one-to-one correspondence with points on the visible surface and, hence, are capable of representing the distribution of light and colour on that surface. In brief, Ibn al-Haytham's idea is that we must distinguish between lines of transmission as determined by the rules of physical optics (in this case they are the lines of rectilinear propagation *and* of refraction), and those on which sensation is carried to the brain, although these two classes of lines may overlap. Let us call the latter group of lines 'sensory lines' (or 'lines of sensitivity'). Within the crystalline, where, as Ibn al-Haytham tells us, sensation "begins",[17] the sensory lines coincide with the radial lines themselves. Within the vitreous humour, they coincide with those lines into which the radial lines are refracted according to the

[17] Cf. *Optics*, II, 2 [10]; Risner 26:1-2.

relative densities of the crystalline and vitreous humours, the purpose of this refraction being to avoid intersection of the incoming rays before they reach the optic nerves. Within the nerves themselves, the sensory lines are those on which the visual spirit extends through the nerve fibres.[18] The idea is that point-forms travelling along the sensory lines will maintain the configuration they have on the crystalline's surface until they reach the juncture of the nerves where the configuration arriving from one eye coincides with that arriving from the other.

We need not go here into questions about the exact details of this series of transmissions from one part of the visual apparatus to the next. The question we need to ask concerns the status of forms within the apparatus. We will call 'sensory point-forms' (f_s's) those point-forms that travel along the sensory lines. We note, first, that these sensory forms retain the optical properties associated with our f_p and f_i. This accords well with Ibn al-Haytham's characterization of the effect of light and colour inside the visual apparatus as one of illumination and coloration.[19] The same forms also retain an optical character within the eye in that they proceed across the tunics and humours in accordance with the laws of optics, though these laws no longer govern their journey through the optic nerves. Furthermore, the distribution

[18] Cf. *Optics*, II, 2 [1-18].

[19] *Optics*, II, 3 [9]: "Now to distinguish between two greens is not the same as the sensation of green, for the latter is due to the eye's becoming green by the [action of] the green; and the eye has become green by the [action of] the two greens; and as a result of becoming green by [the action of] both greens the sense perceives them to be of the same kind. Thus its perception that one of the greens is stronger than the other, and that they are of the same kind, is a discernment of the coloration that has taken place in the eye, and not a sensation of the coloration itself." Risner 30:40-42: "sed distintio inter duas viriditates non est ipse sensus viriditatis, quoniam sensus viriditatis est ex viridificatione visus ab utraque viriditate: et comprehendet quod sunt unius generis." See also *Optics*, II, 3 [47] = Risner 34:58-60, where Ibn al-Haytham speaks of the effect of light and colour on the visual spirit in the optic nerve as one of illumination and coloration: "the forms of colour and light reach the cavity of the nerve only because the sentient body that extends in that cavity becomes coloured by the form of the colour and illuminated by the form of the light/et forma lucis et forma coloris non perveniunt ad concavum nervi, nisi quia corpus sentiens extensum in concavo nervi coloratur a forma lucis et coloris, et illuminatur a forma lucis . . ." See also II, 3 [54, 100].

of sensory point-forms over any plane that cuts across the visual axis will produce a pattern of illumination and colour that can now be said to represent a visible surface in the sense that each of these point-forms corresponds to one and only one point on the surface. Such a pattern may, in other words, be said to constitute a distinct image of the surface, in the sense of 'image' defined above.

But, on the other hand, this pattern is not observable like the pattern of illumination and colour we see projected on the back of a pin-hole (or lens) camera. Rather, it has something in common with the *imago* which Kepler proposed to distinguish from what he called *pictura*. I am referring to the well-known definition in *Ad Vitellionem Paralipomena*:

> While up till now image (*imago*) has been a mental entity (*ens rationale*), let us call "pictures" (*picturae*) those figures of objects which truly exist (*vere existentes*) on a piece of paper or some other screen.[20]

To be sure, Ibn al-Haytham's forms, including those produced inside the eye, are not merely things of the mind; like the pictures on the paper, they truly exist as physical modifications or properties of parts of the eye. But, unlike Kepler's *picturae*, they are distinctly visible only to the mind of the perceiver.

Ibn al-Haytham's account of this part of his theory allows us to distinguish at least three more senses of the term 'form' in addition to those of f_p and f_i. First, there is the sensory point-form (or point-image), call it f_s, as an optical image f_i that has first entered the eye along one of the radial lines and is proceeding through the visual apparatus, crystalline-vitreous-nerve, *along one of the sensory lines.* Thus sensory point-images are a sub-class of the optical point-images inside the eye; they are set apart from other optical images in the eye by virtue of a selective sensitivity with which the composite crystalline-vitreous body is endowed.[21] Second, there is the optical form M_p of an object (or, rather, of the visible surface or aspect of an object) as the pattern of optical properties f_p's displayed by the object (or surface or aspect). Sometimes Ibn al-Haytham refers to M_p as the "form of a visible object".[22] Third, there is the sensory form/image M_s that is made up of all those point-images f_s's that correspond one-to-one with the elements of M_p. And just as M_p is the total optical form (light and colour display) of the visible surface, so also is M_s a total image of the surface. M_s is therefore an optical analogue or copy of M_p; and, as such, it may be said to represent or depict M_p. And it is this quasi-pictorial form/image M_s that is finally presented to the brain. I describe it as quasi-pictorial because while it shares some properties with Kepler's *pictura*, e. g. the one-to-one relationship with the represented optical display, it does not exist in public space – we cannot stand around it and observe it. It can perhaps be compared in some respects to the image on an exposed but not yet developed film, although the analogy may not be quite apt. (In Ibn al-Haytham's theory, "developing" a distinct image from the confused images in the eye remains a psychological, not a physical, process.)

[20] First edition, Frankfurt, 1604, p. 198. See also Catherine Chevalley, trans., *Paralipomènes à Vitellion (1604)*, Paris, p. 352.

[21] See especially *Optics*, II, 2 [11], and the associated note in my Commentary, in *The Optics of Ibn al-Haytham. Bks I–III*. Also A. I. Sabra, "Sensation and inference in Alhazen's theory of visual perception", in P. K. Machamer and R. G. Turnbull, eds, *Studies in Perception: Interrelations in the history of philosophy and science*, Columbus, Ohio, 1978, pp. 160–85.

IV

Ibn al-Haytham did not identify physiology and psychology, nor did he take the seeing of an object to be merely a matter of holding up an image to the mind's eye, so to speak. To present his theory simply as an explanation of how an optical image flows through the visual apparatus would be to report the results

[22] An example is the following: "if from that part that is cut off by the [visual] cone, the crystalline perceives only the form [M_i] that has arrived at that part along lines of the cone, without perceiving from that part of its surface any form [M_i] that reaches it along other lines, it will perceive the form [M_p] of the object as it is, with its own order" (I, 6 [30]). The identity here considered by Ibn al-Haytham is that between a pattern of light and colour on the surface of the object (called "form of that object" = M_p) and the pattern (M_i) produced at the crystalline's surface by the radial lines issuing from points (of light and colour f_p's) on the object. For other examples, see I, 6, [31, 32, 33, 46, 58, 62, 68]. At II, 3 [100] it is asserted that "light and colour constitute the form of the visible object" / "et lux et color sunt forma rei visae" (Risner 43:63).

of Book I of his *Optics*, and to disregard those of the two following Books. Ibn al-Haytham was fully aware that we see things, not images, and he insisted on the distinction between sensation and perception, although he applied the same word, *idrāk/comprehensio/perception* to both, calling the former "perception by pure sensation" (*idrāk bi-mujarrad al-ḥiss/comprehensio solo sensu*). Book II in fact expounds an elaborate theory of what *we* should call visual *perception*.

The theory involves the notion of form as the total visible appearance of a thing or the totality of the thing's visible characteristics. The "form of an object", in this sense, is made up of all the object's "visible properties" (*al-ma'ānī al-mubṣara/intentiones visibiles*), as Ibn al-Haytham calls them.[23] These are indefinite in number, but, according to Ibn al-Haytham, they "generally" fall into twenty-two categories, of which light and colour are two. Some of them coincide with Aristotle's common sensibles (size, shape, number, movement, rest), but they also include such properties as remoteness or distance of the object from the observer, position, beauty and ugliness, similarity and dissimilarity. Let us refer to form in this sense by the symbol F_a (form-as-total-appearance).

Ibn al-Haytham was convinced that no intromission theory would be able to hold its own unless it succeeded in explaining how we come to perceive F_a on the basis of the sensory image M_s. In his view, it was the lack of such an explanation that accounted for the apparent plausibility of the visual-ray hypothesis.[24] In brief, his explanation, to which he devotes the whole of Book II, maintains that we perceive F_a by a series of judgements performed by a faculty of judgement (or discernment or discrimination) (*quwwa mumayyiza/virtus distinctiva*) upon presentation of the sensory image to the *ultimum sentiens* (*al-ḥāss al-akhīr*); and he describes these acts of judgements as inferences (sing. *qiyās, istidlāl/argumentatio, ratio, ratiocinatio, syllogismus*) based *in part* on features, or signs, or clues (*amārāt/signa*) which are contained in the sensory image. Again, there was perhaps no radical innovation here. Plato, Aristotle and, especially, Plotinus all

[23] See below, p. 133.

[24] *Optics*, II, 3 [71]; Risner 38:1–39:4.

recognized a role for judgement in sense-perception; and the Arabic phrase used by Ibn al-Haytham is reminiscent of Aristotle's *dynamis kritikē*.[25] But, again, Ibn al-Haytham's treatment is distinctive, and it amounts to a full theory of the psychology of vision, the like of which is not to be found in any earlier philosopher. Before I indicate something of the character of this theory, it will be necessary to support some of my last statements with one or two quotations.

Concluding a discussion on the role of recognition (*ma'rifa/cognitio*) in identifying objects of vision, as in identifying a particular colour or an individual or a species to which the individual belongs, Ibn al-Haytham writes:

> Now recognition is not pure sensation. For the sense of sight perceives the forms [F_a's] of visible objects from the forms [M_i's as a configuration of f_i's] that come to it from the colours and lights of those objects. And its perception of lights *qua* lights and of colours *qua* colours *is* by pure sensation. But those properties (*ma'ānī/intentiones*) in the form [?F_a] which, or the like of which, it has previously perceived, and which, or the like of which, it remembers having perceived, are at once perceived by recognition from the signs in the form [?M_s, ?F_a]. The faculty of judgement then discerns this form [?M_s, ?F_a], thus perceiving from it all properties in it, such as order, outline, similarity, dissimilarity and all properties in the form [F_a] the perception of which is not effected by mere sensation or by recognition. Therefore, among the properties that are perceptible by the sense of sight, some are perceived by pure sensation, others by recognition, and others still by a judgement and inference that exceeds the inferences of recognition.[26]

[25] The surviving Arabic translation of Aristotle's *De anima* employs *'aql mumayyiz* (discriminating mind) for *nous* and for *nous kritikos* ('A. Badawī, ed., *Arisṭūṭālīs fī al-Nafs, etc.*, Cairo, 1954, p. 75, line 22; p. 85, line 18), but generally renders *krinein* by *qaḍā* (to judge) (e. g. ibid., p. 44, line 15; p. 55, lines 4–8; p. 69, lines 18–19). Mattā ibn Yūnus, in his Arabic version (from Syriac) of Aristotle's *Posterior Analytics*, renders *dynamis kritikē* (at 99b35) as *quwwa mukhtabira*, examining faculty ('A. Badawī, *Manṭiq Arisṭū*, II, Cairo, 1949, p. 463, line 11). See John I. Beare , *Greek Theories of Elementary Cognition from Alcmaeon to Aristotle*, Oxford, 1906, Greek Index, under *dynamis, krinein, kritikos*.

[26] *Optics*, II, 3[25]. The corresponding Latin text in Risner's edition contains only the first sentence and a paraphrase of the last sentence: "*Cognitio autem non est solo sensu. Intentiones ergo quae comprehenduntur a sensu, quaedam per cognitionem, quaedam per rationem et distinctionem*" (Risner

It is passages like this, and they abound in the *Optics*, that force us to attempt distinctions such as those I have proposed here. The word 'form' oscillates between different senses in the same passage, and even in the same sentence. That 'form' could refer to the sum total of the object's visible characteristics is made quite explicit elsewhere. Ibn al-Haytham says, for example at II, 4 [1], that "the forms of visible objects are composed of the particular properties (*al-ma'ānī al-juz'iyya/intentiones particulares*) . . . such as shape, size, colour, position, order . . .". Adding that sight does not perceive any of these properties alone (on the grounds that none of them exists in a material object apart from the others), he concludes that "Sight . . . perceives only the forms of visible objects, and each of these forms is composed of a number of particular properties" [perceptible to the sense of sight]. That, moreover, he wants to maintain a distinction between sense impressions and other objects of perception is also made clear in several places, as in the following:

> . . . that which sight perceives by pure sensation is light *qua* light and colour *qua* colour. But nothing of what is visible, apart from light and colour, can be perceived by pure sensation, but only by discernment, inference and recognition, in addition to sensation; for all visible properties that are perceptible by discernment and inference can be perceived only by discerning the properties in the sensed form [M_s]. Similarly, all perceptions by recognition can be achieved only by perceiving the signs in the form that is sensed [M_s]. Therefore, perception of the visible properties that are perceived by discernment, inference and recognition comes about together with sensing the form [M_s].[27]

In this passage 'form' is used three times in the sense of sensory form. But here, lastly, is a passage in which, I trust, the reader will be able to detect the various meanings of 'form':

> Now all visible properties can be perceived only from the forms [M_i's as configurations of f_i's] produced in the eye by the forms [M_p's as configurations of f_p's] of the colours and lights of the visible object. But it has been shown that the form [$M_i = M_s$] of the light and colour that exist in the surface of a visible object occurs in the

31:48–50). The difficulty in deciding between M_s and F_a in the above passage is due to the fact that Ibn al-Haytham speaks of discernment as an operation applied sometimes to the sensed form, sometimes to the objective form consisting of the object's "particular properties".

[27] *Optics*, II, 3 [52]; Risner 35:48–52.

> surface of the crystalline humour where it has the same order [of parts] which it has [as M_p] in the object's surface; and that the forms [M_i's = M_s's] extend from that surface and pass through the body of the crystalline and through the sentient body [visual spirit] that exists in the cavity of the common nerve, while preserving throughout their extension the order they have on the crystalline's surface, . . . and that the *ultimum sentiens* perceives the forms [M_p's] of visible objects only from the forms [M_s's] that occur in the cavity of the common nerve. It has also been shown that sensation (*iḥsās/sensus*) is accomplished only when the *ultimum sentiens* perceives the forms [M_s's] of visible objects. All that being so, it follows that the discernment and inference which the faculty of judgement applies to the properties existing in the forms [?M_s's, ?F_a's] of visible objects, and also the recognition of forms [?M_s's, ?F_a's] and of signs in the forms [?M_s's, ?F_a's], and all that is perceived by discernment, inference and recognition, are due to the faculty of judgement's discernment of the forms [M_s's] that occur in the cavity of the common nerve when the *ultimum sentiens* perceives them, and to recognizing the signs which are in the forms [M_s's] and which are perceived in this manner.[28]

In these three passages we are told that seeing an object as a thing possessed of a multitude of visible properties is the result of a mental activity over and above the capacity to register the received impressions of light and colour. This activity consists in forming a series of judgements or inferences based, in part, on "properties" and "signs" in the sensory form (M_s) after the latter has reached the cavity of the common nerve or optic chiasma, where it is finally made apparent to the *ultimum sentiens*. Since it is only light and colour that can be objects of "pure sensation",[29] we may call 'judgemental (or inferential) properties' all other properties perceptible to the sense of sight. What are those features or signs in the total sensory image of an object that may serve as clues to the inferential properties? The question is only partially answered in the quoted passages, and a more complete

[28] *Optics*, II, 3 [45]. This is drastically reduced in Risner's text as follows: "*Et cum ita sit, distinctio et argumentatio virtutis distinctivae, et cognitio formarum et signorum eorum non erunt nisi ex cognitione vel distinctione virtutis distinctivae ex formis pervenientibus intra concavum nervi communis, apud comprehensionem ultimi sentientis illas, et ex cognitione signorum formarum illarum*" (Risner 34:31–34).

[29] *Optics*, I, 6 [61]: ". . . that which sight perceives by pure sensation is only the light and colour in that object."

answer would have to be gathered from Ibn al-Haytham's detailed expositions of what he takes to be the modes of inference involved in the perception of each of the judgemental properties. But, for the sake of illustration, it will be sufficient to observe that these signs or clues include such things as: the size of the sensory image, which Ibn al-Haytham, following Euclid and Ptolemy, takes to be a factor in judging the object's size; the shape of the image, which, again, is taken into account in inferences about the object's shape; the light and colour themselves, or rather their modalities, such as intensity and dimness of illumination, or brightness and darkness of colour, which may function as clues to distance from the observer;[30] the order of parts of the image, which replicate the arrangement of the object's own parts; and so on.

It is important to add, however, that Ibn al-Haytham's description of the unconscious processes that underlie our perception of the physical world are not confined to considerations of features internal to the optical image, but include other experiences not all of which are connected to the organ of sight. Tactile and muscular sensations that do relate to the organ of sight are, for example, those experienced when we close or open our eyes – operations which, Ibn al-Haytham says, are at the basis of our judgement that objects of vision lie before us in external space and not inside our eyes or our heads.[31] Another example is the muscular sensation associated with turning the head or orienting the eyeball, which is involved in judgements about directions.[32] Other examples involve other types of experience. Some of the most remarkable explanations in this connection are concerned with estimating the distance of an object, itself an essential factor in estimating the object's size. To convey something of the character of these explanations it will be best to excerpt a few related passages.

Ibn al-Haytham first remarks (II, 3[149]) that "Being much accustomed to judging distances of visible objects, sight, upon sensing the [object's] form [M_s] and the object's distance, will imagine the magnitude of the area occupied by the form and the magni-

[30] *Optics*, III, 7 [19–21, 194].

[31] *Optics*, II, 3 [72–74].

[32] *Optics*, II, 3 [95–96].

tude of the distance, and from both these notions will perceive the object's size." Now, according to Ibn al-Haytham, distances are ascertainable in magnitude only if they extend along a series of connected bodies the size of which can be (roughly) estimated. How, then, is *this* size measured? He first observes (II, 3[150]) that, in most cases, the series of connected bodies include those parts of the ground close to our feet, and these parts, having been repeatedly perceived and measured, will serve as standards for measuring the more remote parts. But how is a close part measured in the first place? One factor is, of course, the visual angle represented by the size of the image projected by this part. But there are other considerations. Here is Ibn al-Haytham's answer:

> . . . sight will perceive the magnitude of that [close] part, and the faculty of judgement will perceive that part and its magnitude, and it will ascertain its extent as a result of measuring it by our body. For we always measure such parts unintentionally by our feet whenever we step upon them, or by our arms whenever we stretch our hands to them. Thus all parts of the ground next to us are always measured unintentionally by our body. . . . Thus when someone, standing, looks at the ground close to his feet, the length of the radial lines will be measured by his height, and the faculty of judgement will comprehend with certainty the distance between his eyes and the part of the ground close to his feet, which is his height.[33]

It will not be possible here to give an account of the inferences by which the information acquired in this manner and attached in memory to images of the ground's close parts is compared with the sensed images of the remote parts. But it will be instructive to quote Ibn al-Haytham's conclusion:

> It is in this manner that . . . the sentient [faculty] and the faculty of judgement acquire [perception] of the magnitudes of the surrounding and neighbouring parts of the ground that lie between the eye and the visible objects. This acquisition takes place at the beginning of childhood, after which the magnitudes of distances of familiar objects existing on the earth's surface are established for the sentient [faculty] and for the faculty of judgement, so that perception of these familiar objects comes to be performed by recognition and by assimilating their distances to one another, at the moment of glancing at the intermediate bodies between these objects

[33] *Optics*, II, 3 [151].

and the eye and without recommencing the [process of] judgement and inference, but rather through recognition and assimilation alone.[34]

The views expressed in these and other, similar passages are repeated or echoed very strongly in the writings of Latin authors such as Roger Bacon, Witelo and Pecham. It is, therefore, surprising to find the "medieval theory of vision" which these authors inherited from Ibn al-Haytham described recently as an "openly physicalistic" theory which "does not . . . attempt to account for vision as a partly noncorporeal process".[35]

V

That sight may fall into error is a consequence of the fact that visual perception is largely a matter of judgement or discernment. Ibn al-Haytham calls "perceived form" the perception consisting of the group of judgements which we form either automatically upon glancing at an object or deliberately by scrutinizing the object. Form in this sense is a product, or construct, of "the faculty of judgement," although it contains the sensory form M_s representing the array of light and colour on the surface of the object and passively received (and selectively sensed) by the visual organ. Let us call form as a product of the faculty of judgement 'F_j'. The form to which F_j corresponds is F_a, that is, the visible object itself with all its visible characteristics as a collection of properties (*ma'ānī/intentiones*) capable of being apprehended by the sense of sight.

In Book II of the *Optics* and, especially, in Book III, F_j is often qualified by a number of attributes, all expressed by the adjective *ḥaqīqiyya*/true, or the passive *muḥaqqaqa*, which has several meanings, including: true, correct, distinct, ascertained, verified. In the Latin translation, *ḥaqīqiyya* is translated by *vera*, and *muḥaqqaqa* is variously rendered as *certa*, *certificata*, *verificata*, and, occasionally, as *manifesta*. Not every true form F_j is a verified or ascertained form (*forma certificata*), says Ibn al-Haytham. To

[34] *Optics*, II, 3 [154].

[35] Cf. Gareth Mathews, "A medieval theory of vision", in Machamer and Turnbull, eds, op. cit. (note 21 above), p. 189.

paraphrase some of his examples: I may truly recognize my neighbour's cat by merely glancing at it. This would be what Ibn al-Haytham calls "perception by recognition" or recognition based upon noticing a particular feature (*amāra/signum*) of the cat. But the form F_j thus obtained, though true, would be confirmed only after my first perceptual judgement has been verified by giving the cat a careful scrutiny. Such a scrutiny (*ta'ammul, tafaqqud/consideratio, intuitio*) can only consist of forming further judgements which, like the first, will result from comparing presently perceived images with others stored in my memory. Clearly, it is the psychologist who is speaking here, not the epistemologist. The verified form will be distinguished from the one obtained by mere glancing (*bi-al-badīha/per aspectum*) by being clearer and more detailed. But not just clearer and more detailed: the verified form is now seen to match, so to speak, a larger number of stored forms or images all labelled "my neighbour's cat".[36]

For Ibn al-Haytham, visual error may occur, not only with regard to the inferential properties, but also, in certain circumstances, with regard to what he calls "perception by pure sensation".[37] As we have seen, there are only two objects of "pure sensation", namely, light "as such" and colour "as such". But what Ibn al-Haytham seems to mean by "perception by pure sensation" is perception of either of these qualities *as a quality of some object*. It is identical neither with the impression produced in the eye nor with awareness of such an impression. (The latter is what Ibn al-Haytham would call, simply, sensation/*iḥsās/sensus*.) Nor is perception by pure sensation the same as recognition of light as illumination of a certain degree of intensity, or of colour as a colour of a certain quality or hue. Such recognitions constitute another mode of perception, called "perception by recognition"/*idrāk bi-al-ma'rifa/comprehensio per cognitionem*, and consists, in this example, in fitting the perceived quality under a certain concept or "universal form" (*ṣūra kulliyya/forma universalis*), an operation which does not come into play in perception by pure sensation. Consider a multi-coloured object in which all

[36] *Optics*, II, 4.

[37] *Optics*, III, 4 [1–5], 5 [1–15].

the colours are strong and similar in quality or hue: it will, in certain circumstances, give the impression of uniform darkness-colour, according to Ibn al-Haytham, being a sort of darkness.[38] Similarly, a multicoloured object in which all the colours are weak and similar in quality or hue will give the impression of uniform shadow. In these cases, sight wrongly takes the object to be of a single, uniform colour, namely, uniform darkness or uniform shadow, and this is an error in "perception by pure sensation." Again, if the multi-coloured parts of an object are too small to be perceived separately, sight will fail to distinguish them and, consequently, it will attribute a single colour to the object as a whole. In all these cases (they comprise the principal cases of error in perception by pure sensation)[39] the error consists in attributing to the object a confused, unrepresentative impression or form. At the root of this error is the inability of the sense faculty to register accurately representative forms in certain circumstances. This is expressible in our terminology by saying that in these situations M_i and M_s are not coincident. And the reason for this lack of coincidence must be due to a certain limitation of the sense-faculty's power of discrimination.[40]

Error, then, occurs in all three modes of perception – by pure sensation, by recognition and by inference. Ibn al-Haytham enumerates eight conditions without the combination of which vision cannot be achieved: that the object is at a certain distance and position from the eye, that it is opaque, of a certain magnitude and endowed with a certain degree of luminosity, the transparency of the intervening medium, soundness or health of the organ of sight, and, finally, suitable duration of the visual perception. He observes further that each of these conditions has a certain latitude (*ʿarḍ/latitudo*) or range (called "the moderate range")[41] which varies with each of the other conditions, and outside which a particular property cannot be perceived as it is. It is the aim of Book III to explain how errors of sight occur in regard to each of the visible properties when any one of the eight conditions falls outside the moderate range. The Book accordingly consists of a series of investigations in each of which it is shown how the perception of a particular property, say size, depends upon a given condition, say distance, while other conditions are kept constant. At the end of the Book, Ibn al-Haytham outlines a procedure in which several conditions are simultaneously varied. This is obviously a psychologist's agenda, the implied assumption of which is that veridical vision is normal vision, and that false or erroneous vision is abnormal vision – an assumption which, it has been said, also underlies the Aristotelian approach to sense-perception.[42]

We need not, therefore, go into epistemological jitters when we read Ibn al-Haytham's definition of moderate range as that within which there is no "sensible discrepancy" between the perceived form (F_j) and the true (or actual) form (F_a) of the object (III, 3 [15]); or his conclusion that a visible object is seen *as it is*, only if each and every one of the conditions for veridical vision falls within the moderate range (III, 3[33]). For even if the visible properties that make up an objective form F_a can be grouped under a finite number of categories, their modalities (like intensities of illumination, shades of colour, visible texture, etc.) will remain indefinite in number, and so will the number of perceptual judgements that make up the constructed form F_j. In these circumstances, I do not believe that the concept of "seeing an object as it is" is intended to have epistemological value; the occurrence of "*sensible* discrepancy" in the definition of moderate range clearly betrays its psychological status.

[38] *Optics*, II, 3 [54], III, 5 [1].
[39] *Optics*, III, 4 [1–5], 5 [1–15].
[40] *Optics*, II, 4 [35].
[41] *Optics*, III, 3 [6–34].

[42] Cf. Irving Block, "Truth and falsity in Aristotle's theory of sense perception", *The Philosophical Quarterly*, 11 (1961), pp. 1–9.

XII

THĀBIT IBN QURRA ON EUCLID'S PARALLELS POSTULATE

Problems connected with the concept of parallel lines were already seriously investigated in the time of Aristotle,[1] but after the composition of Euclid's *Elements* these investigations took a definite and, perhaps, new direction: their aim was to examine the epistemological status of the statement which, as Postulate 5, Euclid had included among the undemonstrated propositions of his system.[2] The postulate asserted that if a straight line falling on two straight lines makes the interior angles on one side together less than two right angles, the two lines, if produced indefinitely, meet on that side. Those after Euclid who did not wish to reject this proposition but who, nevertheless, thought it lacking in self-evidence, set out to establish its truth by more obvious means. Only rarely, it appears, was the truth of the postulate denied outright,[3] but again and again mathematicians had the feeling that it did not qualify for the special position given to it as a premiss. The remarkable thing is, however, that attempts to prove the Euclidean postulate should persistently go on, first in late antiquity, then in the Islamic world down to the thirteenth century. Every one, at least from the ninth century onwards, *assumed* the postulate to be true, but there were always some who could not be satisfied with the reasons hitherto proposed to support it. Not many of the proofs conceived in antiquity have come down to us. Proclus, in his *Commentarii in primum Euclidis Elementorum librum,* cites a demonstration by Ptolemy and, himself not being content with it, proposes a demonstration of his own.[4] Neither of these two proofs seems to have been transmitted to the Arabs and there is no evidence that Proclus's commentary was itself ever translated into Arabic. But the Arabs had access to Greek mathematical works which either have not survived in the original language or have not survived at all. For example, they had in

* This article is dedicated to my friend Alexandre Chryssoverghi.

[1] Cf. Thomas L. Heath, 'On an allusion in Aristotle to a construction for parallels,' in *Abhandlungen zur Geschichte der Mathematik,* ix, 1899, pp. 153–60; *idem, Mathematics in Aristotle,* Oxford 1949, pp. 27–30, 41–4; *idem, The Thirteen Books of Euclid's Elements,* translated from the text of Heiberg with Introduction and Commentary, 2nd ed., Cambridge, 1956, vol. i, Introduction, pp. 190–2; Imre Tóth, 'Das Parallelen-problem im Corpus Aristotelicum,' in *Archive for History of Exact Sciences,* iii, 1966–7, pp. 249–422 (with bibliography appended).

[2] I say 'epistemological', rather than 'logical', to emphasize the fact that terms like truth, evidence and belief were as much basic to the discussions about parallels as were the purely formal-logical considerations.

[3] Proclus has reproduced the arguments of some (un-named) thinkers who sought to show that the postulate was impossible; see *Procli Diadochi in primum Euclidis Elementorum librum commentarii,* ex recognitione Godofredi Friedlein, Leipzig 1873, pp. 368–71 = Proclus de Lycie, *Les Commentaires sur le premier livre des Éléments d'Euclide,* traduits pour la première fois du grec en français avec une introduction et des notes par Paul Ver Eecke, Paris 1948, pp. 315–17. See also Heath, *The Thirteen Books of Euclid's Elements,* i, pp. 206–7.

[4] Proclus, *In primum Euclidis Elementorum,* pp. 365–8, 371–3 = *Commentaires,* tr. Ver Eecke, pp. 312–15, 317–19. See Roberto Bonola, *Non-Euclidean Geometry, a critical and historical study of its developments,* authorized English translation with additional appendices by H. S. Carslaw, with an introduction by Federigo Enriques, etc., New York, 1955, pp. 2–7; Heath, *The Thirteen Books of Euclid's Elements,* i, pp. 204–8.

Arabic translation a commentary by Simplicius on the preliminary propositions of the *Elements,* i.e. the definitions, postulates and common notions (or axioms). Neither the Greek text nor the complete Arabic translation is extant, but extensive passages from this translation survive as quotations in the commentary to Euclid's book, composed in Arabic by the ninth-century mathematician, Abu 'l-'Abbās al-Faḍl ibn Ḥātim al-Nayrīzī.[5] One of the passages quotes in turn a proof of the Euclidean postulate by an unknown 'associate' (*ṣāḥib*) of Simplicius, a certain 'Aghānīs' (or 'Aghānyūs').[6] The Arabic sources also mention a book of Archimedes 'on parallel lines,' which is otherwise unknown.[7] And Thābit Ibn Qurra, the mathematician whose work concerns us here, is quoted by Ibn al-Nadīm as having ascribed to Apollonius 'a treatise on the fact that if two straight lines are drawn [from a transversal] at angles [together] less than two right angles, they will meet.'[8] Thus while it

[5] Of the Arabic text of Nayrīzī's commentary only Books i–vi and a short fragment of Book vii are extant in a unique manuscript at Leiden: *Codex Leidensis 399, I. Euclidis Elementa ex interpretatione al-Hadschdschadsch cum commentariis al-Narizii,* Arabice et Latine ediderunt notisque instruxerunt R. O. Besthorn et J. L. Heiberg, Copenhagen 1893–1932. [In three parts, each comprising two fascicules, pt. iii, fasc. ii (Bks. v–vi) being edited by G. Junge, J. Raeder and W. Thomson]. Gerard of Cremona's translation comprises Bks. i–x: *Anaritii in decem libros priores Elementorum Euclidis commentarii,* ex interpretatione Gherardi Cremonensis in codice cracoviensi 569 servata. Edidit Maximilianus Curtze, Leipzig 1899. *(Euclidis Opera omnia,* ediderunt I. L. Heiberg et H. Menge, *Supplementum).*

[6] *Euclidis Elementa . . . cum commentariis al-Narizii,* ed. Besthorn and Heiberg, pt. i, pp. 118–30; *Anaritii commentarii,* ed. Curtze, pp. 65–73. The identification of Aghānīs (Aganis) with Geminus, suggested by Besthorn and Heiberg, has been refuted by Paul Tannery: 'Le Philosophe Aganis est-il identique à Geminus?' *Bibliotheca mathematica,* 3. Folge, ii, 1901, pp. 9–11, reprinted in *Mémoires,* iii, Toulouse-Paris 1915, pp. 37–41. Further discussion in Heath, *The Thirteen Books of Euclid's Elements,* i, pp. 27–8. The form 'Aghānyūs' is found in an anonymous treatise on parallel lines, preserved in the Istanbul MS. Carullah 1502, fols. 26ᵛ–27ʳ, copied in 894/1488–9. The incipit is quoted by Max Krause, 'Stambuler Handschriften islamischer Mathematiker,' *Quellen und Studien zur Geschichte der Mathematik und Astronomie und Physik,* Abt. B (Studien), iii, 1936, p. 522. The spelling 'Aghānyūs', which does not occur in the extant text of Nayrīzī's commentary, makes plausible a suggestion by Tannery that the underlying Greek name might be Agapius. To obtain 'Agapius' from the Arabic 'Aghānyūs' one would simply have to read a dot under the line, thus forming a letter *b* (= *p*), rather than above the line (as is clear *in the MS.*), forming the letter *n*. But not much information would be gained as a result.—'Alam al-Dīn Qayṣar ibn Abi'l-Qāsim, in a letter to Naṣīr al-Dīn al-Ṭūsī, described a proof of Euclid's postulate by Simplicius which he found in the latter's 'Commentary to the premisses of the Book of Elements' (*fī sharḥihi li-muṣādarāt Kitāb al-Uṣūl)*—see Naṣīr al-Dīn al-Ṭūsī, *al-Risālad al-Shāfiya 'an al-shakk fī 'l-khuṭūṭ al-mutawāziya,* no. 8 in *Rasā'il al-Ṭūsī,* ii, Hyderabad 1359 H, p. 37. The proof is distinct from that of Aghānīs and is not reproduced by Nayrīzī; I plan to publish a number of Arabic documents relating to it. On the sense of the word *muṣādara* as used in Qayṣar's version of the title of Simplicius's work, see Abū 'Abd Allāh al-Khwārizmī, *Kitāb Mafātīḥ al-'ulūm,* ed. G. Van Vloten, Leiden 1895, p. 203. Nayrīzī's version of the title also uses *muṣādara*—see Besthorn and Heiberg's edition, pt. i, p. 40. The *Fihrist,* p. 389 (see following note) has *ṣadr,* i.e. beginning, preface, premisses.

[7] Ibn al-Nadīm, *al-Fihrist,* Cairo n.d., p. 386—see also Ibn al-Qifṭī, *Ta'rīkh al-ḥukamā',* ed. J. Lippert, Leipzig 1903, p. 67.

[8] *Fihrist,* p. 387: 'Thābit ibn Qurra said that [Apollonius] has *(lahu)* a treatise on the fact that, etc.' Though Thābit is not reported here to have explicitly said that he *saw* Apollonius's treatise, one would presume that this was implied. Cf. ibn al-Qifṭī, *Ta'rīkh,* p. 62.

is certain that the Arabs inherited the problem of parallels from their Greek predecessors, it would not be easy to determine exactly the extent to which they were influenced by Greek models. But the tenacity with which they pursued the problem for more than four hundred years, and the impressive variety of the solutions they offered, clearly testify to a great deal of independent thinking.[9]

As far as the problem of parallels is concerned, the twelfth-century Latin translation (by Gerard of Cremona) of Nayrīzī's commentary does not appear to have aroused a great deal of interest. For the renewal of serious and sustained attention to this problem in the West we have to wait until after the publication in 1560 of Barozzi's Latin translation of Proclus's commentary.[10] In the seventeenth century the English mathematician, John Wallis, was responsible for making known a proof of the Euclidean postulate by the thirteenth-century Persian mathematician and astronomer, Naṣīr al-Dīn al-Ṭūsī. This came about in the following way. Ṭūsī prepared in Arabic two revised editions of Euclid's *Elements*, known as *Taḥrīr Uṣūl Uqlīdis*, which we may refer to as the shorter and the longer version respectively. Each contained a proof of Euclid's Postulate 5 which is different from that in the other. The text of the shorter version was published in Rome in 1594.[11] At the request of Wallis, Edward Pocock made a Latin translation of the proof presented in

[9] For a general account of Arabic discussions of the theory of parallels see A. P. Juschkewitsch, *Geschichte der Mathematik im Mittelalter*, Basel 1964 (originally published in Russian, Moscow 1961), pp. 277–88; B. A. Rosenfeld, A. P. Yushkevich, *The Prehistory of Non-Euclidean Geometry in the Medieval East*, Moscow 1960, (xxv International Congress of Orientalists. Papers Presented by the USSR Delegation). The bibliographies appended to these two works give particulars of editions, Russian translations and studies of a number of Arabic treatises on parallels, including those of Thābit, by B. A. Rosenfeld, A. P. Yushkevich and G. D. Mamedbeili. The ascription of a proof of the Euclidean postulate to Shams al-Dīn al-Samarqandī is a misunderstanding —Hamid Dilgan, 'Démonstration du v[e] postulat d'Euclide par Schams-ed-Din Samarkandi. Traduction de l'ouvrage Aschkal-ut-tessis de Samarkandi,' *Revue d'histoire des sciences*, xiii, 1960, pp. 191–96. In fact Samarqandī (*ca.* 1276) believed that the postulate was not in need of any proof whatsoever. The demonstration reproduced by Qāḍī Zāda al-Rūmī (d. 1412–13) in his commentary to Samarqandī's *Ashkāl al-ta'sīs* is by Athīr al-Dīn al-Abharī (d. 1265) who wrote an Emendation *(Iṣlāḥ)* of the *Elements*. I have consulted MS. British Museum, Or. 10 (see particularly fols. 33^{v}–34^{r}) and MS. Princeton University Library, Yah. 373 (particularly 383^{r-v}).

[10] *Procli Diadochi Lycii philosophi platonici ac mathematici probatissimi in primum Euclidis Elementorum librum, commentariorum ad universam mathematicam disciplinam principium eruditionis tradentium libri IIII. A Francisco Barocio patritio veneto summa opera, cura, ac diligentia cunctis mendis expurgati: scholiis et figuris quae in graeco codice omnes desiderabantur aucti: primum jam Romae linguae venustate donati, et nunc recens editi.* Cum privilegio. Patavii, exudebat Gratiosus Perchacinus, 1560. An edition of the Greek text had appeared (Basileae, apud Joan. Hervagium) in 1533. In 1539 Bartholomeo Zamberti completed a Latin translation of this text but it was never published (Ver Eecke, *Commentaire,* pp. xx–xxiv). The English translation of Thomas Taylor, *The Philosophical Commentaries of Proclus . . . on the First Book of Euclid's Elements,* 2 vols., London 1788–9 (re-issued in 1792), was largely based on Barozzi's Latin version.

[11] *Kitāb Taḥrīr Uṣūl Uqlīdis . . . Euclidis elementorum geometricorum libri tredecim, ex traditione doctissimi Nasiridini Tusini, nunc primum arabice impressi.* Romae, in typographia Medicea, 1594. What we call 'shorter' Heath, following H. Suter *(Die Mathematiker und Astronomen der Araber und ihre Werke,* Leipzig, 1900, p. 151), calls 'larger' and our 'longer' is his 'smaller' *(The Thirteen Books of Euclid's Elements,* i, pp. 77–8). According to the nomenclature adopted

this version, which later appeared in the second volume of Wallis's mathematical works.[12] It was from this volume that Gerolamo Saccheri derived his knowledge of Ṭūsī's demonstration, which he discusses in his *Euclides ab omni naevo vindicatus* (1733).

This article presents, in English translation, two of the earliest demonstrations of Euclid's Postulate 5 to be composed in Arabic. They are both attributed to the distinguished mathematician and translator of Greek mathematical works, Thābit ibn Qurra of Ḥarrān, who flourished in Baghdād and who died in 901. Other mathematicians of the ninth century have been credited with proofs of the postulate: al-'Abbās ibn Sa'īd al-Jawharī, active during the reign of al-Ma'mūn (813–33); Banū Mūsā (i.e. the three sons of Mūsā ibn Shākir, Muḥammad, Aḥmad and al-Ḥasan), flourished before 873; al-Kindī, died *ca.* 870; Muḥammad ibn 'Īsā al-Māhānī, flourished *ca.* 860;[13] al-Nayrīzī, flourished under al-Mu'taḍid (892–902).[14] Only two of these

here, 'shorter' and 'longer' simply refer to the number of books in each of the two versions: the shorter version comprises the thirteen books of the *Elements;* the longer version has, in addition, the so-called Bks. xiv–xv, ascribed to Hypsicles. Both versions were based on two earlier Arabic translations, the first by al-Ḥajjāj ibn Yūsuf ibn Maṭar, the second by Isḥāq ibn Ḥunayn and revised by Thābit ibn Qurra. Only two MSS. of the shorter version are extant, both at the Biblioteca Medicea-Laurenziana: Or. 20 and Or. 50 (not Pal. 272 and 313). The former, in 273 fols., ends with Prop. 96 in Bk. x (not Bk. vi) and is not dated. The latter, transcribed at Āmud in 969/1561 from a copy 'in the author's hand' (fol. 208^{r}), is complete, i.e. in 13 Books; it contains 208 fols. The author's conclusion gives 698/1298 as the date in which the work was completed: '. . . let me conclude the discourse by invoking the exalted God . . . and that coincided with the morning of Saturday the tenth of Muḥarram in the year eight and ninety and six hundred *(thamān wa-tis'īn wa-sittimāya),* and here ends the discourse of the author *(al-muṣannif),* may God illuminate his tomb . . .' (fol. 208^{r}). The date is impossible, since Ṭūsī died in Dhu 'l-Ḥijja, 672/1274. It could be made possible by assuming that *wa-tis'īn* (and ninety) was a mis-transcription of *wa-sittīn* (and sixty), which would conveniently push the date of composition back to 668/1269, but we lack further evidence. No such mystery surrounds the longer version (in 15 books) which is represented by numerous MSS., many of which are listed in C. Brockelmann, *Geschichte der arabischen Literatur,* i², Leiden 1943, p. 673, no. 23, *Suppl.* i, Leiden 1937, p. 29. Two 13th-century MSS., both executed in the author's lifetime, state the date of completion of the work as 22 Sha'bān 646/1248: MS. British Museum, Add. 23387, copied in 656/1258, fol. 216^{v}, and MS. Damat Ibrahim Paşa 852, copied in 660/1261–2, fol. 114^{r}. There are a number of printings of this version, none of which is satisfactory (I have not seen the last one in the list): Istanbul 1801, Calcutta 1824, Lucknow 1873–4, Delhi 1873–4, Tehran 1881. An edition of the proof of Postulate 5 as presented in Ṭūsī's longer version together with a facsimile of the proof in the Rome edition of the shorter version was published by A. I. Sabra, 'Burhān Naṣīr al-Dīn al-Ṭūsī 'alā muṣādarat Uqlīdis al-khāmisa,' *Bulletin of the Faculty of Arts of the University of Alexandria,* xiii, 1959, pp. 133–70. G. D. Mamedbeili published a Russian translation of both proofs: *Mukhammed Nasireddin Tusi o teorii parallelnuikh liny i teorii otnosheny,* Baku 1959, pp. 12ff. See also B. A. Rosenfeld in *Istoriko-matematicheskie Issledovaniya,* iv, 1951, pp. 492–500.

[12] *Ioannis Wallis Opera mathematica,* ii, Oxford 1693, pp. 669–673. The Bodleian Library copy of this volume, Savile Gg. 2, has six pages of notes in Wallis's hand, inserted between pp. 672 and 673—information kindly communicated by Dr. Christoph Scriba of the University of Hamburg.

[13] That Banū Mūsā, al-Kindī and al-Māhānī composed such proofs, or at least that they discussed the problem of parallels, can be gathered from the opening sentence of the Anonymous Treatise on the parallels mentioned in note 6 above.

[14] See note 17 below.

proofs, namely those of al-Jawharī and al-Nayrīzī, are now known to be extant. Jawharī's proof is quoted from the author's *Emendation of the Elements* (*Iṣlāḥ al-Uṣūl*, now lost) and discussed by Naṣīr al-Dīn al-Ṭūsī in his *al-Risāla al-Shāfiya ʿan al-shakk fi ʾl-khuṭūṭ al-mutawāziya*. Just for the sake of comparison it may be helpful to give here a translation of Jawharī's alternative to Postulate 5 and of the six propositions proved by him on the basis of this alternative:[15]

> *Principle (mabdaʾ)*: If from the longer of two unequal lines, a half is cut off, and from the [remaining] half another half is cut off, and so on many times *(mirāran kathīra)*, and to the shorter line an equal line is added, and to the sum a line equal to it is added, and so on many times, there must remain of the halves of the longer line a line shorter than the multiples *(aḍʿāf)* of the shorter line.[16]
>
> *Propositions:* (1) If a straight line falling on two straight lines makes the alternate angles equal to one another, then the two lines are parallel to one another; and if parallel to one another, the distance of every point on one from the corresponding *(naẓīra)* point on the other is always the same.
>
> (2) If each of two sides of any triangle is cut into two halves and a line is drawn between the dividing points, then the remaining side of the triangle will be twice as equal as this line.
>
> (3) For every angle it is possible to draw any number of bases (sing. *qāʿida*).
>
> (4) If a line divides any angle into two parts *(bi-qismayn)* and a base to this angle is drawn at random, thereby generating a triangle, and from each of the remainders of the sides containing the angle a line is cut off equal to the side [continuing with it] of the generated triangle, and a line is drawn between the dividing points, then this line will cut off from the line dividing the given angle a line equal to that which is drawn from the [vertex of] the angle to the base of the generated triangle.
>
> (5) If any angle is divided by a line into two parts, and a point is marked on that line at random, then a line may be drawn from that point on both sides [of the dividing line] so as to form a base to that given angle.
>
> (6) If from one line and on one side of it two lines are drawn at angles together less than two right angles, the two lines meet on that side.

The proof of al-Nayrīzī[17] directly derives from that of Aghānīs. Adopting Aghānīs's definition of parallel lines as equidistant lines, Nayrīzī argues as follows: Equality is by nature prior to, and hence more apt to exist than,

[15] al-Ṭūsī, *al-Risāla al-Shāfiya, loc. cit.* (note 6), pp. 17–24; Russian translation of Ṭūsī's *Treatise* by B. A. Rosenfeld, with introduction and notes by B. A. Rosenfeld and A. P. Yushkevich, in *Istoriko-matematicheskie Issledovaniya*, xiii, 1960, pp. 475–532.

[16] This is Jawharī's version of the Eudoxus-Archimedes axiom, utilized by Euclid in x.1 and xii.2—see Heath, *The Thirteen Books of Euclid's Elements*, i, p. 234, iii, pp. 15–16; *idem, The Works of Archimedes*, Cambridge 1897 (Dover reprint, n.d.), p. xlviii. The axiom, in one form or another, became a feature of many demonstrations of the parallels postulate.

[17] I have consulted one of two known copies of his *Treatise on the Demonstration of the well-known Postulate of Euclid*, MS. Bibliothèque Nationale arabe 2467, fols. 89r–90r, where the author's name is misspelt 'al-Tabrīzī': *Risāla li-Faḍl ibn Ḥātim al-Tabrīzī fī Bayān al-muṣādara al-mashhūra li-Uqlīdis*. The MS. is from the 16th century; see Bibliothèque Nationale, *Catalogue des manuscrits arabes*, par Le Baron de Slane, Paris, 1883–95, pp. 436–7.

inequality. Consequently, straight lines which always maintain the same distance between them are more apt to exist than those which do not. Therefore, there must exist straight lines which are everywhere equidistant. His proof then consists of four propositions of which 1–3 correspond to the first three propositions of Aghānīs, while 4 is identical with Euclid's Postulate 5.[18]

That Thābit ibn Qurra composed two treatises, not one, on parallel lines is explicitly stated in the catalogue of his works reproduced by Ibn al-Qifṭī from 'papers in the handwriting of Abū ʿAlī al-Muḥassin ibn Ibrāhīm ibn Hilāl al-Ṣābiʾ.'[19] (Ibn al-Nadīm does not mention Thābit's treatises, but his list is much too short anyway.[20]) Another witness from the thirteenth century is ʿAlam al-Dīn Qayṣar ibn Abi ʾl-Qāsim who, corresponding with Naṣīr al-Dīn al-Ṭūsī from Syria, wrote that among the treatises on the problem of parallels, which he had come across in that part of the world, were *two* treatises by Thābit.[21] All extant copies of these two works (four in all) bear Thābit's name. Neither of the treatises refers to the other, and I have not been able to determine which of them was composed first, though I should like to think that what I call here 'Proof I' was written before 'Proof II'. Proof I is preserved in two manuscripts, one at Paris and the other at Istanbul: MS. Bibliothèque Nationale arabe 2457, fols. 156v–160r, copied at Shīrāz in 359/970;[22] and MS. Carullah 1502, fols. 13r–14v, 894/1488–89.[23] I have consulted both of these, but my translation follows the clearly superior text of the Bibliothèque Nationale MS. The Carullah MS. lacks the introduction preceding the statement of Thābit's postulate (see below) and it often abbreviates and even omits whole expressions. Proof II exists in MS. National Library, Cairo, no. 40 Riyāḍa Mīm, fols. 200v–202r, copied in 1159/1746; and in MS. Aya Sofya 4832, fols. 51r–52r. The latter is not dated but, according to Ritter, belongs to the eleventh century.[24] The Cairo MS. makes up for its modernity by being clearly written in a beautiful *naskhī* hand.

[18] See *Euclidis Elementa . . . cum commentariis al-Narizii*, ed. Besthorn and Heiberg, pt. i, pp. 120–4. Gerard of Cremona renders *mutawāziya* (parallel) as *equidistantes*. This makes nonsense of the distinction between Euclid's definition of parallel lines as non-secant lines and their definition as equidistant. Thus Gerard translates Euclid's and Aghānīs's definitions respectively as follows: 'Dixit Euclides: Linee recte equidistantes sunt, que cum sint in una superficie, si utique etiam in infinitum protrahantur, non concurrunt in aliqua duarum partium' *(Anaritii Commentarii*, ed. Curtze, p. 25). 'Philosophus tamen Aganis diffinivit lineas equidistantes dicens: Linee equidistantes sunt, que cum sint in una superficie, si utique in infinitum protrahantur, erit semper spatium, quod est inter eas unum' (*ibid.*, p. 26). Aghānīs's conception goes back to Posidonius and was shared by many writers in antiquity (Heath, *The Thirteen Books of Euclid's Elements*, i, pp. 190ff); it had a strong influence on Arabic mathematicians though some of them, like Ṭūsī, abandoned it.

[19] *Taʾrikh*, p. 116. Brockelmann *(Gesch. arab. Lit.*, Suppl. i, p. 385, no. 15) lists the two treatises as one work; and so does Krause, 'Stambuler Handschriften,' *loc. cit.* (note 6), p. 454, no. 4.

[20] *Fihrist*, p. 386.

[21] See Naṣīr al-Dīn al-Ṭūsī, *al-Risāla al-Shāfiya, loc. cit.* (note 6, see also note 15), p. 38.

[22] For a description of this very precious MS., almost entirely written by the well-known mathematician ʿAbd al-Jalīl al-Sijzī, see de Slane, *Catalogue des manuscrits arabes* [*dans la Bibl. Nat.*], pp. 432–3, 434.

[23] See note 19 above.

[24] Hellmut Ritter (mit Beiträgen von Martin Plessner), 'Schriften Jaʿqūb ibn Isḥāq al-Kindī's in Stambuler Bibliotheken,' *Archiv Orientální*, iv, 1932, p.363. The MS. came into the possession of a certain Ibn al-Ḥammāmī in Rajab, 568/1173.

One particularly interesting feature of Proof I should be noted. This is Thābit's argument, aimed to support his own alternative to Euclid's postulate, that geometry must be based on the concept of motion. Given the Aristotelian doctrine that motion is a property peculiar to physical bodies, Thābit's view would amount to the proposal that geometry should be founded on physics. But he does not actually say this; and the motion he talks about is something 'imagined'. His argument may be summarized as follows. Geometry is mainly concerned with the equality and difference of certain magnitudes—lines, angles, triangles, etc. The means for assessing such equality or difference between two magnitudes consists in our imagining one of the magnitudes to be moved, while maintaining its shape, and be applied to the other; that is to say, it consists in the operation of displacement without deformation. He remarks that many of Euclid's demonstrations are based on the use of this operation, and as examples he mentions Euclid I.4 and I.8. According to Thābit, these propositions were two of the oldest elements, and knowledge of them had been presupposed in proving other propositions. He does not go so far as to say that Euclid would have done better to assume I.4 as an axiom. But he asserts that this proposition, being independent of the three theorems preceding it, should be considered 'a beginning (or principle, *mabda'*) and a first' in relation to subsequent theorems in Euclid's book.

He observes further that the demonstration of Euclid I.1 ('On a given finite straight line to construct an equilateral triangle') depends on the equality of the radii of each of the two intersecting circles introduced by construction in the demonstration. This equality is in his view established by what we take for granted in the construction of a circle—namely that a straight line of a fixed length completes one revolution while one end of it remains in contact with a given point. The circle's radii are equal because one and the same rigid straight line has been successively applied to all of them.[25] In Proof I Thābit wants to make use of a similar idea, but for his purpose he wants the straight line to move in such a way that one end remains in contact with another, given, straight line, while the other end keeps the same distance from the given line. To control the motion of the line he imagines it to be contained in a moving solid. The rigidity of the solid thus ensures that the line is fixed both in respect of direction and magnitude.

[25] Compare the views which Proclus ascribes to Carpos, the writer on mechanics (ὁ μηχανικός), regarding the priority of problems (in which certain constructions are required to be made) over theorems in geometry—*Procli Diadochi in primum Euclidis Elementorum librum Commentarii*, pp. 241–4 = *Commentaires*, trans. Ver Eecke, pp. 209–11. On Carpos see P. Tannery, 'Sur Carpos d'Antioche', *Revue de Philologie*, xxii, 1898, pp. 93–7, repr. in *Mémoires*, ii, Toulouse-Paris 1912, pp. 249–554. By championing the thesis that the science of geometry depends on the idea of motion Thābit initiated an interesting discussion among Arabic mathematicians which lasted for a long time. Ibn al-Haytham in his *Commentary on the Premisses of Euclid* (*Sharḥ muṣādarāt Uqlīdis*, MS. Feyzullah 1359, fols. 150r–237r, partial Russian translation of this work by B. A. Rosenfeld in *Istoriko-matematicheskie Issledovaniya*, xi, 1958, pp. 743–62) proposed an alternative to Euclid's Postulate 5 which, like Thābit's postulate in Proof I, rests on the idea of displacement without deformation. On the other hand, 'Umar al-Khayyāmī, arguing in Aristotelian terms, wanted to banish motion from geometry altogether—see his *Risāla fī Sharḥ mā ashkala min muṣādarāt Kitāb Uqlīdis*, ed. A. I. Sabra, Alexandria, Munsha'at al-Ma'ārif, 1961, pp. 6–8. An edition of this work, based on one of the two extant MSS., had been published by T. Erani, Tehran 1936. This was translated by B. A. Rosenfeld, with commentaries by B. A. Rosenfeld and A. P. Yushkevich, in *Istoriko-matematicheskie Issledovaniya*, vi, 1953, pp. 67–107, 143–68; reprinted with a facsimile of the Leiden MS. used by Erani (Or. 199,8) in 'Omar Khayyām, *Traktatui*, Moscow 1961, pp. 113–46, 271–97. The English translation by Amir-Móez, *Scripta Mathematica*, xxiv, 1959, pp. 272–303, is to be used with caution.

Another interesting feature of Proof I is that in it we have the first occurrence of the so-called 'Saccheri's quadrilateral', the figure with reference to which the eighteenth-century mathematician formulated his three hypotheses—of the obtuse angle, of the acute angle and of the right angle respectively. Thābit does not enumerate the three hypotheses, but relying on his own postulate he proves the hypothesis of the right angle, from which he then derives Euclid's Postulate 5.

Proof II readily makes use of the application of figures but, in contrast to Proof I, without explicitly basing this operation on the idea of motion. In the course of the demonstration a new postulate is introduced: that straight lines in the same plane and converging together in one direction must diverge in the other direction, and *vice versa*. The postulate is stated as 'evident and admitted'. Though different from one another the two proofs are founded on the conception of parallel lines as equidistant lines.

TRANSLATION OF THĀBIT'S PROOFS OF EUCLID'S POSTULATE 5[26]

Proof I

THE TREATISE OF THĀBIT IBN QURRA ON (THE FACT) THAT IF TWO LINES ARE DRAWN AT LESS THAN TWO RIGHT ANGLES, THEY MEET[27]

Reasoning in the science of geometry is mostly concerned with magnitudes—their equality, difference, sizes, and related matters. The first element among the essential propositions,[28] which is taken for granted in this science, and which is a means for understanding all measurements and estimation of sizes[29] is [the following]: [a] the coincidence[30] of something with something else equal to it, when we imagine [the former] to be moved to [the latter]

[26] Both proofs have been translated into Russian—Proof I by B. A. Rosenfeld in *Istoriko-matematicheskie Issledovaniya*, xv, 1963, pp. 363–80; and Proof II by B. A. Rosenfeld with commentaries by B. A. Rosenfeld and A. P. Yushkevich, *ibid.*, xiv, 1961. Unfortunately, being ignorant of Russian I have not been able to make use of these translations.

[27] *Maqālat Thābit ibn Qurra fī anna 'l-khaṭṭayn idhā ukhrijā 'alā aqall min zāwiyatayn qā'imatayn iltaqayā*—the title in the Bibl. N. MS. arabe 2457, fol. 156v. MS. Carullah 1502, fol. 13r has: *Maqāla li-Thābit fī*, etc. (but omitting *Zāwiyatayn*)—*A Treatise by Thābit on*, etc. The highly abbreviated formulation, leaving it to be understood that the two lines are drawn from two points on a transversal such that the sum of the two interior angles is less than two right angles, is of Greek origin. It is to be found in the title of Ptolemy's work on the parallels as reported by Proclus: περὶ τοῦ τὰς ἀπ' ἐλαττόνων ἢ δύο ὀρθῶν ἐκβαλλομένας συμπίπτειν (*In primum Euclidis Elementorum*, p. 365). As pointed out by Heath, it has a precedent in Euclid i, 29: αἱ δὲ ἀπ' ἐλασσόνων ἢ δύο ὀρθῶν ἐκβαλλόμεναι εἰς ἄπειρον συμπίπτουσιν, *The Thirteen Books of Euclid's Elements*, i, p. 312.

[28] *awwal al-uṣūl min al-qaḍāyā al-ma'khūdha min dhāt al-shay'*

[29] *al-taqdīrāt wa 'l-masāyiḥ*

[30] *inṭibāq*

while maintaining its shape and be placed upon[31] [the latter] so as to measure it; [*b*] the excess of the greater over the smaller when we subject [one of them] to the same operation in our imagination; and [*c*] the smaller being less than the greater, and the number of times [the smaller] is applied to [the greater] when the application is repeated at successive places in it—this being the means for determining the size of a thing. [Therefore] the principles of many demonstrations of those demonstrable first elements among geometrical propositions consist in the use of the said operation—I mean moving one of the two things to be measured by one another and pushing it from its place and transferring it in our imagination without changing its shape by movement so as to place it with its shape [unchanged] upon that which is to be measured by it.

Euclid was obliged to do the same thing in the demonstration of Proposition 4 of Book I of his work on *Elements*, and in the demonstration of Proposition 8 of [the same Book]; for these are two of the oldest elements, knowledge and demonstration of which are premissed and taken as basis for other [propositions]. For though, in respect of numerical order, Proposition 4 is posterior,[32] it is, by being independent of the preceding propositions, a principle and a first in relation to other [propositions following it].

But if we look also at the first of the propositions preceding it, which is first with respect to the whole of that book, and ascertain what is involved in it,[33] we learn that the basis of its demonstration consists in the equality of the straight lines drawn from the centre of the constructed circle[s]—the truth of this [equality] being due to nothing other than what we have understood and established in ourselves regarding the construction and generation of a circle. This understanding we gain by imagining that one straight line having a [determinate] magnitude or something else that preserves its magnitude and dimension,[34] has moved by rotating from some position until it returned to its first place from which it started, while one end of it has remained in contact with one unmoved point. From this we derive the understanding and knowledge of what we said regarding the equality of the lines extending from the centre of the circle. For we have attained that knowledge only by continuously moving a straight line in our imagination in such a way as to apply it to all those distances and extensions.[35]

Now to move this straight line and rotate it about one centre while one end of it is in contact with one unmoved point is something evident that has been made familiar by custom. And thus it is understood that the distances passed through are being measured and estimated[36] by [the rotating line] which remains unchanged. But to imagine the movement of [the line] to be such that one of the extremities continues to touch a fixed straight line, other conditions being present besides—that would be something unfamiliar. Nor would it be safe [to assume] that the shape or position of the moving line would not change in such a way as to bring about some difference in the equality of the distances [covered by it]. Thus it is uncertain to assert their equality unless that [uncertainty] be guarded against by [introducing]

[31] *intabaqa*
[32] *tābi'an fi martabatihi min al-'adad*
[33] *wa-ḥaṣṣalna 'l-ḥāla fīhi*
[34] *al-miqdār wa 'l-bu'd*
[35] *al-masāfāt wa 'l-ab'ād*
[36] *yuqaddiruhā wa-yamsaḥuhā*

something which he who begins to look into it may find difficult or beyond his understanding.[37]

That being so, and since I need for my purpose a line of that description in order to show by its coincidence with the distances it covers that they are equal distances, it occurred to me to qualify [the line's] movement by a condition which would remove the doubt and lead to the certainty that no such change in its shape or character[38] would take place as to produce a difference in the covered distances whose equality I wished to establish, but by having coincided with them [this line] would establish their equality. The best way I could secure that [condition] was to imagine [the line] being placed in a solid[39] which moves while [the line] remains fixed in it, as I shall describe, so that the solid, by maintaining itself[40] while in motion, may also maintain that which we require [to be unchanged] in that line. That was more conducive to the clarification of what I require. Consequently, as a premiss, I have started from something known regarding the solid—which is this:

If any solid is imagined to move as a whole in one direction with one simple and straight movement, then every point in it will have a straight movement and will thus draw a straight line on which it will pass; as for the straight lines that are in [the solid], those lying in the direction of its movement will also pass along straight lines, but not those which are not in the direction of movement.

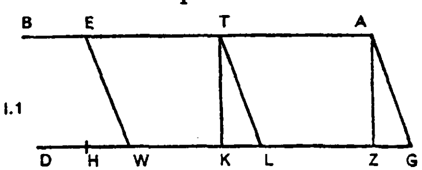
Having premissed this, that which follows from it will easily become evident in this way:

I.1

If two straight lines are in the same plane, and two straight lines equal to one another are drawn across them in such a way as to contain with one of the first two lines two equal angles on one side, then any two perpendiculars falling on that line from two points on the other will be equal.

Let the straight lines *AB*, *GD* be in the same plane; let the straight lines *AG*, *EW* be drawn across them and let them be equal; and let the angles *AGD*, *EWD* be equal.

I.1



I say that any two perpendiculars falling on the line *GD* from two points on the line *AB* will be equal.

Demonstration: We imagine that a solid surrounds the line *AG* so as to be cut by *GZ* which is part of *GD*. Then [*AG* and *GZ*] are within the solid.

We imagine further that the solid has moved as a whole from the side of *G* to that of *D*—this being a straight simple movement along *GD*.

Let analogues[41] to the lines *AG*, *GZ* be drawn in the solid and let them remain and preserve their shape in it. Thus the line that is drawn in the solid

[37] *falā yūthaq bi'l-ḥukm 'alayhā* (the distances) *min qibalihi* [referring to what?] *bi 'l-tasāwī illā bitaḥarruz min dhālik bi-mā yab'ud 'an fahm al-mubtadī bi'l-naẓar fīh wa-ya'sur 'alayh*
[38] *hay'atihi wa-ṣifatihi*
[39] *mujassam*
[40] *bi-annahu ḥāfiẓun li-nafsihi*
[41] sing. *mithāl*

as an analogue to AB [*sic*; read AG] will not be situated along the solid's movement; but the other line drawn in the solid as an analogue to GZ will be situated along that movement.

Therefore, the analogue drawn for the line GZ will throughout the solid's movement pass along GD and will always be placed on it.

If we imagine that the point G on the analogue drawn for the line AG has reached the point W as a result of the solid's movement, then the position of the analogue drawn in the solid for the line GZ will be that of WH, for it has moved along GD.

But the angle EWH is equal to the angle AGZ; therefore the analogue to the line AG in the solid will fall on WE when the point G on it comes to W.

And since AG is equal to WE, it coincides with it, the point A in AG falling on the point E in WE.

Therefore, the point A in the solid passes to the point E as a result of the movement of the solid, by its passage draws a straight line, for this is the case with every point in the solid. Therefore, the passage of the point A will be along the line AEB since there is no other straight line passing through the points A, E.

Let a point T be marked at random on AEB; and from it let TK be drawn perpendicular to GD.

Then, either the angle AGD is right or it is not. If it is right then when the point A on the analogue drawn in the solid for the line AG arrives at point T, the analogue will coincide with the perpendicular TK and be equal to it.

For suppose it did not coincide with (the perpendicular) but took (instead) a position such as that of the line TL. Then the angle TLK would be right because it is equal to the angle AGD in view of the fact that the analogues of AG, GZ within the solid have moved with their arrangement as it is and the fact that the analogue of GZ always keeps to its position along GD.

But the angle TKL also is right, for we have drawn TK perpendicular to GD.

Thus in the triangle KTL there would be two right angles—which is impossible since any two angles in a triangle must together be less than two right angles.

Line AG will therefore coincide with the perpendicular TK and will be equal to it. AG will likewise coincide with any perpendicular falling on the line GD from a point on the line AB.

Now suppose that the angle AGD is not right. Then from the point A we draw AZ perpendicular to GD. And we imagine that when the point A on the analogue drawn in the solid for the line AG has reached the point T, the analogue will take the position of the line TL, and the analogue of the line GZ will take the position of LK.

Then in the triangles GAZ, LTK, the sides AG, TL are equal since they coincide with one another; and the angles AGZ, AZG in the one are respectively equal to the angles TLK, TKL in the other.

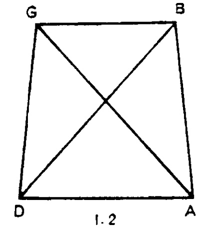
Therefore, the remaining sides and angles in these two triangles are respectively equal to one another.

Therefore, the perpendicular TK is equal to the perpendicular AZ, and also equal to any perpendicular falling on GD from a point on the line AB.

All these perpendiculars are therefore equal. And that is what we wanted to prove.

I.2

In any quadrilateral plane[42] if two angles on one side[43] are equal and the two sides joining that side are equal, then the remaining two angles are equal.



I.2

Let the quadrilateral plane be $ABGD$, let its two angles BAD, GDA be equal and likewise the sides AB, DG.

I say that the angles ABG, DGB are equal.

Demonstration: We draw the diagonals[44] AG, DB.

Then the line AB is equal to the line DG, and AD is common; therefore the two sides BA, AD in the triangle BAD are respectively equal to the two sides GD, DA in the triangle ADG.

But angle BAG [*sic*; read BAD] is equal to angle BGD [*sic*; read GDA]. Therefore the base BD is equal to the base AG.

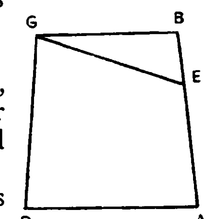
And AB was equal to DG. Therefore sides BA, AG in the triangle ABG are respectively equal to sides GD, DB in the triangle BGD.

And the base BG is common to both. Therefore the angle ABG is equal to the angle DGB.

And that is what we wanted to prove.

I.3

In any quadrilateral plane if two angles on one side are equal, and the two other angles are equal, then the two sides joining its first side are equal.



I.3

Let the quadrilateral plane be $ABGD$, let its two angles BAD, GDA be equal, and likewise the angles ABG, BGD.

I say that its sides AB, DG are equal.

Demonstration: If the side AB is not equal to the side DG, then one of them is longer than the other. Let the longer one be AB, from which we cut off AE equal to DG, and draw the line EG.

Then the plane $AEGD$ is a quadrilateral whose angles EAD, GDA are equal. Therefore its angles AEG, DGE are also equal.

But the angle AEG which is exterior to the triangle GEB is greater than the opposite and interior angle EBG.

Therefore, the angle DGE is greater than the angle ABG.

And the angle DGB is greater than the angle DGE. Therefore the angle DGB is much greater than the angle ABG. But they were [supposed] equal—this is impossible.

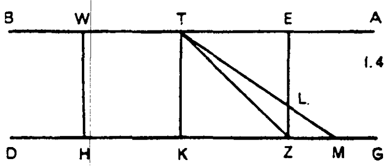
Thus neither of the sides AB, DG is greater than the other, and they are therefore equal.

And that is what we wanted to prove.

42 *saṭḥ dhū arbaʿat aḍlāʿ*
43 *ḍilʿ*
44 sing. *quṭr*

I.4

If any two straight lines are in the same plane, and if two perpendiculars drawn from two points on one of them to the other are equal, then they will also be perpendicular to the first line, and all perpendiculars falling from each one of the two lines upon the other—whatever the points they are drawn from—will be perpendicular to the companion[45] line and will be equal among themselves and equal to the first two perpendiculars.



Let the straight lines *AB*, *GD* be in the same plane, and from the points *E*, *W* on the line *AB* let *EZ*, *WH* be drawn perpendicular to *GD*, and let them be equal.

I say that they are also perpendicular to *AB*, and that every line drawn from a point on one of the two lines *AB*, *GD* perpendicular to the other will also be perpendicular to the line from which it has been drawn and equal to each of *EZ*, *WH*.

Demonstration: We mark a point *T* on the line *EW* at random, and from this point draw *TK* perpendicular to *GD*.

Then the perpendicular will meet *GD* without meeting either of the lines *EZ*, *WH*.

For if it met one of them, as does the line *TZ*, then the greater angle *EZD* would be equal to the smaller angle *TZD* (both being right)—which is impossible.

And if it met one of them cutting it in the point *L*, as the line *TLM* also meets the line *EZ*, then the angle *EZD* that is exterior to the triangle *MLZ* would be equal to the interior and opposite angle *LMZ*—which is impossible.

Therefore, the perpendicular *TK* meets the line *ZH* without meeting either of the perpendiculars *EZ*, *WH*.

Thus because the straight lines *AB*, *GD* are in the same plane and the straight lines *EZ*, *WH* have been drawn across them so as to meet them, and these latter lines are equal and they contain with *GD* two equal angles on the same side (being both perpendicular to *GD*), then any two perpendiculars falling on *GD* from a point on the line *AB* will be equal.

Consequently, the perpendicular *TK* will be equal to each of the perpendiculars *EZ*, *WH* since the angles *EZK*, *TKZ* in the quadrilateral plane *ETKZ* are equal, being both right.

And it has been shown that the two sides *EZ*, *TK* are equal; therefore, the angles *ZET*, *KTE* are equal.

Similarly, we prove the angles *KTW*, *HWT* in the quadrilateral *TWHK* to be equal, and the angles *ZEW*, *HZE* in the quadrilateral *EWHZ* to be equal.

Consequently, the angles *KTE*, *KTW* are equal, and therefore each of them is a right angle.

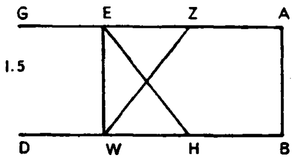
But we have shown that they are equal to the angles *ZET*, *HWT*; therefore these two angles are also right and the lines *ZE*, *HW* are perpendicular to *AB*.

[45] *a'mida 'alā ṣāḥibihi*

And so it is with the perpendicular *TK* and with every other perpendicular drawn from either of the lines *AB*, *GD* to the other, and all these perpendiculars will be equal to the perpendiculars *EZ*, *WH*. And that is what we wanted to prove.

I.5

If any two straight lines are drawn from the extremities of a straight line in the same plane so as to contain with it two right angles, then any perpendicular drawn from a point on one of the two lines to the other will also be perpendicular to the first, and will be equal to the line from whose extremities the two lines have been drawn.



From the extremities of the straight line *AB* let us draw the lines *AG*, *BD* in the same plane at right angles, and from the point *E* on one of them let us draw *EW* perpendicular to the other, viz. *BD*.

I say that *EW* is also perpendicular to *AG* and that it is equal to the line *AB*.

Demonstration: Either the line *AE* is equal to the line *BW* or it is longer or shorter. I say that it is equal to it, nothing else being possible.

If something else were possible, let the line first be longer.

From [*AE*] let *AZ* be cut off equal to [*BW*] and draw the line *WZ*. Then, from the points *W*, *Z* on the line *ZW*, two perpendiculars to *AB* have been drawn, viz. *ZA*, *WB* which are equal. They are therefore also perpendicular to *ZW*, and the angle *ZWB* is right.

But the angle *EWB* also was right—the greater being equal to the smaller, which is impossible. Therefore *AE* is not longer than *BW*.

Now, if possible, let it be shorter.

From *BW* cut off *BH* equal to [*AE*]. Then from the points *E*, *H* on the line *EH* two perpendiculars *EA*, *HB* have been drawn to *AB* which are equal.

They are therefore perpendicular to *EH*, and the angle *EHB* exterior to the triangle *WEH* is right and equal to the interior and opposite angle *EWH*—for this was right—which is impossible.

Therefore the line *AE* is not shorter than the line *BW*.

But we have shown that it was not longer; therefore they are equal.

It follows that in the quadrilateral *ABWE* the angles *EAB*, *WBA* are equal, and so are the sides *AE*, *BW*; therefore the angles *AEW*, *BWE* are equal; and since *BWE* is right, the angle *AEW* is also right; and *EW* is perpendicular to *AG*.

Thus in the quadrilateral plane *ABWE*, the two angles on the side *BW* are equal and the other two angles on *AE* are equal.

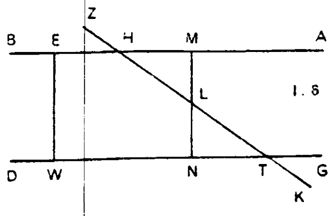
Therefore, the side *EW* is equal to the side *AB*, and so are all the perpendiculars drawn from the line *AG* and falling on the line *BD*.

And that is what we wanted to prove.

I.6

If a straight line falling on two straight lines in the same plane is perpendicular to both, then any straight line cutting the two lines will make the

alternate angles equal to one another and the exterior angle equal to the opposite interior angle.



Let *AB*, *GD* be the two straight lines in the same plane, and let the line *EW* fall on them so as to be perpendicular to both, and let the two lines be cut by the line *ZHTK*.

I say that the alternate angles *AHT*, *DTH* are equal and that the exterior angle *ZHB* is equal to the interior and opposite angle *DTH*.

Demonstration: Let us divide the line *HT* into two halves in *L*, and from point *L* draw *LM* perpendicular to *AB* and produce it rectilinearly to *N*. Then it will meet the line *TW*.

For if it did not, it would meet the line *EW*. But this is impossible since they [*MN* and *EW*] are both drawn at right angles to *ME*. Let it [*MN*] meet it [*TW*] in point *N*, and therefore be perpendicular to it as we have shown before.

Then the angles *LMH*, *MLH* in the triangle *MHL* will be respectively equal to the angles *LNT*, *TLN* in the triangle *NTL*.

And the side *LH* in the first triangle will be equal to the side *LT* in the second triangle.

Therefore, the remaining sides and angles are respectively equal—and the angle *MHL* will be equal to the angle *LTN*, these being the alternate angles.

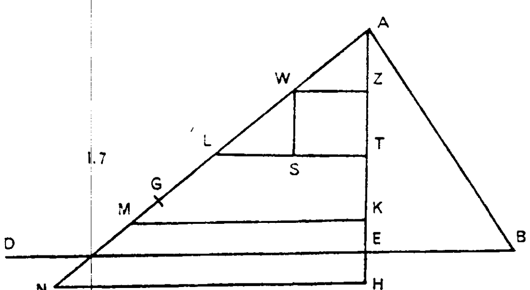
But the angle *MHL* is equal to the opposite angle *ZHB*.

Therefore, the exterior angle is equal to the opposite and interior angle.

And that is what we wanted to prove.

I.7

If two straight lines are drawn from the extremities of a straight line in the same plane at less than two right angles, then the two lines meet in that direction.



From the extremities of the straight line *AB* let us draw the two straight lines *AG*, *BD* in the same plane, and let the angles *BAG*, *ABD* be together less than two right angles.

I say that the lines *AG*, *BD* meet when produced in the direction of *G*, *D*.

Demonstration: One of the angles *BAG*, *ABD* must be less than a right angle.

Let it be the angle *ABD*.

Then from *A* let us draw *AE* perpendicular to *BD*, mark a point *W* on *AG* at random and from it draw *WZ* perpendicular to *AE*.

Then the lines *AZ*, *AE* are finite, and *AE* is longer than *AZ*, and therefore it is possible to multiply the smaller, viz. *AZ*, until its multiple becomes greater than *AE*.

Let *AH* be the multiple which is greater than *AE*.

From *ZH* let us cut off equals to the line *AZ*, viz. *ZT*, *TK*, *KH*.

And from the line *WG* let us cut off equals to the line *AW* as many times as *ZT*, *TK*, *KH*, viz. *WL*, *LM*, *MN*. If *WG* is less than is sufficient we prolong it until it is enough.

I say that the line *AZ* has cut the line *BD*.

Demonstration: From the point *T* let us draw *TS* perpendicular to *AE*, and from *W* we draw *WS* perpendicular to this perpendicular.

Then *WS* is also perpendicular to *WZ* and equal to *ZT*.

But *ZT* was equal to *AZ*.

Therefore, *WS* is equal to *AZ*.

And it is evident that the line *WL* will fall outside the (area) between *WS*, *ZT*—for the angle *ZWS* is right, and the angle *AWZ* is less than a right angle since the angle *AZW* is right and there cannot be two right angles in the same triangle.

Again, the line *WZ* has fallen on the two lines *AT*, *WS* in such a way as to be perpendicular to both, and the straight line *AG* has also fallen on them.

Therefore, the exterior angle *LWS* is equal to the interior and opposite angle *WAZ*.

Therefore these two angles in the triangles *AWZ*, *WLS* are equal.

But we have shown that their sides *AZ*, *WS* are also equal, and side *AW* in one of the two triangles is likewise equal to the side *WL* in the other.

Therefore, the two bases are equal, and the remaining angles are respectively equal to one another.

And the angle *WSL* being equal to the angle *AZW*, and the angle *AZW* being a right angle, then the angle *WSL* is right.

But the angle *WST* was also right. Therefore the lines *TS*, *SL* are joined in such a way as to make one straight line. Therefore the line that joins the points *T*, *L* is the line *TSL* itself, and it is perpendicular to *AH*.

And likewise we show that the straight line *KM* joining the points *K*, *M* is perpendicular to *AH*, and that *HN* is perpendicular to *AH*.

Therefore the angle *AHN* is a right angle.

But the angle *BEH* also was a right angle, being equal to the angle *AED*.

Therefore on the straight lines *HN*, *BD* a straight line *AEH* has fallen making the alternate angles equal.

The two lines are therefore parallel and will not meet even if produced indefinitely.

But one of them, *HN*, has been met by the line *AG* in *N*, and therefore *AG* has gone across to the other side of *BD*.

Therefore the line *AG* has met the line *BD*, cutting it and crossing it.

And that is what we wanted to prove.

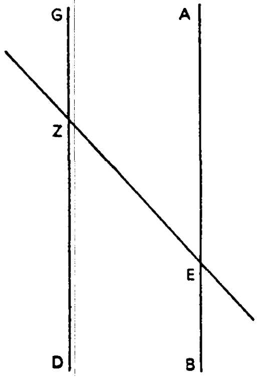
Ends the treatise of Thābit ibn Qurra on the fact that if two lines are drawn at less than two right angles they will meet in that direction—much praise be to God. I copied it in Shīrāz on Wednesday, two nights before the end of Rabī II, in the year 359 [A.D. 970]. I checked it against the original.

Proof II

THE TREATISE OF THĀBIT ON THE FACT THAT IF A STRAIGHT LINE FALLING ON TWO STRAIGHT LINES MAKES THE TWO ANGLES ON ONE SIDE LESS THAN TWO RIGHT ANGLES, THEN THE TWO LINES, IF PRODUCED ON THAT SIDE, MEET.[46]

II.1

If a straight line falls on two straight lines and the two alternate angles are equal to one another, then those two lines do not converge or diverge[47] on either side.



II.1

For example, the line EZ has fallen on the two lines AB, GD so that the angles AEZ, EZD are equal;

I say that AB, GD do not converge or diverge either on the side of A, G or on the side of B, D.

Demonstration: If we apply EA upon ZD by placing the point E upon Z, EZ upon itself, and the angle AEZ upon the angle EZD, GZ will coincide with EB, the angle GZE with the angle ZEB, and the line GZ will coincide with the line EB, always continuing with it, and the line ZD will also (coincide) with the line AE.

If this were not so, an angle would be greater than the(angle) equal to it; and that is impossible.

And from this it is seen that if the two lines EB, ZD converge, if produced, on the side of B, D, then the two lines AE, GZ will also similarly converge on the side of A, G—because of (their) coincidence.

But it is evident and admitted that, if a straight line falls on two straight lines so that the two lines converge on one side, then they diverge on the other side, and their convergence increases on the side of convergence and their divergence increases on the side of divergence.

And thus if we assume that the two lines EB, ZD converge on the side of B, D, it is necessary that the two lines AE, GZ should diverge on the side of A, G.

But the two lines AE, GZ have coincided with the two lines EB, ZD on the side of B, D; and if EB, ZD converge, AE, GZ will diverge and not coincide with them.

(But) if they coincide with them and do not diverge on the side of A, G, it remains that either the two lines AE, GZ converge on the side of A, G like the

[46] *Kitāb Thābit fī annahu idhā waqaʿa khaṭṭ mustaqīm ʿalā khaṭṭayn mustaqīmayn fa-ṣayyara ʾl-zāwiyatayn allatayn fī jiha wāḥida aqall min qāyimatayn, fa-inna ʾl-khaṭṭayn idhā ukhrijā fī tilka ʾl-jiha iltaqayā* (MS. Aya Sofya 4832, fol. 51ʳ). The title in the Cairo MS. National Library, no. 40 Riyāḍa Mīm, fol. 200ᵛ is: *Maqāla fī Burhān al-muṣādara al-mashhūra min Uqlīdis li-Thābit ibn Qurra al-Ḥarrānī, raḥimahumā Allāh taʿālā:* A Treatise on the Demonstration of the well-known postulate of Euclid by Thābit ibn Qurra of Ḥarrān, may the exalted God have mercy upon them. He [Thābit] said: etc.

[47] *la yaqrubān wa-lā yabʿudān*

assumed convergence of the two lines EB, ZD on the side of ZD,[48] or[49] they neither converge nor diverge on the side of A, G.

If they converge on (that side) the accepted premiss would be false, for there would exist two (straight) lines that converge on both sides.

And if they keep the [same] distance between them, then they do not coincide with EB, ZD;

but they have coincided with them;

therefore, the assumption that EB, ZD converge on the side of ZD, if the alternate angles AEZ, EZD are equal, is impossible.

And it is also impossible that they should diverge on that side.

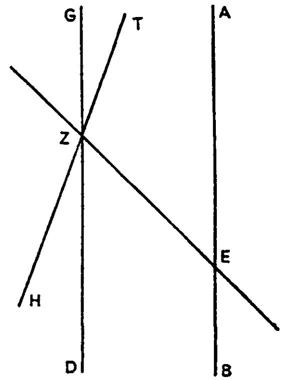
Therefore, they do not converge or diverge on that side.

And the same can be shown for the two lines AE, GZ.

And that is what we wanted to prove.

II.2

If a straight line falls on two straight lines which do not converge or diverge on either side, then the two alternate [angles] are equal to one another.



II.2

For example, the two lines AB, GD do not converge or diverge on either side, and EZ has fallen on them;

I say that the two alternate angles AEZ, EZD are equal.

Demonstration: If the angles are not equal, let AEZ be smaller, and let the angle EZT be equal to the angle AEZ;

and we produce TZH.

Then the two lines TZH, AB do not converge or diverge because of the equality of the two alternate angles, as we have shown;

and, by hypothesis, the two lines AB, GD were neither convergent nor divergent;

and GD has cut TH in the point Z;

and each of (the two lines) neither converges with nor diverges from AB;

but ZT is nearer to EB than is ZD, since it lies between the one and the other:

this is absurd;

therefore, the two angles AEZ, EZD are equal.

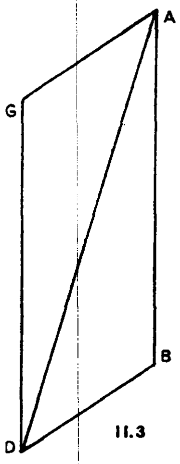
And that is what we wanted to prove.

II.3

If the extremities of two straight and equal lines which do not converge or diverge are joined by two straight lines, then these also are equal and do not converge or diverge.

[48] ZD in both MSS. B, D would make better sense.

[49] *or* omitted in the Cairo MS.



II.3

For example, the two lines *AB, GD* are straight and equal, and they do not converge or diverge, and their extremities have been joined by the two lines *AG, BD;*

I say that *AG, BD* are equal and they do not converge or diverge.

Demonstration: The two alternate angles *ADG, DAB* are equal;

therefore, the two lines *AB, AD* are equal to the two lines *GD, DA* each to the one corresponding to it.[50]

And the two angles *ADG, DAB* are equal, and they are alternate angles;

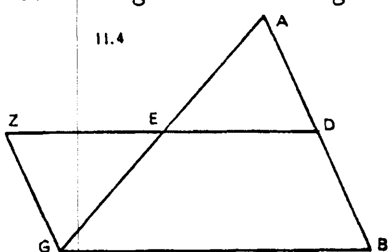
therefore, the two lines *AB, GD* do not converge or diverge.

And similarly the two lines *AG, DB* also do not converge or diverge, and they are equal.

And that is what we wanted to prove.

II.4

In every triangle (if) two of the sides are each divided into two halves, and the two points at which they have been divided are joined by a straight line, then (this straight line) will be half the other [i.e. third] side and it does not converge with or diverge from it.



II.4

For example, in the triangle *ABG, AB* has been divided into two halves in *D,* and *AG* into two halves in *E,* and the straight line *DE* has been joined;

I say that it is half of *BG* and it does not converge with or diverge from it.

Demonstration: We produce *DE* to *Z* so that *EZ* will be equal to *DE,* and we join *GZ.*

Thus the two triangles *ADE, GEZ* will be equal, and the two lines *AD, GZ* will be equal.

And similarly the two lines *DB, GZ* will also be equal.

But the two angles *ADE, EZG* are equal, and they are alternate angles;

therefore, the two lines *AB, GZ* do not converge or diverge.

And similarly the two lines *BD, GZ* also do not converge or diverge;

and they are equal;

and their extremities have been joined by the two lines *BG, DZ;*

therefore, they are equal and they do not converge or diverge.

But *DZ* is twice *DE;*

therefore, *BG* is twice *DE* and it does not converge with or diverge from it.

And that is what we wanted to prove.

And in this way we can show that if we divide each of *AB, AG* into any number of parts, provided their number is even,[51] and we join the [dividing]

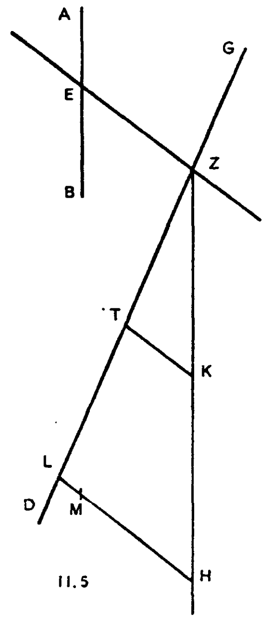
[50] *kullu wāḥid li-naẓīrihi*

[51] *bi-aqsām kam kānat ba'da an yakūna azwājā*

point next to *A* on the line *AB* with the corresponding point on *AG* by a straight line, then [the joining line] will be the same part of *BG* as the part of the line [*AB*] lying between *A* and the [dividing] point next to *A* on the line *AB.*

II.5

If a straight line falling on two straight lines makes the two angles on one side [together] less than two right angles, then the two lines, if produced on that side, meet.



II.5

For example, the line *EZ* has fallen on the two lines *AB, GD,* and the two angles *BEZ, DZE* are [together] less than two right angles;

I say that the two lines *AB, GD,* if produced on the side of *B, D,* meet.

Demonstration: From the point *Z* we draw the line *ZH* neither converging with nor diverging from the line *AB,* and we mark on *ZD* the point *T* at random, and from it we draw to *ZH* the line *TK* neither converging with nor diverging from *EZ.*

It (*TK*) may happen to be greater than *EZ.* If not, we cut off *TL* equal to *ZT* and *KH* equal to *ZK,* and we join *LH:* it is seen that *LH* will be twice *TK* and also that it will not converge with or diverge from *TK.*

Thus if *TK* is smaller than *EZ* and we double it, then double its double, and continue in this always,[52] it is necessary that we should reach at the end of this multiplication a line greater than *EZ*—let it be *LH.*

Thus the line *LH* is longer than *EZ* and it does not converge with or diverge from it.

Let us cut off from *LH* an equal to *EZ,* which is *HM.*

Then the two lines *ZE, HM* are equal and they do not converge or diverge.

Therefore, the two (lines) joining their extremities are equal, and they do not converge or diverge as [has been shown] before.

But *ZH* has joined *Z* and *H;*

therefore *EB,* if produced in a straight line on the side of *B,* will proceed to *M;*

or else, it would happen that if the (line) joining *E* and *M* were other than *EB* produced, the (line) joining *E* and *M* would neither converge with nor diverge from *ZH.*

But *EB* was neither convergent with nor divergent from *ZH;*

and the (line) joining *E* and *M* (lies) between *EB* and *ZH:* this is absurd;

therefore *EB,* if produced, will proceed to *M;*

thus it is necessary that it should meet a point on the line *GD* before it meets the point *M.*

Therefore *AB, GD,* if produced on the side of *G, D,* meet.

And that is what we wanted to prove.

[52] *wa-mararnā 'alā hādhā dā'iman*

The treatise is completed with the help of the exalted God, and with His guidance. Praise be to God for that, and blessing and peace upon His beloved Muḥammad and all his good family and companions.

The ink was dry on this copy on the eve of Saturday the twentieth of Dhu 'l-Qa'da of the year nine and fifty and a hundred after one thousand of the Hijra of the prophet, to him the best salutation.[53]

[53] 'The treatise is completed, etc.' as in the Cairo MS.

SIMPLICIUS'S PROOF OF EUCLID'S PARALLELS POSTULATE

A commentary by Simplicius on the premisses to Book I of Euclid's *Elements* survives in an Arabic translation of which the author and the exact date of execution are unknown. The translation is reproduced by the ninth-century mathematician al-Faḍl ibn Ḥātim al-Nayrīzī in the course of his own commentary on the *Elements*. Of Nayrīzī's commentary, which is based on the earlier translation of the *Elements* by al-Ḥajjāj ibn Yūsuf ibn Maṭar, we have only one manuscript copy at Leiden and Gerard of Cremona's Latin translation, both of which have been published.[1]

The passages quoted by Nayrīzī, owing to their extensiveness and consecutive order, would strongly lead one to assume that they together make up the whole of Simplicius's text. In what follows, however, I shall argue that they suffer from at least one important omission: a proof by Simplicius himself of Euclid's parallels postulate. Since the omission occurs both in the Leiden manuscript and in Gerard's translation, it cannot simply be an accidental feature of the former. My argument will consist in (1) citing evidence (Document I) to the effect that such a proof was known to some Arabic mathematicians, and (2) producing a hitherto unnoticed text (Document II) which, in the light of the evidence cited, may well be taken to be the missing proof. In addition, I shall show how Simplicius's proof entered Arabic discussions on parallels, first, by being made subject to criticism (Document I), and then by being incorporated into a new proof which was designed to take that criticism into account (Document III).

The title of Simplicius's work in question appears in the Arabic sources in slightly different forms. Nayrīzī concludes the last citation from that work with the following words: 'There end the matters which Simplicius has put forward in the commentary to the *muṣādara* of Euclid for the first part of the book of *Elements*.'[2]

The word *muṣādara* has here something a little unexpected about it. Usually, as in translations of Euclid and Aristotle, it corresponds to the Greek αἴτημα, and it is used in this sense in the body of Simplicius's commentary itself. (The Arabic verb *ṣādara* appropriately means 'to demand'.[3] *Muṣādara:* demanding or that [proposition] which is demanded.) But the commentary is not restricted to the αἰτήματα or postulates at the beginning of the *Elements*, but also treats of the common notions (κοιναὶ ἔννοιαι: *'ulūm muta'ārafa*) and the definitions (ὅροι: *ḥudūd*). Could *muṣādara* be used here in a general sense

[1] *Codex Leidensis 399, 1. Euclidis Elementa ex interpretatione al-Hadschdschadschii cum commentariis al-Narizii* . . . in three parts, edited by R. O. Besthorn, J. L. Heiberg, G. Junge, J. Raeder and W. Thomson, Copenhagen 1893–1932; *Anaritii in decem libros priores Elementorum Euclidis commentarii*, ex interpretatione Gherardi Cremonensis in codice cracoviensi 569 servata. Edidit Maximilianus Curtze, Leipzig 1899. These two editions will subsequently be referred to simply as Nayrīzī and Gerard respectively. For more information about editions and manuscripts mentioned in this article the reader may refer to my article on Thābit ibn Qurra in this *Journal*, XXXI, 1968, pp. 12ff.

[2] *tammat al-ma'ānī allatī qaddamahā s(i)nb(i)liqyūs fī tafsīr muṣādarat ūqlīd(i)s li 'l-maqāla 'l-ūlā min kitāb al-uṣūl*. Nayrīzī, pt. 1, p. 40. Gerard simply has (p. 42, l. 20): Explicit expositio prologi.

[3] *al-Qāmūs al-muḥīṭ: ṣādarahu 'alā kadhā ṭālabahu bihi.*

that covers all three groups of Euclid's premisses? Such a hypothesis would derive at least partial support from a statement in Proclus that some ancient writers applied the term αἰτήματα to axioms (or common notions) as well as to postulates. Proclus quotes Archimedes as an example.[4] In agreement with this usage the titles of at least two Arabic works on geometry employ the plural *muṣādarāt* as a collective term for the axioms, definitions and postulates.[5] And it was probably this sense which the eleventh-century scholar Abū ʿAbd Allāh al-Khwarizmī had in mind when he gave the following explanation in his *Keys of the Sciences*: '*al-muṣādara* are those premisses of the question which are put at the beginning of a book or chapter of geometry.'[6]

The tenth-century bibliographer Ibn al-Nadīm gives a somewhat different version of the title of Simplicius's book: 'A commentary on the *ṣadr* of the book of Euclid, which is the introduction to geometry.'[7] *Ṣadr* means fore-part or front and is frequently used to refer to the introductory part of a book; it might have rendered the Greek προοίμιον. The latter part in this version, 'which is the introduction to geometry', looks like a description of the book supplied, perhaps, by Ibn al-Nadīm himself, but it may also have been an alternative title of the book. Nayrīzī's version of the title agrees with Khwārizmī's definition in applying the singular *muṣādara* to a multitude of premisses, but we shall see that the thirteenth-century author of Document I cites the same title with *muṣādarāt* in the plural.

Simplicius prefaces his comments on the individual postulates of Euclid with a long passage on the meaning and function of postulates in general. It will be useful to quote this passage here in full since it is one of the channels through which Greek discussions of mathematical methodology were transmitted to the Islamic world—particularly discussions connected with the question of parallels.

SIMPLICIUS ON POSTULATES[8]

Euclid said: The postulates (*muṣādarāt*) are five.

Simplicius said: Euclid, after having stated the definitions which signify the essence (*jawhar*) of each one of the defined things, went on to enumerate the postulates. Postulates, in general, are those things which are not granted

[4] Proclus, *In Euclidem comment.*, ed. Friedlein, Leipzig 1873, p. 181, ll. 16ff.; Ver Eecke's trans., Paris 1948, p. 159.

[5] These are *Sharḥ muṣādarāt Uqlīdis* (Commentary on the *Muṣādarāt* of Euclid) by Ibn al-Haytham, and *Sharḥ mā ashkala min muṣādarāt Kitāb Uqlīdis* (Explanation of the Difficulties in the *Muṣādarāt* of Euclid's Book) by ʿUmar al Khayyāmī. The first is a commentary on *all* of Euclid's premisses; I have consulted MS. Feyzullah 1359, dated 869 h. The latter consists of three parts of which the second is concerned with Euclid's *definitions* of ratio and proportion; see this *Journal*, XXXI, 1968, p. 18, n. 25.

[6] *Mafātīḥ al-ʿulūm*, ed. Van Vloten, Leiden 1895, p. 203: *al-muṣādara mā yuṣaddaru bihi ʾl-kitāb aw al-bāb min abwāb al-handasa min muqaddamāt al-masʾala*. Al-Khwārizmī appears to imply a connection between *muṣādara* and the verb *ṣaddara*, to preface.

[7] *Fihrist*, Cairo n.d., p. 389: *sharḥ ṣadr kitāb Uqlīdis wa-huwa ʾl-madkhal ila ʾl-handasa*. The version in Ibn al-Qifṭī's *Taʾrīkh al-ḥukamāʾ* (13th century; see Lippert's edition, Leipzig 1903, p. 206) agrees with that in the *Fihrist* but omits the word *ṣadr* and thus implies that Simplicius wrote a commentary on the whole of the *Elements*. There is nothing in the literature to support this implication and we must therefore assume that the omission was not intended.

[8] Nayrīzī, pt. 1, pp. 12–16.

(*laysa muqarran bihi*), but the student is asked (*yufāraqu ʿalā*)[9] to grant them as a permissible hypothesis (*aṣl mawḍūʿ*) which is laid down between himself and the teacher and which is granted.[10]

Now this hypothesis (*aṣl*) may be impossible, as for instance the postulate which Archimedes asked to be granted him, namely to postulate that he stands outside the earth. For he promised (*taḍammana*) that if this were conceded to him (*sullima lahu*), he would prove[11] that he could move the earth—saying: Young man, grant me that it is possible that I rise and stand outside the earth, and I will show you that I can move the earth. This he said when he boasted of having discovered mechanical power,[12] demanding that that [assumption], though impossible, be postulated and laid down for the sake of teaching.

Thus a postulated thing may be impossible, as we said, or it may be possible and known to the teachers but unknown to the students and required to be used at the beginning of teaching. For the things that are subject to demonstration (*allatī tubarhanu*) are also known to the teachers and unknown to the students, but they are not laid down by postulation (*ʿalā ṭarīq al-muṣādara*) since they are not primary premisses (*awāʾil*: πρῶτα) but are capable of demonstration (*tubarhanu*).

But as for postulates, whoever posits them demands that they should be postulated as principles (*mabādiʾ*: ἀρχαί).[13] Some are demanded to be postulated as necessary for teaching only—such as the first three postulates. Some require an easy proof[14] in order to be assented to and accepted by themselves.[15] The difference between these and axioms[16] is that axioms are accepted by themselves as soon as thought takes hold of them. Whereas postulates are by nature intermediate (*mutawassiṭa fi ʾl-ṭabʿ*) between, on the one hand, the principles taken from the first science (*al-ʿilm al-awwal*), whose reasons are unknown to those who use them—such as definitions, and on the other, axioms which all people equally accept.[17] For postulates are known, not however to all people, but to the teachers in each one of the arts.

[9] *Lisān al-ʿarab*: *wa-min kalāmi kuttābi ʾl-dawāwīni an yuqāla ṣūdira fulānun al-ʿāmilu ʿalā mālin yuʾaddīhi ay fūriqa ʿalā mālin ḍaminahu.*

[10] Gerard, p. 28, ll. 14–17: Sed ea, que premittuntur, sunt ea, que non sunt concessa; non tamen dimittetur discipulus, qui non cogatur concedere. Exempli gratia ut sit vis magisterii et quasi radix posita et concessa.

[11] Reading *yubayyina* for the edition's *t(a)bayy(a)na*. The MS. lacks the diacritical points for the first two letters.

[12] *al-quwwa ʾl-handasiyya*. Gerard, p. 28, l. 25: virtutem geometricam.

[13] Gerard, p. 29, ll. 5–6: Ea autem, que premittuntur, non ob aliud querantur premitti, nisi quia sunt sua principia.

For the suggested Greek equivalents of *awāʾil* and *mabādiʾ*, see, for example, Mattā's translation of the *Posterior Analytics* in A. Badawī (ed.), *Manṭiq Arisṭū*, ii, Cairo 1949, p. 313, 71b21, 24, p. 314, 72a5f, and *passim*. The translation, made from a Syriac version by Isḥāq ibn Ḥunayn, was not always consistent, sometimes using *awāʾil* for both πρῶτα and ἀρχαί; see *Manṭiq Arisṭū*, ii, p. 338, 76a31, and p. 339, 76b14.

[14] Reading, with the MS., *bayān yasīr*, for the edition's *bayyin yasīr* (pt. 1, p. 14, l. 8).

[15] Gerard, p. 29, ll. 8–10: et eorum sunt quedam, que parum sunt declaranda, donec concedantur et recipiantur per se.

[16] *al-ʿulūm al-mutaʿārafa* (lit. recognized notions): either ἀξιώματα or κοιναὶ ἔννοιαι. The latter, which is the one used in Euclid's *Elements*, was sometimes rendered as *al-ʿulūm al-mutʿārafa*, as is the case for example in Hajjāj's translation, but also as *ʿilm jāmiʿ* (common knowledge), or *ʿilm ʿāmm* (universal knowledge). Mattā's Arabic version of Aristotle's *Anal. post.* has *al-ʿulūm al-mutʿārafa* for ἀξιώματα; see *Manṭiq Arisṭū*, *ed. cit.* (note 13), ii. p. 339, 76b14.

[17] Gerard, p. 29, ll. 12–16: sed petitiones

Some have thought that geometrical postulates are only intended for granting the matter (*al-'unṣur:* τὸ στοιχεῖον) in as much as not all operations are applicable to it. For an opponent may raise an objection on account of the matter, saying: I cannot produce a straight line on the surface of the sea,[18] nor can I produce a straight line to infinity since the infinite does not exist.[19] But those who say this, first, think that postulates are only for those whose geometry is material (*'unṣuriyya*). But what would they say about the equality of right angles?[20] How would they establish[21] that this is postulated on account of matter? And similarly with the postulates that come after this.

It is, therefore, better to say that postulates are those things which are not accepted by the student when he first hears them, and which are required in the demonstration. Some are impossible and, therefore, are not as easily accepted as the first three, but are demanded to be granted for carrying out the process of teaching, as I said. Others are known to the teacher and accepted by him, but to the student are at the outset remote and not evident, and, accordingly, he is asked to grant them, as is the case with the postulates that come after the first three.[22] The utility of the first three is not to allow the demonstration to be hindered by an incapacity or failing in the matter. Whereas those that follow upon the first three are required for certain demonstrations.

[End of Simplicius's text]

The 'five postulates' to which these general remarks apply are the same as those in Heiberg's edition of the *Elements*.[23] According to Simplicius the first three (to join two points by a straight line; to produce a straight line; to describe a circle with any radius) are only required 'for the sake of teaching'; their utility lies in 'not allowing the demonstration to be hindered by an incapacity or failing in the matter' in which these operations are to be effected. Postulates 4 and 5, however, constitute a separate group. Unlike the axioms they are not self-evident statements accepted by everyone, but they are

sunt naturaliter medie inter per se nota et alia, quorum cause ignote sunt discipulis, sicut diffinitiones, que sunt medie inter probabilia, que ab omnibus recipiantur, et inter <per> se nota . . .

Gerard, it seems, read: *allatī 'ilaluhā majhūla 'inda 'l-muta'allimīna lahā* (quorum cause ignote sunt discipulis) where the Leiden MS. has: *allatī* . . . *'inda 'l-musta'milīna lahā* (whose reasons are unknown *to those who use them*). I do not understand the meaning of the sentence as a whole either in the Arabic or in Gerard's translation.

[18] Gerard, p. 29, ll. 23–25, adds: et impossibile est mihi, ut protraham lineam rectam supra locum, in cuius medio est civitas aut flumen.

[19] the infinite does not exist (*lā-nihāyata ghayru mawjūdin*). Gerard, p. 29, l. 27: infinitum . . . non reperitur.

In medieval Latin translations from the Arabic *reperire* was often used to render cognates of the verb *wajada* (to find), even when these have the sense of *existence*, as in the case of the passive *mawjūd*, found or existent.

[20] I.e. Euclid's Postulate 4.

[21] *kayfa yūjidūnana*: thus in the edited text and in the Leiden MS. Gerard, p. 29, ll. 30–31, has: quomodo reperit. See note 19 above.

[22] as is the case with the postulates that come *after* the first three. Gerard, p. 30, l. 10: sicut tria, que premittuntur.

[23] Simplicius quotes a *sixth* postulate ('two straight lines cannot enclose a surface') but adds: 'This postulate is not found in the ancient copies, perhaps because it is manifest and evident, and accordingly the postulates have been described to be five' (*ed. cit.*, pt. 1, p. 24). He then goes on to give a demonstration which he attributes to 'the moderns' (*ibid.*, pp. 24–26).

indispensable for certain geometrical demonstrations. Their truth is known to the teacher but not, at the outset, to the student. How then, we may ask, does the teacher gain possession of this knowledge, and how is he eventually going to convey it to the student? Simplicius seems to be inconsistent when he says that some postulates require an 'easy proof' in order to be accepted by themselves, for he also states that postulates, being principles of demonstration, are not themselves matter for demonstration.[24] Maybe he would want to distinguish between an 'easy proof' and a demonstration, though the Arabic term translated here by 'proof' (*bayān*) is the one regularly used in translating Euclid's words that always come at the end of the demonstration of a theorem: ὅπερ ἔδει δεῖξαι (*wa-dhālika mā aradnā an nubayyin*).

However that may be, Simplicius's judgement on Postulate 5, the parallels postulate, is perfectly unequivocal: 'This postulate, he says, is not all that evident but needs a geometrical proof . . .'[25] Nayrīzī quotes this sentence and says that he has inserted Simplicius's 'commentary (*tafsīr*) with the additions of Aghānīs' after the demonstration of Proposition I.26.[26] The inserted passage in fact occurs after Prop. I.28, where it is announced by the following heading: '*Premisses* (muqaddamāt) *and Propositions* (ashkāl), *required for Proposition 29 of Book I, by Simplicius and Aghānīs*.'[27] These words, missing in Gerard's translation, would lead us to expect some 'premisses' or 'propositions', perhaps even a proof consisting of 'premisses and propositions', which Simplicius had contributed, in addition to those of Aghānīs. But what we have immediately following upon this announcement is Aghānīs's proof preceded by a few introductory lines by Simplicius:[28]

The premiss[29] which is used in the demonstration of Proposition 29 of Book I, namely that every two lines drawn [from a transversal] at less than two right angles will meet,[30] is not one of the accepted propositions.[31] Concerning this Simplicius said: This postulate is not all that evident, but

[24] Avicenna expounds and rejects the opinion of those who thought that postulates, as distinguished from hypotheses, needed a 'little thinking' in order to be accepted. He argued that if the 'thinking' was for the purpose of grasping the meaning of the words involved, this would not distinguish postulates from axioms proper. And if it consisted in seeking to establish the truth of postulates through middle terms, then they would not be distinguishable from theorems. See *Kitāb al-Burhān*, ed. A. 'Afīfī, Cairo 1956, pp. 113–114. Simplicius appears to be in the same dilemma.

[25] Nayrīzī, pt. 1, p. 24. Leiden MS.: *inna hādhihi 'l-muṣādarata laysat bi-ẓāhiratin kulla dhāka, lākinnahā qad uḥtija fīhā ilā bayānin bi 'l-khuṭūṭ* (lit. but needs a proof by means of lines διὰ γραμμῶν). What Simplicius means is that the responsibility for demonstrating Postulate 5 does not fall on a science other than geometry.

[26] Gerard, p. 35, l. 4: post probationem figure 29e.

[27] Nayrīzī, pt. 1, p. 118.

[28] *Ibid.*, pp. 118–20.

[29] *al-muqaddama*. MS. adds in margin: *al-qaḍiyya*. Gerard, p. 65, l.17: Postulatum.

[30] On this excessively abbreviated form of Euclid's Postulate 5, see this *Journal*, XXXI, 1968, p. 19, n. 27.

[31] *al-qaḍāya 'l-maqbūla*, i.e. the axioms. Earlier in the text Euclid's common notions are called *al-qaḍāya 'l-maqbūla wa'l-'ulūm al-muta'ārafa*. A third alternative appellation, added in the margin of the Arabic manuscript, is: *'ilm jāmi'* (Nayrīzī, pt. 1, p. 26). The last expression is used by Avicenna in his version of the *Elements*, forming the geometrical part of the mathematical section of *al-Shifā'*.

needs to be proved by means of lines,[32] so that Abzīnyāṭūs[33] and Dh(i)yūdhūr(u)s [Diodorus] proved it (*bayyanāhu*) by many different propositions. Ptolemy, too, constructed a proof and demonstration of it,[34] using for this purpose Propositions 13, 15 and 16[35] of Book I of the *Elements*. This is not objectionable, for Euclid did not use this postulate until Proposition 29 of this Book. But this notion [Post. 5] is in itself also worthy of being examined and discoursed upon and [it requires us] to prove that just as when two lines are drawn [from a transversal] at two right angles they are parallel, they meet if drawn at less than two right angles. * But as for Aghānīs, our associate,[36] it was not his opinion to propose this notion as a postulate, for it is in need of demonstration, but used other propositions instead of those in the *Elements* until he demonstrated Proposition 29 without making this notion a postulate, and he then demonstrated this postulate by geometrical methods.[37] This is his own discourse. [There follows Aghānīs's demonstration.]

I am inclined to think that the place which I have marked with an asterisk in this passage is that from which Simplicius's proof has been removed. Was it Nayrīzī who decided to leave the proof out? If so, why? At the end of Aghānīs's proof we read the following comment:[38]

> All that he [Aghānīs] has described in this proposition [Euclid's Postulate 5 presented as the final conclusion of the proof], and in the premisses (*muqaddamāt:* lemmas?) thereof which he put forward, is acceptable as necessary in accordance with the *muṣādara*[39] of Book I, and in accordance with the propositions which Aghānīs has arranged and added to those of Euclid. Nothing of what he has brought forward is in any way subject to objection.

Who is the author of this comment? Not Simplicius, whose own concluding remarks begin immediately afterwards, introduced by the words: 'Simplicius said.' Assuming therefore that it was Nayrīzī who inserted it, we may venture to guess why he ignored Simplicius's proof; he was so convinced of Aghānīs's demonstration that he thought it not worthwhile to quote Simplicius's inferior effort beside it. Our guess gains confirmation from the fact that a proof bearing the name of Nayrīzī is found to be largely based on that of Aghānīs.[40]

[32] See note 25 above.

[33] Gerard and the editors of the Arabic text read: Abthiniatus, but no further guesses about him have been forthcoming. See Nayrīzī, pt. 1, p. 119, also p. 25; Gerard, p. 65, l. 23, also p. 35. Cf. Heath, *The Thirteen Books of Euclid's Elements*, Cambridge 1956, i, pp. 203–4.

[34] *'amila bayānahu wa 'l-burhāna 'alayhi*. See Proclus, *op. cit.* (note 4), pp. 365ff.; Ver Eecke's trans., pp. 312ff.

[35] Gerard has: figura 13ª et 15ª et *18ª* (*ed. cit.*, p. 66, ll. 1–2).

[36] But this notion is in itself worthy . . . our associate: *wa-qad kāna hādha 'l-ma'nā fi nafsihi mustaḥiqqan li 'l-naẓari wa 'l-qawli fīhi wa-an nubayyina annahu kamā anna 'l-khaṭṭayni idhā ukhrijā . . . fa ammā Aghānīsu ṣāḥibunā . . .* (Nayrīzī, pt. 1, p. 118). For Aghānīs see this *Journal*, XXXI, 1968, p. 13, n. 6.

[37] *bi-madhāhiba wa-subulin handasiyya*, Nayrīzī, pt. 1, p. 120.

[38] Nayrīzī, pt. 1, p. 130.

[39] Here again we see *muṣādara* used in the singular to denote the sum of the preliminary propositions. See above, p. 2.

[40] Cf. this *Journal*, XXXI, 1968, pp. 16–17.

But before we examine our documents, here is the full text of Simplicius's final remarks:[41]

> This then is the discourse of Aghānīs in his own words. It may be that Euclid used this notion[42] in the postulates as being more accessible than this [notion of Aghānīs]. For if parallel lines are those lines which lie in one surface and which, when continually produced on both sides, always maintain the same distance between themselves,[43] then the converse of this statement will be true: viz. that lines which lie in one surface, if the distance between them is not the same, are not parallel, and if they are not parallel they meet. For Euclid used this notion in this proposition as one of the propositions that must be accepted.[44] Now the lines that are drawn at less[45] than two right angles do not maintain the same distance; therefore, they meet. And it is evident that they meet on the side of their inclination to one another, for on the other side they diverge and widen and the distance between them increases. But since the statement 'that the two lines, if they are not parallel, will meet' needs to be argued (*yuqūsā*) and proved, and further because the conic sections are not parallel but do not meet, Aghānīs has stated this premiss and used these propositions.
>
> Furthermore, this notion is the converse of the proposition in which it is said that if on two straight lines there falls a straight line making the two interior angles equal to two right angles, the two lines are parallel.[46] For since this proposition has been shown by a demonstration, that notion also needs to be shown by a demonstration.[47] We have now presented everything that can be said concerning parallel lines, and the matter regarding them has been ascertained.

It is to our first document that we now turn for the evidence that a proof by Simplicius of the parallels postulate was originally included in his commentary to the premisses of the *Elements*.[48] The document is an extract from

[41] Nayrīzī, pt. *1*, pp. 130–32.

[42] I.e. the notion involved in Postulate 5.

[43] This is the definition of parallel lines adopted by Aghānīs and, following him, by many of the Arabic mathematicians who tackled the problem of parallels. Euclid defines parallel lines as those lines which, being in the same plane, do not meet in either of the directions in which they may be produced. The meaning of this part of Simplicius's passage is lost in Gerard's version as a result of translating *khuṭūṭ mutawāziya* (parallel lines) by *linee equidistantes*; cf. Gerard, p. 73.

[44] Gerard, p. 73, ll. 15–16: 'quod Euclides posuit in figura 29ª, ac si esset necessario recipiendum . . .'

[45] Gerard, p. 33, l. 17: maiores.

[46] A reference to Eucl. I. 28.

[47] Proclus mentions a similar objection to Postulate 5 which he derives from Geminus, but for him the demonstrable converse to Postulate 5 is Eucl. I. 17: In any triangle two angles taken together in any manner are less than two right angles. Proclus, *op. cit.* (note 4), pp. 183f., also p. 364; Ver Eecke's trans. p. 161, also p. 312. The reference to conic sections is also obviously implied in another passage in Proclus; see Heath, *The Thirteen Books of Euclid's Elements*, i, pp. 202–3.

[48] The author of the anonymous treatise on parallels, MS. Carullah 1502, fols. 26ᵛ–27ʳ, dated 894/1488–9, mentions two 'ancient' thinkers whose treatment of Euclid's postulate he had examined: Simplicius and Aghānyūs. (See M. Krause, 'Stambuler Handschriften islamischer Mathematiker', *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abt. B: Studien, iii, 1936, p. 522; and this *Journal*, XXXI, 1968, p. 13, n. 6.) He does not explicitly state that what he had seen by Simplicius was a *demonstration* of the postulate; but it should be noted that Proposition 3 in his own proof is the same as

a thirteenth-century letter sent from Syria to Naṣīr al-Dīn al-Ṭūsī, and in the manuscripts it is found attached to al-Ṭūsī's separate treatise on parallels: *al-Risāla al-Shāfiya ʿan al-shakk fī 'l-khuṭūṭ al-mutawāziya*.[49] The author of this letter, ʿAlam al-Dīn Qayṣar ibn Abi 'l-Qāsim ibn ʿAbd al-Ghanī ibn Musāfir, al-Ḥanafī, known as Taʿāsīf, was born at Asfūn (or Aṣfūn) in Upper Egypt in 574/1178–9.[50] A Ḥanafite jurist who was versed in the science of Koranic readings, he studied mathematics in Egypt and in Syria before he went to Mosul to read music under the renowned teacher Kamāl al-Dīn ibn Yūnus (d. 1242). He returned after one year to Syria[51] where he resided for some time at Ḥamā, teaching and serving its Ayyūbid ruler Taqī al-Dīn Maḥmūd for whom he built observation towers, a water-mill on the river al-ʿĀṣī and a celestial globe. (The latter was in the collection of Cardinal Borgia at Velletri in 1809.) His reputation as a mathematician is revealed in the fact that when the Ayyūbid sultan al-Malik al-Kāmil[52] received a series of the so-called 'Sicilian questions'[53] propounded by Frederick II and particularly dealing with matters of 'philosophy and mathematics', Qayṣar's help was sought as 'the authority' in this field.[54] He died in Damascus in 649/1251, and it was probably from there that he had corresponded with Naṣīr al-Dīn al-Ṭūsī.[55]

DOCUMENT I

Extract from a letter of ʿAlam al-Dīn Qayṣar ibn Abi 'l-Qāsim to Naṣīr al-Dīn al-Ṭūsī[56]

[1] One thing that may be proposed for your elevated consideration is what occurred to me regarding a proposition which Simplicius, in his commentary to the premisses of the book of *Elements*, mentioned among lemmas of the

the proposition ascribed to Simplicius in Document I, paragraph [3]: If two straight lines, such as *AG*, *BD*, are drawn from a straight line, such as *AB*, at two [interior] angles of which one is a right angle, such as *A*, and the other acute, such as *B*, then the two lines will meet in the direction in which they have been drawn.

[49] Cf. *Rasāʾil al-Ṭūsī*, ii, Hyderabad 1359h, *Ris.* no. 8, pp. 36–37.

[50] See H. Suter, *Die Mathematiker und Astronomen der Araber und ihre Werke* (*Abhandlungen zur Geschichte der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*, X.), Leipzig 1900, no. 358, p. 143; *idem*, 'Nachträge und Berichtigungen zu "Die Mathematiker . . ." ', in *Abhand. z. Gesch. d. math. Wiss.*, XIV, 1902, p. 175; Krause, 'Stambuler Handschriften', *loc. cit.* (note 48), p. 496, no. 10; C. Brockelmann, *Geschichte der arabischen Literature*, i², Leiden 1943, p. 625, Suppl. i, Leiden 1937, p. 867. The following may be added to the Arabic sources referred to by Suter and Brockelmann: Jaʿfar ibn Thaʿlab al-Adfuwī (d. *c.* 748h), *al-Ṭāliʿ al-saʿīd al-jāmiʿ li-asmāʾ al-fuḍalāʾ wa 'l-ruwāt bi-aʿla 'l-Ṣaʿīd*, Cairo 1914, no. 367, pp. 259–60. See also al-Ziriklī, *Aʿlām*, vi, 2nd edn., Cairo? 1955, p. 62.

[51] Qayṣar told Ibn Khallikān that he had been acquainted with musical theory before he went to Mosul, but he wished to be associated with Ibn Yūnus. See Ibn Khallikān, *Biographical Dictionary*, trans. de Slane, iii, Paris 1868, pp. 471–2; Wüstenfeld's edition of the *Wafayāt, Vitae illustrium virorum*, fasc. ix, Göttingen 1840, no. 757, p. 25. Also: *al-Ṭāliʿ al-saʿid, ed. cit.* (note 50), p. 259.

[52] He ruled in Egypt from 1218 until his death in 1238, and in the last year of his life extended his rule over Damascus.

[53] C. H. Haskins, *Studies in the History of Medieval Science*, New York 1960, pp. 264f. (Reprint of the second edition, 1927.)

[54] *al-Ṭāliʿ al-saʿid, ed. cit.* (note 50), p. 259.

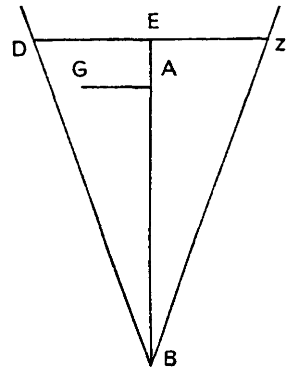
[55] In MS. Aya Sofya 2760, at the beginning of his first letter to Ṭūsī (fol. 9ʳ) Qayṣar is described as 'of Damascus'.

[56] An edition of the Arabic text of Documents I and III and a facsimile of Document II will be found at the end of this article.

famous proposition, which is this: If a straight line falling on two straight lines makes the two interior angles on one side equal to less than two right angles, then the two lines, if produced on that side, will meet.

[2] He said that every angle can have infinitely many chords of increasing magnitudes[57] each of which cuts off equal [segments] from the two lines containing that angle.

[3] He used this in [the following]: If the line *AB* falls on the two lines *BD*, *AG*, and the angle *GAB* is a right angle, and the angle *ABD* is acute, then the two lines *AG*, *BD* will meet on the side of *G*, *D*.



[4] For he constructed on the point *B* of the line *AB* an angle *ABZ* equal to the angle *ABD*. Then there are infinitely many chords of increasing magnitudes which subtend the angle *DBZ*. And one of the chords will fall beyond point *A*—such as the chord *ZED*. Therefore the two angles *A*, *E* being right angles, the line *AG*, if produced, will not meet the line *ED*. It will therefore meet the line *BD*.

[5] But assuming the line *BD* to be at the beginning of its deviation from being in a straight line with *BZ*, every chord that subtends the angle *ZBD* will fall between the two points *A*, *B*; for *AB* is infinitely divisible.

[6] Thus if it is possible to find a demonstration showing that one of the chords will fall outside of the point *A*, the conclusion will follow, and our master will have generously and graciously added further to his previous benefactions.

I have divided Qayṣar's letter into sections for ease of reference. Section [1] explicitly refers to Simplicius and is also specific in stating that the reported proof occurred in his *Commentary to the' Premisses of the book of Elements* (*fī Sharḥihi li-muṣādarāt Kitāb al-Uṣūl*). There can be no doubt that this is the same work listed, though with a slightly different title, in the *Fihrist* and utilized by Nayrīzī. The same section tells us that Simplicius's proof consisted of *several* lemmas (*maqaddamāt*) from which Postulate 5 was deduced.

In [2] Qayṣar cites a statement which, according to what he says in [3], formed a basis for the proof.

[3] sets forth the proposition to be proved on the basis of the statement in [2]. This proposition is a special case of Euclid's Postulate 5, i.e. the case in which one of the two angles described in the Euclidean Postulate (quoted in [1]) is acute whilst the other is a right angle.

[4] describes Simplicius's demonstration of the special case of Postulate 5.

In [5] Qayṣar raises an objection to Simplicius's demonstration. In words close to Qayṣar's the objection is that owing to the infinite divisibility of the transversal *AB* the angle *DBZ* may be such that 'chords' drawn across *BD*, *BZ* will all fall on the near side of the point *A*. The problem which he accordingly puts to Naṣīr al-Dīn al-Ṭūsī in [6] is to provide a demonstration to the effect that one of the chords will in fact fall on the far side of *A*.

[57] of increasing magnitudes. More literally: of which some are greater than others (*baʿḍuhā aʿẓam min baʿḍ*).

Al-Ṭūsī replied[58] that he had never seen Simplicius's proposition, but that his own long preoccupation with the problem of parallels had led him to devise a method, partly derived from others before him and completed by his own ideas, which solved the problem satisfactorily and from which Simplicius's proposition could be demonstrated. The solution was contained in *al-Risāla al-Shāfiya* of which a copy accompanied the letter to Qayṣar.[59] With the exception of one reservation Qayṣar agreed that Ṭūsī's demonstration of Euclid's Postulate was superior to all the proofs which he had seen: two by Thābit ibn Qurra, one by Ibn al-Haytham and one by Yūḥannā al-Qass. Qayṣar's reservation stimulated a final reply from Ṭūsī. In this exchange the main difficulty which Qayṣar had pointed out in Simplicius's demonstration was set aside to make way for problems arising from *al-Risāla al-Shāfiya* in which Ṭūsī discussed ideas of al-Jawharī, of Ibn al-Haytham and of ʿUmar al-Khayyāmī.[60]

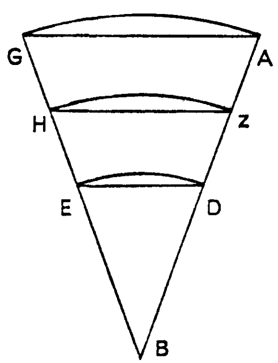
Let us now look at Document II. It is a demonstration of the Parallels Postulate which occupies less than one page of an Arabic manuscript preserved at the Bodleian Library, Oxford. Although the copyist tells us that he transcribed this demonstration from a book by an Arabic author, we shall see that it corresponds exactly to Qayṣar's description of the proof attributed to Simplicius.

DOCUMENT II

Bodleian Library MS. Thurston 3,[61] fol. 148r

Statement of the Demonstration of the Postulate Mentioned at the Beginning of the Book [of *Elements*?]—which Consists of Four Propositions.

1



Let *ABG* be an angle taken at random (*kayfa ittafaqa*); then it has many chords infinite in multitude (*fa-lahā awtār kathīra lā nihāyata li-kathratihā*).

About *B* as centre let us draw any number of circles, such as *DE*, *ƵH*, *AG* and let us draw (*naṣil*) their chords.

Thus every angle has chords which are infinite in multitude.

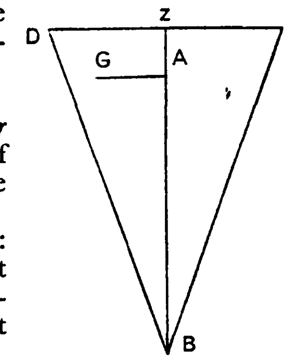
[58] *al-R. al-Shāfiya, loc. cit.* (note 49), pp. 37ff.

[59] The 'solution' is identical with Ṭūsī's proof of the postulate in the 'longer version' of his edition (*Taḥrīr*) of the *Elements* which he completed in 1248. See this *Journal*, XXXI, 1968, pp. 14–15, n. 11; also p. 16, n. 15. The proof of the 'shorter version' which later became known to Saccheri is quite different.

[60] This *Journal*, XXXI, 1968, p. 14, n. 9, pp. 15–16 and p. 18, n. 25.

[61] J. Uri, *Bibliothecæ Bodleianæ codicum manuscriptorum orientalium . . . catalogus*, pars prima, Oxonii 1787, *Codices manuscripti arabici*, p. 198, cod. CMXIII. The catalogue gives only a very inadequate account of the contents of this codex; it does not list our document.

2



Let the angle *ABD* (MS. *ABG*) be acute and the angle *BAG* be right; then the lines *AG*, *BD*, if produced, will meet.

Let the angle *ƵBE* be equal to the angle *ƵBD*;

let *BD*, *BE* be produced indefinitely (*ilā ghayr nihāya*), and from them let an unlimited number of multiples (*aḍʿāf ghayr maḥṣūra fī 'l-ʿadad*) of *BA* be cut off;

let the chords joining their extremities be drawn:

Then one of these chords must fall beyond point *A*, towards point *D*. For the multiples taken are unlimited and the line *AB* is limited (*maḥṣūr*). Let it [this chord] be as the line *EƵD*.

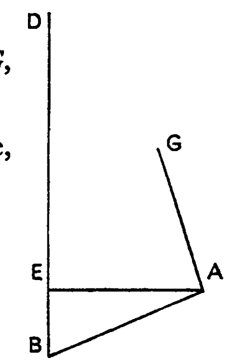
Then, since the angle *B* has been halved and *BD* is equal to *BE*, the angles at *Ƶ* are right angles.

Therefore, the line *AG* will not meet *ƵD*, for if it did, the angle *A* which is exterior to the triangle made up of the lines *AƵ*, *ƵD* and *AG* would be greater than the right angle *Ƶ*; but the two angles are equal—this is impossible.

Therefore, the line *AG* will not meet the line *ƵD*, but as it goes out (*bi-khurūjihi*) will meet the line *BD*.

Which was to be proved.

3



Let each of the angles *A*, *B* be acute; then the lines *AG*, *BD*, if produced, will meet on the side of *G*, *D*.

We draw *AE* perpendicular to *BD*.[62]

Then, the angle *E* being right and the angle *EAG* acute, the lines *AG*, *BD* will meet if produced.

4

The line *AB* [falling] on the lines *AG*, *BD* makes[63] the angle *ABD* obtuse and the angle *BAG* acute, but these two angles are less than two right angles. Therefore, the two lines, when produced, will meet on the side of *G*, *D*.

Let us divide *AB* into two halves at *E* and draw the perpendicular *EƵ* and produce it to *H*.

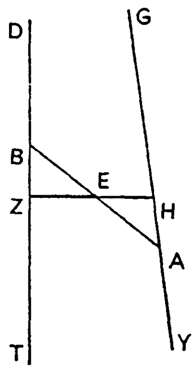
Then the angle *ƵHG* is acute.

For, if it were right, then the angles at *E* being opposite to one another

[62] The MS. adds: *mathalan* (?).

[63] *wa-yuṣayyir* (?), *wa-naʿtabir* (?).

(*mutaqāṭiʿatān*), the acute angle at A would be equal to the acute angle at B, since AE, EB are equal.



And since the angles at B are equal to two right angles, the angles GAB, ABD are equal to two right angles;

but they are less;—this is impossible.

And if the angle ZHG is obtuse, then the angle AHE is acute;

but the angle HZT is a right angle;

therefore, the lines GY and DT will meet when produced on the side of T, Y—which is impossible because the angles YAB, ABT are greater than two right angles.

Therefore, the angle ZHG is neither obtuse nor right; then it must be acute.

And the angle Z is right.

Therefore, the lines AHG, BD, if produced, will meet on the side of G, D.

Which was to be proved.

Transcribed from the Book of Euclid of Muḥyi 'l-Dīn al-Maghribī al-Andalusī. He produced (*awrada*) these four propositions after Proposition 24 (*kd*), viz. if a line falling on two lines makes the alternate angles equal to one another—to the end of the Proposition. Written by the same scribe,[64] in the year 763 [i.e. A.D. 1361–2].

It is obvious that a very close relationship exists between Documents I and II. The statement cited in I.[2] is the same as Proposition II.1. According to Qayṣar, this statement was the basis of Simplicius's proof.

There are several alternative expressions in Arabic for describing the 'chords' of the angle in question as infinite in number: *lā nihāyata li-ʿadadihā*, *lā-mutanāhiyat al-ʿadad*, etc. It may therefore be interesting to observe that I.[1] and II.1 concur in using the expression *lā nihāyata li-kathratihā* (infinite in multitude). But since the last expression was not itself rarely used, this linguistic agreement may not necessarily imply a dependence of Document I on Document II. More indicative of this dependence is the use of 'chord' (*watar*) as applied to an angle in I.[2]. Since the angle is not here expressly associated with arcs subtending it, it is not immediately clear why 'chord' is used.[65] It is Proposition I in Document II that provides the explanation by telling us how the 'chords' have been drawn: namely by joining the corresponding points where the 'circles' DE, ZH, AG intersect the sides of the angle ABG. The construction accords with Eucl. I. 3.

But the most important similarity is that between I.[3], the proposition proved by Simplicius, and Proposition II.2. Both are expressions of the same special case of Euclid's Postulate 5. Moreover, the demonstration of II.2 corresponds exactly to the description of Simplicius's proof as given in I.[4];

[64] That is, the scribe whose name appears at the top of the page at the end of the preceding item in the manuscript.

[65] See Eucl. I. 5, where βάσις (*qāʿida*) is applied to angle; cf. T. L. Heath, *Euclid in Greek*: Book I, with introduction and notes, Cambridge 1920, p. 169, n. 26; *idem*, *The Thirteen Books of Euclid's Elements*, i, p. 252; and Ḥajjāj's translation in Nayrīzī, pt. 1, p. 58. Also, this *Journal*, XXXI, p. 16.

the construction is the same in both and the figures associated with it are identical—with the sole inessential difference of interchanging the positions of points E and Z.

The same objection which Qayṣar raised against Simplicius's proof in I.[5] could certainly be addressed in an identical manner to the demonstration of Proposition II.2. For the author of this demonstration provides no means for *constructing* a chord ED such that it falls on the far side of point A. He simply hopes that as he goes on drawing lines that join the extremities of multiples of AB on the sides of the angle DBE, one of these lines will fall beyond A. A proof is indeed required here that such a line exists, exactly as Qayṣar pointed out in his letter. The author of Document II appears to have anticipated the objection and, accordingly, has provided an answer: we will eventually go beyond A because 'the multiples taken are unlimited and the line AB is limited'. This, of course, is a futile application of the Eudoxus-Archimedes axiom.[66] For what this axiom would warrant is that by taking multiples of AB on BD, BE (in the figure for II.2), segments greater in length than AB will be cut off from BD, BE. The axiom does not guarantee that a line joining the extremities of any of these pairs of segments will fall beyond A. But the proposed way out of the difficulty seems to have influenced the formulation of Qayṣar's objection in II.[5]. As we have seen, he wrote that AB being 'infinitely divisible' it could accommodate an infinite number of chords by making the angle in question sufficiently large. The *infinite divisibility* of AB thus appears to have been adduced to counteract the statement in II.2 that AB is *limited* in magnitude.

So far we have had no difficulty in establishing a correspondence between all the main points in Simplicius's proof as described in Document I and features of the demonstration in Document II. But there remains a part of the latter demonstration to which nothing corresponds in Document I. This is the part consisting of Propositions 3 and 4. Why does Qayṣar not say a word about these propositions if indeed he had the demonstration in Document II before him when he composed his letter to Ṭūsī? The answer is obvious. Three special cases of Euclid's Postulate 5 are considered in Document II. Propositions 3 and 4 merely reduce two of these cases to the case proved in Proposition 1. Euclid's postulate will therefore have been proved if the demonstration of Proposition 1 could be made to work. It thus makes sense that Qayṣar should concentrate his attention on this proposition. It may be pointed out, moreover, that his description clearly implied that Simplicius's demonstration consisted of *more than one* proposition.[67]

In the light of the preceding analysis, I believe one is entitled to conclude that when Qayṣar wrote his description of the proof which he found in Simplicius's *Commentary* he was referring to a demonstration identical with the one we have in Document II.

We cannot, however, ignore the fact that the four propositions in Document II were transcribed from 'the Book of Euclid by Muḥyi 'l-Dīn al-Maghribī al-Andalusī'. According to the copyist, the propositions occurred after Proposition 24 in that book; and to be even more helpful he quotes the opening words in this proposition. The author referred to here is no doubt

[66] See below, note 80.

[67] See Document I. [1] and p. 9 above.

Muḥyi 'l-Dīn Yaḥyā ibn Muḥammad ibn Abi 'l-Shukr al-Maghribī al-Andalusī, a thirteenth-century mathematician, astronomer and astrologer, many of whose works in these fields are extant.[68] His double *nisba*, 'al-Maghribī al-Andalusī', indicates his origin. He was in Syria, in the service of the Ayyūbid al-Malik al-Nāṣir, when, in 1260, he was captured by Hūlāgū's men. In retaliation for a recent defeat of the Mongol army, al-Nāṣir, his brother al-Ṭāhir, and subsequently their immediate entourage, were drawn into an ambush and massacred. Al-Maghribī, however, was spared on the plea that he was a man who had knowledge of the stars and who could therefore be of use to the 'lord of the earth'.[69] Hūlāgū had already been persuaded of the usefulness of astrology by Nāṣir al-Dīn al-Ṭūsī whom he had allowed to start work on a new series of astronomical observations at Marāgha. He ordered Maghribī to be sent to Marāgha to join Ṭūsī's team.[70] The story of this narrow escape was told by Maghribī himself to Barhebraeus in Marāgha.

The 'Book of Euclid' mentioned by the copyist must be one of those revised editions of the *Elements* which it was not unusual for serious students of geometry to prepare. In the Islamic world these redactions were given such titles as *Taḥrīr* (recension), *Mukhtaṣar* (summary), *Iṣlāḥ* (emendation), etc., and they formed a distinct group from another category, the *Shurūḥ* (commentaries) which might treat of the whole of the *Elements* but more often were confined to a part of it. Now among the extant works of al-Maghribī there is one *Taḥrīr Uqlīdis* or *Taḥrīr Uṣūl Uqlīdis* (Recension of the Elements of Euclid) of which two manuscript copies have been known to exist in Istanbul.[71] Some time ago I was able to identify a third copy at the Bodleian Library which turned out to be older than either of the Istanbul manuscripts.[72] The introduction explains how the author came to write this work. The *Elements*, he

[68] Cf. Suter, *Mathematiker, ed. cit.* (note 50), no. 376, pp. 155–6; Krause, 'Stambuler Handschriften', *loc. cit.* (note 48), pp. 505–6; Brockelmann, *Gesch. d. arab. Lit.*, i², Leiden 1943, p. 626, Suppl. i, Leiden 1937, pp. 868–869. See also Max Krause, *Die Sphārik von Menelaos* . . . (*Abhandlungen der Gesellschaft der Wissenschaften zu Göttingen.* Philologisch-historische Klasse, 3 Folge, no. 17), Berlin 1936, pp. 74ff., where the most complete bibliography on Maghribī is to be found.

[69] Barhebraeus, *Ta'rikh mukhtaṣar al-duwal*, ed. Ṣālḥānī, Beirut 1890, pp. 489–90.

[70] *Géographie d'Aboulféda* [i.e. *Kitāb Taqwīm al-buldān*], texte arabe publié . . . par M. Reinaud et M. Le B^on MacGuckin de Slane, Paris 1840, p. 399; *Géographie d'Aboulféda*, traduite de l'arabe en français et accompagnée de notes par M. Stanislas Guyard, tome ii, seconde partie, Paris 1883, p. 151; Barhebraeus, *op. cit.* (note 69), p. 501. According to Ṣafadī, *Wāfī*, i, ed. Ritter, Istanbul 1931, p. 182, work on the new astronomical tables, the *Zīj-i Īlkhānī*, began in 1259; see also Ibn Shākir al-Kutubī, *Fawāt al-Wafayāt*, ii, Cairo 1951, p. 311. Cf. E. S. Kennedy, 'A Survey of Islamic Astronomical Tables', *Trans. of the Amer. Philos. Soc.*, New Series, vol, xlvi, pt. 2, Philadelphia 1956, no. 6, p. 125; no. 108, p. 137. In the introduction to the *Zīj-i Ilkhānī* Ṭūsī does not mention Maghribī among those who assisted him in its preparation; see J. A. Boyle, 'The Longer Introduction to the *Zīj-i Īlkhānī* of Naṣīr al-Dīn al-Ṭūsī', *Journal of Semitic Studies*, iii, 1963, p. 247.

[71] Cf. Krause, 'Stambuler Handschriften', *loc. cit.* (note 48), p. 506, no. 11.

[72] *Bibliothecæ Bodleianæ codicum manuscriptorum orientalium catalogi partis secundæ volumen secundum arabicos complectens. Confecit Alexander Nicoll. Editionem absolvit et catalogum Urianum aliquatenus emendavit E. B. Pusey*, Oxford 1835, Cod. Arab. Moham. no. CCLXXX, pp. 260ff. The MS. lacks the title; the catalogue quotes the introduction but does not identify the author whose name does not appear in the MS. For dates of the three MSS. of Maghribī's *Taḥrīr* see edition of Document III at the end of this article.

says, suffers from certain imperfections affecting both its matter and its form. Nevertheless, in the whole time that had elapsed since Euclid no one had produced a satisfactorily emended version. Maghribī himself had seen three editions of the *Elements*: one by Avicenna, another by a certain Nīsābūrī, and a third by Abū Ja'far al-Khāzin. Avicenna had left out the enunciations and many of the lemmas (?) (*muqaddamāt*), and he refrained from resolving doubts (*shukūk*: ἀπορίαι). Al-Nīsābūrī did likewise; he also made redundant additions whilst omitting important matters. Al-Khāzin did include the enunciations, but he altered the number of propositions and re-arranged their order; he, too, ignored the *shukūk*.[73] Maghribī's aim in writing his own *Taḥrīr* was to produce a truly improved edition which explained matters in need of explanation, omitted redundancies, answered objections and added lemmas (*muqaddamāt*) required for certain demonstrations.

In a book of this description it is not surprising to find a proof of Euclid's postulate. But nowhere in it do we find a proof *identical* with that of Document II, neither after Proposition 24 as we are told by the copyist of that document, nor elsewhere. In fact the proposition quoted in part by this copyist occurs in Maghribī's text as Proposition 27 (*kz*): If a straight line falling on two straight lines at random makes the alternate angles equal to one another, or the exterior angle equal to the interior angle, then the two (lines) are parallel. This corresponds to Eucl. I. 27 and part of Eucl. I. 28. But what is intriguing is that, following Proposition 34 (*ld*),[74] we do find a proof of the Euclidean postulate which is based on Simplicius's proof and which obviously aims to obviate the sort of objection raised by Qayṣar in his letter to Ṭūsī. But Maghribī mentions neither Simplicius nor Qayṣar. Let us turn to his proof.

DOCUMENT III

Extract from the *Recension of the Elements of Euclid* by Muḥyi 'l-Dīn ibn Abi 'l-Shukr al-Maghribī

Lemma

If a line falling on two straight lines makes the two angles on one of the two sides less than two right angles, then the two lines, if produced indefinitely on that side, meet.

Example:[75] The line AG has fallen on the two lines AB, GD, making the two angles BAG, DGA less than two right angles:

I say that if the two lines are indefinitely produced, they meet.

Demonstration: If one of the two angles is right, we complete the demonstration as will be shown.

But if not, we produce DG indefinitely and let fall on it the perpendicular AE (12)[76] which we then produce indefinitely on the side of T.

[73] Of the three editions mentioned by Maghribī only that of Avicenna is known to be extant in several manuscripts of his *al-Shifā'*.

[74] The enunciation of this proposition is quoted below, p. 17.

[75] *Mithāl*, which in this context corresponds to ἔκθεσις, the setting out of the conditions of the problem in a concrete figure.

[76] Such numbers in parentheses refer to preceding theorems in Maghribī's book. Throughout this proof the theorems referred to coincide with propositions bearing the same numbers in Bk. I of the *Elements* in Heiberg's edition and in Heath's translation; they will not therefore be quoted.

Then, on the point A of the line AE, we construct the angle KAE equal to the angle BAE (23);

and we indefinitely produce the lines AB, AK in the directions of B, K (Post.)[77] [i.e. Eucl. Post. 2 = Maghribī's Post. 2];

we mark on AB the point L and cut off AM equal to AL (3) and join ML.

Then, because the angles KAE, BAE are acute (Post.),[78] the line ML cuts AT in N;

and since the two sides AL, AM are equal, and the side AN is common, and the two angles at A are equal, then the two angles N in the two triangles ANL, ANM are right (4 and Post.).[79]

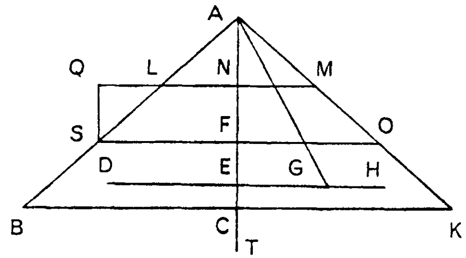
Now if point N lies between the points E, T, we complete the construction;

otherwise, we cut off the lines MO, LS equal to AM, AL (3), and join SO which cuts AT in F.

We then prove as before that the two angles F in the two triangles AFO, AFS are right angles (4).

Now if point F falls between the points E, T, that will be sufficient for us;

otherwise, we produce the line NL indefinitely and from point S draw SQ perpendicular to it (12).



Then, since the angle ANL is right, and the angle LQS is right, and the angles ALN, SLQ are equal (15), and the lines AL, LS are equal, then the lines AN, QS are equal (26).

Further, since the angle QNF is right, and the angle NFS is also right, as we showed, and the angle Q is right,

then the surface FQ is a parallelogram,

and, therefore, the lines QS, NF are equal (34).

We continue with this construction, cutting off from AT multiples (*amthāl*) of the line AN until we reach a multiple (of AN) greater than AE;[80]

let it [this multiple] be AC and let the line which has cut off AC from AT be the line KC.

[77] Occurrences of the abbreviation 'Post.' (*mṣ* = *muṣādara*) are not specified by numbers. Moreover, 'Post.' may refer to a definition or an axiom as well as to a postulate proper. I have added interpretations in square brackets or in footnotes by references to the *ḥudūd* (definitions), to *al-ashyā' allati taḥtāj ila 'l-ittifāq 'alayhā* (the things that need to be agreed upon, or postulates), and to the *'ilm 'āmm muttafaq 'alayh* (common knowledge that is agreed upon, or axioms) at the beginning of Maghribī's book.

[78] I.e. Maghribī's 'definition': The angle that is smaller than a right angle is called acute.

[79] The 'postulate' referred to here is Maghribī's 'definition': If a straight line stands on a straight line making the two angles on both sides equal to one another, then each of the angles is a right angle.

[80] Here Maghribī applies the Eudoxus-Archimedes axiom which, in contrast to Euclid's *Elements*, he explicitly states among the common notions: For any two magnitudes of which one is greater than the other, if the smaller is multiplied indefinitely (*mirāran ghayra mutanāhiya*) it will reach by multiplication a magnitude greater than the given, greater magnitude.

Then, the angles at C are right—as we showed for the angle ANL,

and the angles at E are also right,

therefore, ED does not meet CB (28), nor does it meet AE (Post.),[81]

therefore, it necessarily meets AB.

It is evident from this and from *kḥ* [Eucl. I. 28] that every two lines in a plane surface are either secant or parallel;

for if a straight line falls on them, making the two angles on one of the two sides less than two right angles, they meet;

but if it makes (the angles) equal to two right angles, then the two lines are parallel.

Thus the doubt concerning this question has been removed by virtue of what we have posited as an emendation of the Postulate [the premisses ?]

As in Document II Maghribī attempts to prove the special case described in II.2 to which the two remaining cases are reduced. But in carrying out this proof he shows himself fully aware of the difficulty pointed out by Qayṣar. Maghribī in fact provides precisely that which was lacking in Simplicius's demonstration: a method for constructing the line ZD in the figure for Document I, or ED in the figure for II.2, or KB in Maghribī's figure. Here he relies on the Eudoxus-Archimedes axiom which Document II tried, unsuccessfully, to apply.

The reader will have noticed that at a crucial stage in the proof, where Maghribī concludes the equality of QS and NF, he refers to Eucl. I. 34: 'In parallelogrammic areas the opposite sides and angles are equal to one another, and the diameter bisects the areas.' But for the demonstration of this proposition Euclid relies on I.29 which in its turn presupposes Postulate 5. Maghribī was not, however, guilty of an unconscious *petitio principii*. His book does *not* include the parallels postulate among the premisses; but one of his 'definitions' reads as follows: 'Parallel straight lines are those that lie in the same plane surface and are such that if a straight line falls on any two of them at random it makes the two angles on one side equal to two right angles.' The latter part of this statement is part of what Euclid proves in Proposition 29 on the basis of Postulate 5. Maghribī's 'solution' thus consists in detaching this part and establishing it as a premiss which he strengthens with the Eudoxus-Archimedes axiom in the process of his demonstration of the parallels postulate. It may be remarked here that a common feature of the medieval Arabic treatment of this problem was to deduce Postulate 5 from equivalents to it. In consequence of this, Arabic mathematicians were barred from getting a glimpse into a non-Euclidean system. A crucial step was taken when Saccheri set out to deduce the postulate *directly from its negation*. A study of the history of this problem in Arabic geometry could thus provide an instructive example of those 'obstacles épistémologiques' which have often played a negative role in the progress of scientific thought, but the problem of identifying the 'obstacle' in the present case cannot be discussed in this article.

The puzzle that remains is that posed by the copyist's statement at the end of Document II. How do we explain the fact that we do not find in Maghribī's book the four propositions which are stated to have been copied from it? In

[81] I.e. Maghribī's 'axiom': Two straight lines cannot enclose a surface.

the absence of evidence we can only guess, and one guess in particular seems to me not implausible. The copyist may have seen the propositions in an earlier version of Maghribī's *Taḥrīr* which the author revised after he had become acquainted with the objection raised by Qayṣar. The three extant manuscripts of this book would therefore be copies of the revised edition. In the original edition, Maghribī may or may not have attributed the four propositions to Simplicius. For the copyist simply tells us that they occurred at a certain place in the book. The verb he used, *awrada,* is quite neutral and it neither means nor necessarily implies that Maghribī was the author of the propositions in question. In any case we cannot, in the light of Document I, ascribe them to him.

In considering this suggestion it must be remembered that our three authors were not only contemporaries but also were in touch with one another. Qayṣar informed Ṭūsī of the character of Simplicius's proof and pointed out a difficulty. It is not unlikely that the idea in Qayṣar's letter should have at one time or another been passed on to Maghribī who, we know, collaborated with Ṭūsī. This would mean that Maghribī prepared the extant version of his *Taḥrīr* after he joined Ṭūsī at Marāgha in 1260.[82] But I should not wish to insist on this conjecture. Other possibilities, of course, exist. Maghribī may have learnt of Qayṣar's objection while in Syria, before he went to Marāgha; or he may have thought of the difficulty on his own. One can only hope that further documents will come to light which will help to resolve this question and others connected with it. But what we need to explain is not only the fact that we do not find the proof of Document II in Maghribī's *Taḥrīr,* but also the fact that we do find in that book a proof designed to justify the principal proposition in Document II.

Postscript: In my article on Thābit ibn Qurra (this *Journal,* XXXI, p. 15, n. 11) I quoted from the Laurentian Library MS. Or. 50 the date of completion of 'Ṭūsī's shorter version' of the *Elements.* The date, which does not figure in the Rome edition of 1594, is: Saturday, 10 Muḥarram 698/18 October 1298. I noted the impossibility of this date, since Ṭūsī died in 672/1274, and I suggested that 698 might have been a mis-transcription of 668/1269. It has been kindly pointed out to me by H. Hermelink and B. A. Rozenfeld that my suggestion overlooks the fact that 10 Muḥarram 668 was not a Saturday. The suggestion should be dropped. But then, given the quoted date, Ṭūsī's authorship of this version of the *Elements* must be put in doubt. I now realize that it has already been doubted on other grounds by B. A. Rozenfeld, A. K. Kubesov and G. S. Sobirov in their article 'Kto buil avtorom rimskogo izdaniya "Islozheniya Evklida Nasir ad-Dina at-Tusi"', in *Voprosui istorii estestvoznaniya i tekhniki,* xx, 1966, pp. 51–53.

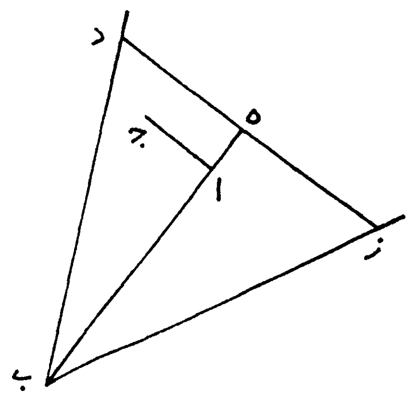
[82] Maghribī does not, however, refer to Ṭūsī's 'longer version' of the *Elements* which had been completed in 1248.

DOCUMENT I

Extract from a letter of
'Alam al-Dīn Qayṣar ibn Abi 'l-Qāsim al-Ḥanafī
to Naṣīr al-Dīn al-Ṭūsī

Sigla of manuscripts: A = Aya Sofya 2760, fol. 9^{r-v}, 845h; C = Carullah 1502, fol. 12^{r}, 894h; F = Fatih 3440, fol. 172^{r-v}, 671h; H = Hyderabad edition, *Rasā'il al-Ṭūsī,* ii (1359h), *Ris.* no. 8, p. 37; P = Bibliothèque Nationale, arabe 2467, fol. 87^{v}, 16th c. A.D.

ومما يعرض على الآراء العالية ما وقع لي في قضية ذكرها سنبليقيوس
في شرحه لمصادرات كتاب الأصول في مقدمات القضية المشهورة وهي ما إذا وقع
خط مستقيم على خطين مستقيمين فصير الزاويتين الداخلتين في جهة واحدة مساويتين
لأقل من قائمتين فإن الخطين إذا أخرجا في تلك الجهة التقيا
فقال كل زاوية يمكن أن يوجد لها أوتار لا نهاية لكثرتها بعضها أعظم
من بعض، وكل واحد منها يفصل من الخطين المحيطين بتلك الزاوية متساويين،
واستعمل ذلك فيما إذا وقع خط اب على خطي بد اج وكانت زاوية جاب
قائمة وزاوية ابد حادة فإن خطي اج بد يلتقيان في جهة ج د،

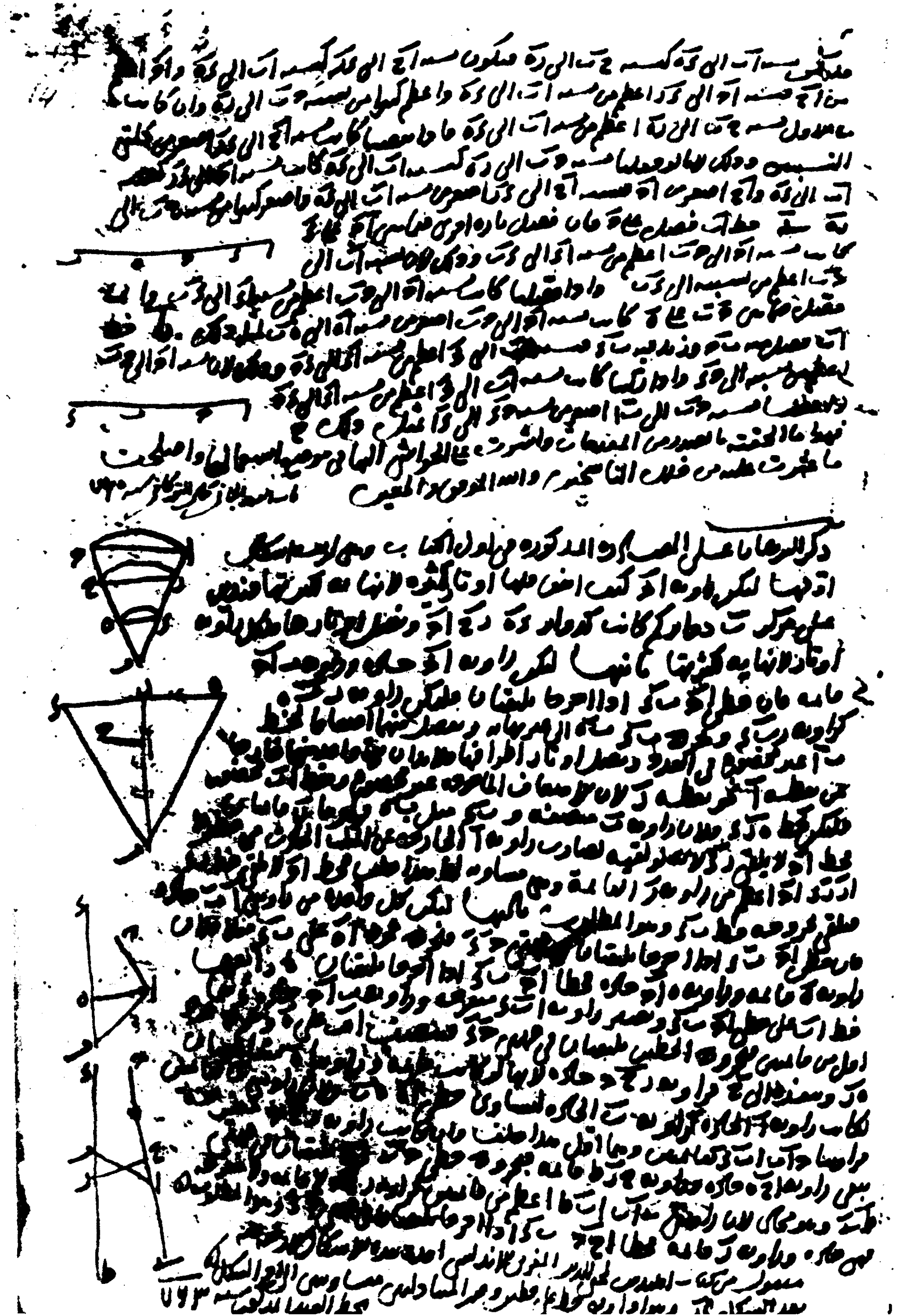


بأن عمل على نقطة ب من خط اب
زاوية ابز مساوية لزاوية ابد،
فزاوية دبز يوترها أوتار لا
نهاية لكثرتها بعضها أعظم من بعض،
فيقع أحد الأوتار خارجاً عن نقطة ا،
مثل وتر زهد، فتكون زاويتا ا ه
قائمتين، فخط اج إذا أخرج لا

يلقى خط هد فيلقى خط باد
فعلى تقدير أن يكون خط باد في مبدأ زواله عن استقامة
18 خط باز فإن كل وتر يوتر زاوية زباد يقع فيما بين نقطتي
آ ب، إذ آب ينقسم إلى غير نهاية
21 فإن أمكن أن يوجد برهان يدل على وقوع أحد الأوتار
خارجاً عن نقطة آ ليحصل المطلوب، فيضيف مولانا إلى سابق
فوائده منعماً متفضلاً

1 سينليتيوس P 2 وهي : وهو ACFP | ما : اما A
3 متساويتين F 4 أخرجا : اخرج C خرجا P 5 كل : om. P | 6-12 وكل واحد ... من بعض : om. P | نهاية : + لها H
6 متساويتين AF 7 فيما إذا : فما اذا A فما اذا (?) C فما اذا F | 9 جاب : ب اج F
بأن عمل : مان عمل A بان عمل C مانه عمل F فان عمل H 12 نهاية : + لها H | وبعضها H 13 فسيقع (?) A فستقع C فسيقع F | خارجة F
16 باد : هد P 17 فعلى : فعلى P | تقدير : تقدر A تقدر C تعدر P | في مبدأ زواله عن : في مبداء زواله عن C فى مبدا زواله عن A فى مبدا زواله عن F فى مبدأ زواله على H فى مبداء زاله P
18-19 آ ب : نقطتي آ نقطة F 19 إذ آب : om. C
21 ليحصل : ليحصل C | فيضيف : فنصف (?) A فتصيف C فيضيف F فتضيف H فنصف P | مولدنا : مرللا (?) C حرلاما P
22 متفضلاً : مفصلا C

Document II
Oxford, Bodleian Library, MS. Thurston 3, fol. 148r (*p.* 10)



DOCUMENT III

Extract from *Taḥrīr Uṣūl Uqlīdis* by
Muḥyi 'l-Dīn Yaḥyā ibn Abi 'l-Shukr al-Maghribī

Sigla of manuscripts: A = Aya Sofya 2719 (not 1719), pp. 21–23 (p. 1 being the title-page), 714h; B = Bodleian Library, Or. 448, fols. 9ᵛ–10ʳ, 659h, Marāgha; M = Mihrişah 337, pp. 21–22 (p. 1 being the title-page), 1159h.

مقدمة

إذا وقع خط على خطين مستقيمين فصير الزاويتين اللتين في إحدى
الجهتين أصغر من قائمتين فإنهما إذا أخرجا في تلك الجهة
التقيا

مثاله خطا اب جد وقع عليهما خط اج فصير زاويتي
باج دجا أقل من قائمتين

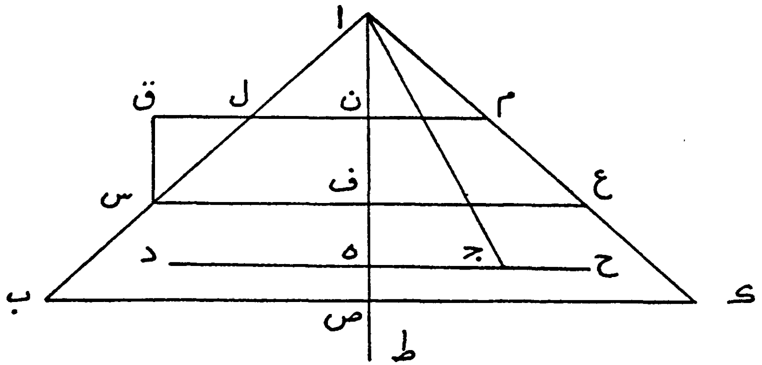
فأقول إنهما إذا أخرجا بغير نهاية التقيا

برهانه إن كانت إحديهما قائمة فإنا نتمم البرهان كما
سيأتي ذكره

وإن لم تكن قائمة أخرجنا دج بغير نهاية وأسقطنا
عليه عمود اه (يب) ونخرجه بغير نهاية من جهة ط

ونعمل على نقطة ا من خط اه زاوية كاه كزاوية
باه (كج)

ونخرج خطي اب اك في جهتي ب ك بغير
نهاية (مص)



ونعلم على اب نقطة ل، ونفصل ام مثل ال (ج)، ونصل م ل

فلأن زاويتي ك اه باه حادتان (مص) فخط م ل يقطع اط على ن

فمن أجل أن ضلعي ال ام متساويان وضلع ان مشترك وزاويتي ا متساويتان تكون زاويتا ن من مثلثي ان ل ان م قائمتين (د و مص)

فإن كانت نقطة ن فيما بين نقطتي ه ط تممنا العمل

وإلا فصلنا خطي م ع ل س مثل ام ال (ج)

ووصلنا س ع، يقطع اط على ف

ونبين كما بينا أن زاويتي ف من مثلثي اف ع اف س قائمتان (د)

فإن وقعت نقطة ف فيما بين نقطتي ه ط كفانا ذلك

وإلا فنخرج خط ن ل بغير نهاية ونخرج إليه من نقطة س عمود س ق (يب)

فمن أجل أن زاوية ان ل قائمة وزاوية ل ق س قائمة وزاويتي ال ن س ل ق متساويتان (يه) وخطي ال ل س

متساويان يكون خطا ان ق س متساويين (كو)

وأيضاً فلأن زاوية ق ن ف قائمة وزاوية ن ف س أيضاً قائمة كما بينا، وزاوية ق قائمة

فسطح ف ق متوازي الأضلاع

فخطا ق س ن ف متساويان (لد)

ولا نزال نعمل بمثل هذا العمل فنفصل من اط أمثالاً لخط ان حتى ننتهي إلى أمثال له هي أعظم من اه، وليكن اص (مص)

والخط الذي فصل اص من اط هو خط ك ص

فزوايا ص قائمة كما بينا في زاوية ان ل

وزوايا ه قائمة أيضاً

فخط ه د لا يلقى ص ب (كح) ولا يلقى اه (مص)

فهو يلقى اب ضرورةً

وقد استبان من هذا ومن كح أن كل خطين في بسيطٍ مستوٍ فهما متلاقيان أو متوازيان

لأنه إذا وقع عليهما خط مستقيم فإنه إن صير الزاويتين اللتين في إحدى الجهتين أصغر من قائمتين تلاقيا، وإن صيرهما كقائمتين فالخطان متوازيان (مص)

فقد زال الشك عن هذا المطلب بما وضعنا من إصلاحنا

المصادرة

2 زاوية دجا ان كانت قائمة : برهانه ... قائمة 8 M وصير : فصير

10 دج : دجا M | برهانه ان A | إحديهما : inser. B

10–11 ونخرجه ... بغير نهاية : mg. B 11 يب : om. B 14 خطا M

15 مص : om. B 18 مص : om. AM inser. B 20 ضلعي:

زاويتا : متساويين | M | مشتركه AM | 21 متساويتين : M |
26 زاويتي 22 د و مص : د بج و مص A
29 مثلث AM | 27 قائمتين M | د : inser. B . د بج A
33 إليه : عليه B 30 بب : inser. B 32 يه : om. B
متساويين
+ آنل متساويان : AM | كد : om. B 35 كما بينا :
زاوية
فسنفصل في A 37 لد : om. B 38 فنفصل :
AB | أمثالاً : امثال B 39 اص
فزوايا اه اعظم من امثالا اط الخط الذى فصل هو كص وليكن : om. M
خط وليكن add. mg. B | مص : inser. B 40-41 فزوايا ...
اص فصل الذى والخط : فزوايا اه اعظم من امثالا اط الذى فصل
الخط هو
45 كح صب ك خط وليكن M 43 كح : om. B
كح ومن : om. A mg. B 45-46 وقد استبان ... متوازيان :
متلاقيين فهما غير متوازيين
متوازيين فهما متلاقيان فان كل خطين
46 غير خطين كل ان هذا من استبان وقد A
متلاقيان او متوازيان
50 اما متوازيان او متلاقيان : متلاقيان او متوازيان M
المطلب : المطلوب A المطلب mg. A | وضعنا : وضعناه AB |
اصطلاحنا : إصلاحنا M

XIV

AN ELEVENTH-CENTURY REFUTATION OF PTOLEMY'S PLANETARY THEORY

I. INTRODUCTION

Two medieval passages purporting to constitute a refutation of the Ptolemaic models for the moon and the five planets are here presented in translation. Their author, the eleventh-century polymath al-Ḥasan Ibn al-Ḥaytham (Alhazen, d. ca. 1040) was well versed in Ptolemaic astronomy to which he devoted a sizable portion of his writings including a mathematical commentary on the *Almagest*, a descriptive account of the planetary motions in terms of the physical spheres associated with them, and a number of short treatises dealing with specific topics[1]. The two passages come from a work entitled *al-Shukūk ʿalā Baṭlamyūs* ("Dubitationes in Ptolemaeum"), which Ibn al-Haytham must have composed late in his career[2]. The commentary to the *Almagest*, apparently an earlier effort, was intended to be simply an explanatory introduction paralleling the thirteen books of Ptolemy's treatise. The descriptive *Configuration of the World*[3] is in the tradition of Ptolemy's *Planetary Hypotheses*. In neither of these works, nor in a treatise "On the Motion of the Moon"[4], do we find an inkling of the objections formulated in the *Shukūk*. The first sign of a critical attitude appears in a reply to an unidentified scholar

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[1] For a bibliography of Ibn al-Haytham's works see my article in *Dictionary of Scientific Biography*, ed. C. C. Gillispie, vol. VI (New York, 1972), p. 189–210. The commentary on the *Almagest* in extant in a unique manuscript (Seray. Ahmed III 3329, Istanbul) which is incomplete, comprising only five out of thirteen books. See Additional Work no. 3 in the *DSB* bibliography. *

[2] A critical edition of the *Shukūk* based on two MSS, has been published by A. I. Sabra and N. Shehaby (Cairo, 1971), See no. III 64 in the *DSB* bibliography.

[3] The Arabic title is *Maqāla fī Hay'at al-ʿālam*. Hebrew and Latin translations of this work were made in the Middle Ages. A Latin version from the thirteenth or the fourteenth century is published in Millás Vallicrosa, *Las traducciones orientales en los manuscritos de la Biblioteca Catedral de Toledo* (Madrid, 1942), App. II, pp. 285–312. See no. III 1 in the *DSB* bibliography.

[4] No. III 82 in the *DSB* bibliography.

who had written a criticism of Ibn al-Haytham's treatise on the movement of *iltifāf*[5]. This is the movement assumed in *Almagest* (XIII, 2) to account for the latitudinal deviations of the five planets and generated by the oscillation of the planes of the epicycles[6]. We learn from Ibn al-Haytham's reply that in his earlier treatise on the movement of iltifāf (now lost), he had expressed dissatisfaction with Ptolemy's treatment or rather omission of this movement in the *Planetary Hypotheses*, and that he had proposed a new physical arrangement which, he believed, successfully produced it. Also in this reply he made known his intention to compose a critique exposing "errors" and "contradictions" which he had noted in three of Ptolemy's works: the *Almagest*, the *Planetary Hypotheses* and the *Optics*. There can be no doubt that it is this critique which has come down to us under the title *al-Shukūk ʿalā Baṭlamyūs* and in which the translated passages occur.

Ibn al-Haytham's criticisms are remarkable in that they are supported by an explicit rejection of Ptolemy's authority. In the same reply he characterized the attitude of his correspondent, who apparently had frowned upon this questioning
* of the authority of Ptolemy, as that of the Traditionalists (*aṣḥā bal-ḥadīth*), not that of the mathematician. It was appropriate for a Traditionalist to follow authority, but nothing less than a demonstration should satisfy the mathematician. At the beginning of the *Shukūk* he briefly formulates a theory of scientific research in which he rejects the doctrine of the manifestness of truth, asserts the fallibility of men, even great men, and prescribes criticism of one's own views and those of others as the only method of advancing scientific knowledge[7]. But apart from the underlying critical attitude, the objections formulated by Ibn al-Haytham, particularly those raised against the Ptolemaic models for the moon and the five planets, have come to acquire a certain significance in the history of Islamic science as a result of recent research mainly concerned with the work of astronomers in the thirteenth and fourteenth centuries. It will be the purpose of the remainder of this Introduction to indicate this significance.

In 1893 Carra de Vaux published a French translation of a chapter from the *Tadhkira* ("Memorandum") of the thirteenth-century Persian astronomer Naṣīr al-Dīn al-Ṭūsī (d. 1274), in which the author proposed non-Ptolemaic models for

[5] The reply, No. III 63, is entitled "Solution of the Difficulties (*shukūk*) Concerning the Movement of *Iltifāf*". It is extant in two MSS, one in Leningrad, no. 192, fols. 1r–20v, the other in Istanbul, Atif 1714, fols. 139r–148v. Ibn al-Haytham's "Treatise on the Movement of *Iltifāf*" (no. III 61) is not known to have survived. It was, however, known to Naṣīr al-Dīn al-Ṭūsī in the thirteenth century and, possibly, to Ibn al-Shāṭir in the fourteenth century.

[6] The name "iltifāf", which means turning or enveloping, seems to refer to the motion of
* the epicycle's two diameters, passing through apparent apogee and trough mean distances respectively, round a small circle which in the case of the first diameter is perpendicular to the eccentric, and in the case of the second, perpendicular to the ecliptic. Al-Bīrūnī calls it *iltiwā'*, twisting; see his *Elements of Astrology*, ed. and trans. R. Ramsay Wright (London, 1934), p. 103.

[7] See S. Pines, "Ibn al-Haytham's Critique of Ptolemy", *Actes du X^e Congrès international d'histoire des sciences* (Ithaca, 1962; published Paris, 1964), I, p. 547–550.

the moon, Venus and the three superior planets[8]. The new models readily made use of deferents and epicycles. But while Ptolemy had to assume a point, the equant, from which the unequal motion of the epicycle in its deferent appeared to traverse equal arcs in equal times, Ṭūsī's models successfully represented the apparent motion of a planet as a combination of uniform motions. Thus they strictly adhered to the uniformity principle which Ptolemy's corresponding constructions had manifestly violated. To achieve this, Naṣīr al-Dīn introduced a device, now known as the "Ṭūsī couple", which consisted of a circle tangentially placed inside another circle twice its size. He proved that if the circles rotate about their centres in opposite senses, the smaller with a velocity twice that of the larger, then any given point on the smaller circle will continually move up and down the diameter of the larger circle passing through that point. By means of this device Ṭūsī was able to conceive of the apparent motion of a planet as the resultant motion of the end-point in a series of connected vectors each rotating with constant angular velocity. Or, to put the matter in Ṭūsī's own terms, the planet's apparent motion was ultimately produced by a series of uniformly moving spheres. Such a system for planetary motions was obviously in accord with the uniformity principle. Only the difficult case of Mercury was left out of the *Tadhkira*, and whether or not Ṭūsī was elsewhere able to deal successfully with this case is not known.

Carra de Vaux recognized the would-be originality of Ṭūsī's endeavor, but the lesson he drew from it emphasized quite a different character of the scientific achievement in medieval Islam. In his view, Ṭūsī's attempt to replace the Ptolemaic models by others more in agreement with an accepted principle of ancient astronomy in fact reveals a certain poverty and niggardliness ("mesquinerie") of Arabic science. The Islamic astronomers, he observed, did not lack a critical spirit, but their criticism did not cut deep enough and, as a result, they were not able to go very far. If they failed to produce a Copernican revolution before Copernicus, this, Carra de Vaux concluded, was due to their lack of that gift, that "power of genius" which earlier manifested itself in the achievements of the Greeks and later in the work of Renaissance and seventeenth-century thinkers.

More recently, however, the work of Ṭūsī and his successors has been viewed in a new light. First it was noted that a lunar model devised by the fourteenth-century Damascene astronomer Ibn al-Shāṭir (d.ca. 1375) was identical with that presented in Copernicus' *De revolutionibus*, except for slight differences in parameters[9]. Soon it was realized that other striking similarities existed between the planetary theories of Copernicus and Ibn al-Shāṭir, and the continuity of the latter's endeavors with those of Ṭūsī and his collaborators and pupils at the Marāgha

[8] Carra de Vaux, "Les sphères célestes selon Nasîr-Eddîn Attûsî", Appendice VI in P. Tannery, *Recherches sur l'histoire de l'astronomie ancienne* (Paris, 1893), pp. 337–361.

[9] This was pointed out by O. Neugebauer on the basis of a study of Ibn al-Shāṭir's models by Victor Roberts. See Victor Roberts, "The Solar and Lunar Theory of Ibn al-Shāṭir: a Pre-Copernican Copernican Model", *Isis*, XLVIII (1957), 428–432.

observatory, the so-called "Marāgha School", also became apparent[10]. For example, both Ibn al-Shāṭir and Copernicus make use of the "Ṭūsī couple", and in the planetary theories of all these astronomers, the equant is replaced by combinations of uniform motions. In the case of Ibn al-Shāṭir and Copernicus these combinations are in some respects identical. Most notable is the close similarity between their respective models for Mercury. In the light of these observations historians of astronomy have been led to investigate the question whether any of the writings of the Marāgha astronomers and Ibn al-Shāṭir could have reached Copernicus and through what channel.

The results of this recent examination of late medieval astronomy allows us to make at least one observation which is not without significance for the historical understanding of Copernicus' work. They demonstrate that a dissatisfaction with the equant, such as that cited by Copernicus as one of the reasons that had set him on the way to his new system, could and did in fact lead to planetary models essentially identical with their Copernican counterparts. That is to say, Copernicus' account of his own discovery must be taken more seriously than it might have been without our knowledge of what Ṭūsī and his successors had achieved.

But what about their significance for the history of Islamic astronomy? For, surely, the interest of the work done at Marāgha and Damascus does not lie solely in its possible influence on later developments in Europe. What motive or motives drove Ṭūsī and his successors to undertake the task of reforming planetary theory; what elements made up the problem-situation which they faced; to what extent was this problem-situation of their own creation; how should their work be characterized? It was questions like these, I think, which Carra de Vaux had in mind when he made the remarks referred to above. With regard to the question of motivation it has been recognized that the impulse behind the activity of the Marāgha School and Ibn al-Shātir was theoretical, not practical; that their reform did not stem from the pressure of new observations in conflict with the Ptolemaic models; that their aim was to remove a certain contradiction between Ptolemy's use of the equant and the principle of uniform motion to which Ptolemy himself subscribed[11]. Theirs was an act aimed at bringing astronomical theory closer to its ideal. But though the ideal had already been determined, declared and accepted, the decision to take it seriously enough to construct new planetary models cannot be taken for granted.

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[10] See E. S. Kennedy and Victor Roberts, "The Planetary Theory of Ibn al-Shāṭir", *Isis*, L (1959), 227–235. Further results and observations are contained in the following articles: Fuad Abbud, "The Planetary Theory of Ibn al-Shāṭir: Reduction of the Geometric Models to Numerical Tables", *Isis*, LIII (1962), 492–499; Victor Roberts, "The Planetary Theory of Ibn al-Shāṭir: Latitudes of the Planets", *Isis*, LVII (1966), 208–219; E. S. Kennedy, "Late Medieval Planetary Theory", *Isis*, LVII (1966), 365–378; Willy Hartner, "Naṣir al-Dīn al Ṭūsī's Lunar Theory", *Physis*, XI (1969), 287–304; *idem*, "Copernicus, the Man, the Work, and its History", *Proceedings of the American Philosophical Society*, CXVII (1973), 413–422.

[11] See E. S. Kennedy in *Isis*, LVII (1966), 366–368.

It is a fact to be explained that, as far as we know, in the thirteenth century, for the first time, a certain group of astronomers decided to resolve a difficulty which must have been apparent from the time of Ptolemy.

The problem lies, of course, in this word "apparent". For awareness of a difficulty may be combined with any one of a number of different attitudes. One may defend the difficulty as inevitable, or one may reinterpret the whole theoretical system in such a way that the difficulty ceases to exist as such, or one may simply ignore it[12]. Hence the important role which Ibn al-Haytham's criticisms must have played. By explicitly and forcefully stating the difficulties involved in Ptolemy's planetary models, and by insistently demanding a solution, he helped to inculcate an attitude which must be part of the explanation of the thirteenth-century astronomical reform.

Ibn al-Haytham's objections assume that astronomy is ultimately a theory of what actually exists in the heavens and not merely an instrument for making accurate predictions. He takes exception to Ptolemy's models because they attribute to the celestial spheres an irregular motion which, by their own nature, they cannot have. His criticisms have been described here as a "refutation" because he believed his a arguments to have shown the planetary arrangements in the *Almagest* to be *
false. They were false because they could not have physical existence, and the sensible motion of a physical body can only be produced by a physical arrangement. His final conclusion was that there existed a true arrangement which Ptolemy had failed discover[13]. This means that the problem he posed for astronomers was no *

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[12] Thus Ptolemy in the *Almagest* (IX, 2) proffers arguments in support of what might appear in his planetary theory to go against the rules (παρὰ τὸν λόγον), while competent astronomers like al-Battānī (ca. 877), al-Farghānī (d. after 861) and al-Bīrūnī (d. after 1050) say nothing about the equant as a problematic feature.

[13] Capitalizing on Ptolemy's apologetic statements towards the end of the *Almagest* (IX, 2), Ibn al-Haytham went as far as to say that Ptolemy had proposed a planetary theory which he knew was false (*Shukūk*, ed. cit., pp. 37–38). "Ptolemy", he added, "assumed an arrangement (*hay'a*) that cannot exist, and the fact that this arrangement produces in his imagination the motions that belong to the planets does not free him from the error he committed in his assumed arrangement, for the existing motions of the planets cannot be the result of an arrangement that is impossible to exist" (*ibid.*, p. 38). Again, he quotes Ptolemy's statement that "if something assumed without proof is later found to agree with the phenomena, then it could not have been discovered without following one of the ways of scientific knowledge, even though it would be difficult to describe how it was apprehended", and comments: "the way Ptolemy followed was indeed a legitimate beginning, but since it led him to what he himself admitted to be παρὰ τὸν λόγον he should have declared his assumed arrangement to be false (*ibid.*, p. 39). And again, at the end of the section dealing with the *Almagest*: "...[Ptolemy] gathered all that his predecessors and he himself had verified regarding the motions of each one of the planets. Then he searched for an arrangement that could belong to existing bodies having these motions. Having failed in his search he assumed an imaginary arrangement in terms of imaginary circles and lines to which these motions are attributed... But for a man to imagine a circle in the heavens and to imagine the planet moving in it does not bring about the planet's motion... And therefore the arrangements assumed by Ptolemy

longer to devise physical models that simply accommodated the motions described in the *Almagest*, as Ibn al-Haytham himself had done in the earlier *Configuration of the World* and as Peurbach would do in the *Theoricae*[14], but to invent, or rather discover, new models more in accord with physical reality. That is precisely the task which the Marāgha astronomers set themselves.

As far as I have been able to determine, Ibn al-Haytham is the only Islamic astronomer mentioned by name in Ṭūsī's *Tadhkira*. In the course of the discussion of physical models for planetary motions in latitude, Ṭūsī writes: "Ibn al-Haytham composed a treatise (*maqāla*) in which he described the bodies (*al-ajsām*) that produce these motions [of the diameters of the epicycles] adding for each epicycle two spheres (*kuratayn*) to account for the inclination [*mayl*, i.e. the inclination of the epicycle's diameter passing through the epicycle's apogee and perigee], and in the case of both inferior planets two more spheres for the slant [*inḥirāf*, i.e. the inclination of the epicycle's diameter passing through mean distances]..."[15]. The context of this and a later reference[16] makes it clear that the "treatise" mentioned must be Ibn al-Haytham's "Treatise on the movement of *iltifāf*" mentioned above. Thus a connection is established between Ṭūsī's and Ibn al-Haytham's physicalist programs, at least as regards planetary latitudes models. The same conclusion applies to Ibn al-Shāṭir who, in a similar context in his *Nihāya*, also refers to Ibn al-Haytham's "*Risāla*"[17]. And though Ibn al-Shāṭir may have been relying here solely on Ṭūsī's *Tadhkira*, it is interesting to note that when he enumerates Ptolemy's "successors", the only name he mentions before Ṭūsī is that of Ibn al-Haytham.

Whether or not Ṭūsī was also acquainted with Ibn al-Haytham's criticism in the *Shukūk* of Ptolemy's lunar and planetary longitudes models cannot be decided with equal certainty. He makes it clear, however, that he was aware of earlier objections against these models. He begins the discussion of the difficulties involved in the lunar model as follows:

An objection (*ishkāl*, problem) has been raised (*warada*) against the movement of the center of the (moon's) epicycle in the circumference of the eccentric about the center of the world and the inclination (*muḥādhāt*, prosneusis) of the (epicycle's) diameter towards a point other than the eccentric's center. The explanation of this is as follows: If the deferent moves the epicycle with a simple and uniform movement, then the distances of the epicycle's center from the (deferent's) center must always be equal, because of the equality of the angles at this center, and because the diameter passing through the apogee and perigee always points to it. Thus if one of these three things differs, this must be due to a composition in the motion. But we find that these matters differ in the case of the moon, for while the center of its epicycle keeps the same distance from the deferent's center, the angles are equal at the center of the world and the diameter is directed towards the point of prosneusis. Now the practitioners of the art [of astronomy] have not shown the manner in which this composition comes about, nor have they explained anything relating to this matter. I shall (later) set forth what I have on this subject, God willing[18].

for the five planets are false, and he asserted them knowing them to be false, and there exists for the planets a true arrangement in existing bodies which Ptolemy failed to grasp. For there cannot be a sensible, permanent, and ordered motion unless it has a true arrangement in existing bodies" (*ibid.*, pp. 41–42).

14 On the relation of Ibn al-Haytham's *Configuration* and Peurbach's *Theoricae* see W. Hartner, "The Mercury Horoscope of Marcantonio Michiel of Venice, a Study in the History of Renaissance Astrology and Astronomy", in A. Beer, ed., *Vistas in Astronomy*, I (London-New York, 1955), 84–138, esp. 122–127; reprinted in Willy Hartner, *Oriens-Occidens* (Hildesheim, 1968), pp. 440–495, esp. 478–483.

15 British Museum MS Add. 23, 394, fol. 30r.

16 *Ibid.*, fol. 30v.

17 For Ibn al-Shāṭir's *Nihāyat al-sūl* I have used an unpublished English translation by Victor Roberts based on Bodleian MSS Marsh 139 and Marsh 290.

In these words there are unmistakable echoes of the first text which we shall reproduce from Ibn al-Haytham's *Shukūk*.

Later in the *Tadhkira*, Ṭūsī reports that

In connection with the moon's prosneusis point (*nuqṭat muḥādhāt al-qamar*), a certain scientist (*baᶜḍ ahl al-ᶜilm*) maintained that there must be assumed for the moon an additional sphere whose center coincides with that point, so that the diameter of the [moon's] epicycle passing through mean apogee and perigee would, as a result of this sphere's motion, always point to its center. But he did not show (*lam yubayyin*) how this motion could take place without interfering with the existing motions (associated with) the moon[19].

The suggestion of a sphere whose center is the prosneusis point does not occur in the text of the *Shukūk*, where, in fact, no solutions are suggested for any of the difficulties raised. It is clear, however, that the scholar whose views are reported here was grappling with the kind of problem pointed out in Ibn al-Haytham's work. Ṭhūsī's researches can therefore be said to have started from a recognition of the problems formulated in the *Shukūk* as well as in the treatise on the motion of *iltifāf*, and already discussed among astronomers, even though he may not have been directly acquainted with the text of the former treatise. It can also be asserted that Ṭūsī was looking for solutions that would satisfy conditions laid down by Ibn al-Ḥaytham. *

Islamic astronomers are sometimes divided into two camps with different interests, approaches and research programs. One camp consisted of Mathematicians working in the tradition of the *Almagest*, the other gravitated toward considerations which properly belonged to natural philosophy. Al-Battānī, al-Bīrūnī, and the Marāgha astronomers, for example, belonged to the first, Ibn al-Haytham to the second. The distinction is sometimes merged into another distinction, which does reflect a situation that obtained in twelfth-century Muslim Spain, between followers of Ptolemy and followers of Aristotle. This way of looking at the history of astronomy breaks down when we remember that Ptolemy was the author of the *Planetary Hypotheses* as well as the *Almagest*. When Ibn al-Haytham compared the results

18 British Museum MS Add. 23, 394, fol. 17v. Carra de Vaux, *op. cit.*, pp. 343–344.

19 British Museum MS Add. 394, fol. 29r. Carra de Vaux, *op. cit.*, p. 353. The word "baᶜḍ" (some) could refer to one or more persons, but Tūsī later uses the third person singular.

of these two works, arguing that either the representations in the former were incomplete or the account in the latter was inaccurate, he was being no more Aristotelian than Ptolemy had been. As for the Marāgha astronomers and Ibn al-Shāṭir, it is doubtful whether they would have embarked on the research that led to their accomplishment in theoretical astronomy if they had not taken the *Planetary Hypothesis* as seriously as they had taken the *Almagest*. Their problem was only generated as a result of bringing these two works together, and the aim of their search for new mathematical models was to enable them to reconstruct the heavens in accordance with accepted principles of physics.

Attempts at such reconstructions are explained in detail in the works of the Marāgha astronomers and in Ibn al-Shāṭir. They remain to be studied by historians of astronomy. Here we shall only quote one passage expounding Ṭūsī's views on the relation of astronomy to physics. Regarding the subject-matter of astronomy and the nature of its principles and the problems it investigates, he states the following in the first Book of the *Tadhkira*:

> Every science must possess the following: a) a subject which this science investigates, b) principles which are either self-evident or need to be proved in another science but are taken for granted in this science, and c) problems (*masā'il*) which are proved in this science. Now the subjects of astronomy (*hay'a*) are the simple bodies, both superior and inferior, in respect of their quantities, qualities, positions and inseparable motions. The principles of astronomy that need proof are demonstrated in three sciences: metaphysics, geometry and physics. The problems of astronomy are aimed at gaining knowledge of these bodies themselves, and of their shapes, arrangements (*naḍd*), motions, and their quantities, distances and the reasons (*ʿilal*) for the difference in their positions[20].

Such expressions were, of course, not new in the history of astronomy. Here they constitute the basis of inquiries that went beyond paying lip service to a dominant natural philosophy.

II. TRANSLATION OF IBN AL-HAYTHAM'S OBJECTIONS IN *AL-SHUKŪK ʿALĀ BAṬLAMYŪS*[21]

II. 1. Concerning the lunar model

[Ptolemy] says in Book V, Chapter 5, which is on the inclination (*muḥādhāt*) of the diameter of the moon's epicycle, that the diameter of the moon's epicycle whose extremity is the epicycle's apogee (*al-buʿd al-abʿad*), alawys inclines towards
* (*yuḥādhī*) a point below the center of the world, a point whose distance from the center of the world is as the distance of the center of the world from the eccentric's center.

[20] British Museum MS Add 23394, fol. 1v.

[21] The following two passages occur on pages 15–19 and 24–29 respectively in the edition cited in note 2 above. Figure 1 has been added to make it easier to follow the argument in the first passage. Figure 2 is copied from the Isḥāq-Thābit version of the *Almagest*, British Museum MS 7475, fol. 68r.

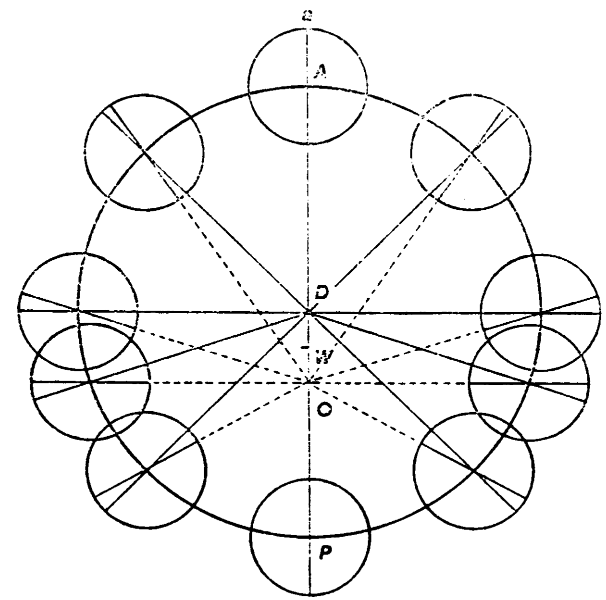

Fig. 1. *a*: apogee of epicycle, *A*: apogee of deferent, *D*: deferent center, *W*: world center, *O*: prosneusis point, *P*: perigee of deferent

But since [*idhā*] the eccentric sphere moves the epicycle, the epicycle's diameter, whose extremity is the apogee when the epicycle is at the eccentric's apogee, will always point to the eccentric's center. [For] when the eccentric deferent moves, thereby moving the epicycle, there will move, together with the epicycle, the eccentric's diameter that passed through the epicycle's apogee, or a line that can be imagined to take its place, and there will move with it the epicycle's diameter that passes through the apogee. This diameter cannot therefore be directed at any time to a point other than the eccentric's center unless it moved and changed its position so as to be oriented towards another point.

Now the epicycle's diameter is an imagined line; and an imagined line does not by itself move with a sensible movement that produces something (*maʿnā*) existing in the world. Similarly, the plane of the epicycle is an imaginary plane; and an imaginary plane does not have a sensible motion. Nor does anything move with a sensible movement that produces something existing in the world unless it be a body that exists in the world. From this it follows that it is the body of the epicycle that moves, thereby giving rise to the change of position of the epicycle's diameter in such a way as to be directed towards a point other than that towards which it would [otherwise] be directed.

Now Ptolemy in the *Planetary Hypotheses* (*Kitāb al-Iqtiṣāṣ*) assumed for the moon's epicycle a sphere (*kura*) or disc (*manshūr*) which moves the planet on (*ḥawl*)

the epicycle, which is a circle, with a circular movement about (*ḥawl*) the circle's center. Thus the diameter whose extremity is the apogee may be imagined to move as a result of this movement, or it may be imagined to remain stationary while the body [of the epicycle] moves. And if the body is imagined to be moving while the diameter is moved by its movement, then this diameter moves with a circular movement about the center of the circle which is the epicycle. Thus its extremity which is the perigee (*al-buʿd al-aqrab*) moves on the circumference of the epicycle, and therefore never points as a result of this movement to one and the same point. Therefore if this diameter always points, as he assumed, to one and the same point, while the body of the epicycle moves with a circular, uniform (*mustawī*) and continuous movement, then this diameter needs another mover which always orients it towards the assumed point.

But no sensible movement exists in the world unless it belongs to a body. That movement therefore needs another body, whether a sphere or disc, which moves the epicyciic sphere with a movement that brings about the movement of its diameter (in such a way as to) diiect it towards the assumed point.

Ptolemy, however, does not assume in the *Planetary Hypotheses* a body that brings about this movement. Moreover, if, for the sake of this movement, a body is assumed to move the epicycle, then this body must needs possess two opposite movements. For when the epicycle moves away from the eccentric's apogee, the diameter whose extremity is the apogee will move so as to be directed towards the assumed point (*nuqṭat al-muḥādhāt*) situated below the center of the world. And this movement results from the movement of the epicycle's apogee towards the eccentric's apogee until its [the epicycle's] perigee points to the assumed point, so that if this diameter extends (*intahā*) to the assumed point, it will contain with the diameter drawn from the center of the eccentric deferent to the epicycle's center, an angle at the epicycle's center. Thus the body that brings about the movement of this diameter moves towards the eccentric's apogee.

Then as the epicycle moves as a result of the movement of the sphere that carries it [i.e. the eccentric sphere], the body that moves the diameter whose extremity is the apogee continues to move in its own direction. For the previously mentioned angle contained by the two diameters widens, as was made clear in Ptolemy's discourse on the sun's eccentric. And the body continues to move and the angle continues to widen until the line whose extremity is the apogee becomes perpendicular to the world's diameter that passes through all the centers. Afterwards the body's movement comes to an end, for at this position the mentioned angle at the epicycle's center is a maximum.

Then when the epicycle subsequently moves, the body moving the diameter will move in the opposite direction to that in which it [hitherto] moved, I mean towards the perigee of the deferent sphere; for the previously mentioned angle at the epicycle's center will subsequently become smaller. Thus the apogee on the epicycle's diameter approaches the apogee on the diameter that points to the center of the deferent sphere that moves the epicycle. Then this body continues to move towards the deferent's perigee until the epicycle's center coincides with the [deferent's] perigee, at which time the two diameters fall upon the world's diameter, and the three diameters become one line. Then when the epicycle subsequently moves, the body which moves the diameter will move in the direction in which it has been moving. For the extremity of the diameter of the deferent sphere that moves the epicycle will be nearer to the deferent's perigee than the extremity of the epicycle's diameter which is the apogee, and the apogee of this [latter] diameter will continually move farther from it [the extremity of the former diameter]; for the angle at the epicycle's center widens at this position, and the apogee of the epicycle's diameter will move away from the deferent's perigee, and therefore this movement of the epicycle's diameter takes place in the direction in which [the diameter] moved when [the epicycle] arrived at the deferent's perigee.

Then the body that moves this diameter will continue to move in this direction until [this diameter] becomes perpendicular to the world's diameter on the other side of the deferent, for the angle at the epicycle's center widens until this position is reached.

Then, after reaching this position, when the epicycle moves as a result of the deferent's movement, the deferent body that moves the diameter will move in the opposite direction to that in which it moved before the diameter was perpendicular [to the world's diameter], that is in the direction in which it moved at first; for the angle at the epicycle's center will in this case become smaller.

Therefore, the epicycle's diameter will not move in such a way as to be always directed to the assumed point unless there exists a body which moves the body of the epicycle with a movement other than that due to the deferent sphere, namely the movement of longitude, and other than that responsible for the movement of the planet about the epicycle's center, which is the movement of anomaly (*ḥarkat al-ikhtilāf*); and unless that body has two opposite, natural and permanent motions.

Now this is an absurd impossibility: I mean that one and the same body should possess two opposite, natural and permanent motions. And if it is said that the two motions are voluntary (*ikhtiyāriyyatān*), it will follow that one part of the heaven makes two opposite choices (*yakhtār ikhtiyārayn*) and therefore its substance must consist of two opposite substances or of a multitude of opposite substances. And this is regarded as impossible by all philosophers.

And if it is said that these two motions belong to two bodies rather than one, then it follows that each one of these bodies will move for a certain interval of time, then stops for some time before moving again. This entails that each of the two bodies consists of two opposite substances, which is impossible. And every hypothesis (*waḍʿ*) assumed for the movement of this diameter leads to an absurd impossibility. But if it is impossible to assume the existence of a body of this description, then it is impossible that the epicycle's diameter should move in such a way as to be directed towards the assumed point.

II. 2. Concerning the Five Planets

[Ptolemy] says in Book IX, Chapter 2, which is on the principles that need to be laid down for the wandering planets:

Since it is our aim to show in the case of the five wandering planets, as we showed in the case of the sun and the moon, what all their apparent irregularities are, and that they are brought about by means of regular and circular motions, inasmuch as such motions are conformable to the nature of divine bodies and do not admit (*mubāyina*) of disorder and irregularity[...]

And he says in Book IX, Chapter 5, which is on what needs to be put forward with respect to the principles employed for the five wandering planets, that he assumed for each of the five planets an eccentric sphere and an epicyclic sphere, and that he made the eccentric move the epicycle. Then he said at the end of this Chapter:

And we also found that the centers of the epicyclic spheres move on circles equal to the eccentric spheres, I mean those that produce the irregularity. But these circles are not about the same centers. Rather, in the case of all the five planets except Mercury, they are about centers that bisect the straight lines between the centers of eccentrics and the ecliptic's center. And, in the case of Mercury, [the circle is] about a center distant from the center that turns it around by as much as this [latter] center is removed towards the apogee from the center about which the motion of anomaly takes place, of as much as this [last] center is removed from the center where the eye is placed.

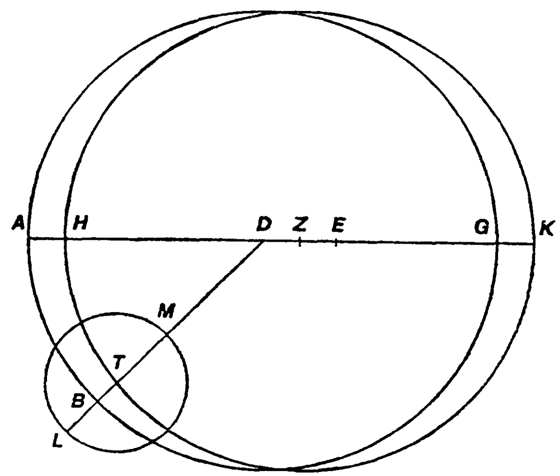

Fig. 2. D = equant center, Z = deferent center, E = ecliptic center, T = epicycle center

Then, in Chapter 6, he assumes a circle in which he draws a diameter and takes the center as given. He then assumes the ecliptic's center below the circle's center, then bisects the distance between the two centers by a point about which as center

he draws a second circle equal to the first. He then takes a point on the circumference of the second circle and draws about it an epicycle. Then, from the first center he draws a line to the epicycle's center and extends it rectilinearly until it reaches the circumference of the epicycle. Then he says that this whole plane revolves uniformly about its own center in the direction of the zodiacal signs. He then says that the epicycle's diameter, referring to the diameter joined to the line drawn from the first center D, which is farther from the center of the world, is made to turn by it also uniformly in the direction of the signs by as much as is required by the planet's longitudinal return; and that by means of its revolution it always turns the two points which are the epicycle's apogee and perigee; and that point T, meaning the epicycle's center, always moves on the circle whose center is nearer to the ecliptic's center; and that the planet itself always moves uniformly on the epicycle and in accordance with the diameter's inclination always towards point D, meaning the first center which is farther from the ecliptic's center.

In the case of Mercury's movement he introduced one more center to be added to these two centers and to the motions about them, without abolishing the motions that belong to the other planets, but rather [assumed similar motions to take place] about two centers similar to those assumed for the other planets.

That which we have mentioned is the truth of what Ptolemy asserted regarding the motions of the five planets. But this is a notion that leads to contradiction. For if the epicycle's diameter moves uniformly about the farther center, then every point on this diameter will consequently move uniformly about the farther center. For the points on the line which is drawn from the farther center, and which moves the diameter with the regular movement, will by virtue of this motion traverse similar arcs in the same time; and thus every point on the diameter will traverse equal arcs in equal times; and therefore every point on the diameter will have a regular motion about the farther center. The epicycle's center will therefore have a regular movement about the farther center, and in consequence will produce at the farther center equal angles in equal times.

Now if the [epicycle's] center produces at the farther center equal angles in equal times, then it will produce at the nearer center unequal angles in equal times. For that has been made clear in Ptolemy's discourse on the sun's eccentric. For since the sun's center cuts off equal arcs on the eccentric's circumference, it produces unequal angles at the center of the world. From this it follows that if it produces equal angles at the center of the world, then it will produce unequal angles at the eccentric's center. From this it follows with regard to the five planets that if the epicycle's center produces equal angles in equal times at the eccentric's center, then it will produce unequal angles in equal times at the nearer center. And if the angles produced in equal times at the nearer center are unequal, then the arcs that the epicycle's center cuts off in equal times from the eccentric's circumferences whose center is nearer to the center of the world, *i.e.* [the circumference of] the deferent sphere, will be unequal. Therefore the epicycle's motion on the eccentric

deferent will be irregular, and consequently the motion of the eccentric deferent about its own center will be irregular, for it is this [deferent] that moves the epicycle. And that is in contradiction with what [Ptolemy] asserted regarding the motion [of the planets], for he asserted that all their motions are regular.

But if the motion of the epicycle's center about the center of the deferent sphere is uniform, then it follows that the motion of the epicycle's center about the farther center will be irregular. For the same reason which entails the irregularity of the [epicycle's] motion about the nearer center because of its regular motion about the farther center will itself entail the irregularity of its motion about the farther center in consequence of its regular motion about the nearer center. Thus it follows from the two spheres which he assumed for the five planets that the motion of the one or the other must be irregular. But this is impossible and in contradiction with the true principles.

Moreover, if the epicycle's diameter always moves uniformly about the farther center, then there must exist a body which produces this motion, for an imaginary circumference does not by itself move with a sensible motion. Again, if the diameter moved by itself, its extremity would not always be the apogee, for the apogee is a point on the epicyclic body. Thus the apogee would not always move with the diameter unless the body of the epicycle moved with the diameter, and unless there moved with the diameter the circle on whose circumference revolves the center of the planet. Therefore the diameter cannot always move about the further center unless there exists a body which moves the epicycle about this center with a different movement from that of the epicycle about its own center. And if the epicycle is carried by the deferent sphere, and the latter moves it about its own center which is nearer to the center of the world, then there cannot exist another body which moves the body of the epicycle about the farther center. For if such a body existed, then the distances of the epicycle's center from the farther center would be equal. And that is impossible. For the distances of the epicycle's center from the nearer center are equal and therefore its distances from the farther center must be unequal.

And if a body existed which moved both the deferent sphere and the epicycle around the farther center, then the deferent's center would move. And he says that this center is fixed and unmoved in the case of the four planets other than Mercury. And if he assumes for the epicycle a body which moves it in such a way as to direct its diameter towards the farther center, as we assumed in the case of the moon' epicycle with respect to the opposite point (*nuqṭat al-maḥādhāt*), it will follow that this body has two opposite motions, just as this followed in the other case. It will also follow that [the assumed body] will move the diameter about the farther center with an irregular motion, given that the epicycle's motion about the center of the deferent is regular, as was shown earlier. But to assume the existence of a body of this description is impossible.

Again it will follow that each of these two contrary motions will be irregular, given that the angles produced at the farther center are equal. For if equal angles

are produced at the epicycle's center by the diameter's motion about the same center so as to be directed towards the farther center, then the angles produced at the farther center will be unequal, as the equations of the epicycle make clear. For equal arcs on the epicycle subtend unequal angles at the center of the world. And thus if the angles produced at the farther center are equal, the arcs subtending them on the epicycle will be unequal. Therefore the angles produced at the epicycle's center by the diameter which points to the farther center must be unequal, and the motion of the body which is responsible for this motion of the epicycle must in itself be irregular.

XV

The Andalusian revolt against Ptolemaic astronomy

Averroes and al-Biṭrūjī

I

The episode referred to in the title of this chapter as "the Andalusian revolt" is the well-known anti-Ptolemaic program of research that was conceived and defended by twelfth-century scholars in Muslim Spain. This was not a widely characteristic or long-lasting phenomenon of Arabic science, being definitely limited both geographically and in time. Nor was it in any way representative of the high degree of mathematical accomplishment already reached and subsequently maintained in the Islamic world. It was, nonetheless, an intriguing phenomenon that was associated with towering figures such as Averroes and Maimonides, and its very confinement to Andalusia under one rule, that of the Almohads, gives rise to rather interesting historical questions. I shall argue that there existed in twelfth-century Spain a certain cultural situation without reference to which the surprising position taken by Averroes and al-Biṭrūjī would be difficult if not impossible to explain.

The Middle Ages witnessed no revolutions in science, at least not in the sense we have come to associate with certain features of the combined achievement of men like Copernicus, Kepler, Galileo, Descartes, and Newton. But this does not mean that the ideal of innovation was beyond the imagination of medieval scholars or that they lacked a critical attitude toward their admittedly respected predecessors. In Medieval Islam the concept of innovation in intellectual endeavor found expression in terms like *istikhrāj* or *istinbāṭ (discovery)*, which denoted accomplishments that went beyond merely elucidating, emending, or completing an earlier contribution to knowledge;[1] and a critical attitude clearly revealed itself in the not infrequent composition of *shukūk* (*aporiai, dubitationes*), a form of argument in which difficulties or objections were raised against ancient authorities. Indeed, it would be impossible to explain the high quality of much of Islamic scientific writings without noting the intellectual ambition and independence of mind their authors often possessed to a remarkable degree.

One example, which is of particular relevance to the subject of this chapter, is provided by the mathematician Ibn al-Haytham, who flourished in Cairo some

This research was completed during tenure of grants from the National Science Foundation and the National Endowment for the Humanities. Grateful acknowledgment is made of this support.

150 years before Averroes. Ibn al-Haytham wrote a series of *shukūk* or objections mainly directed against certain aspects of Ptolemaic astronomy, in particular what (rightly) appeared to him as an inconsistency generated by Ptolemy's introduction of the equant hypothesis.[2] It is not that he (or the later Islamic critics of Ptolemy) failed to appreciate the predictive function of that hypothesis. But, being convinced (as Ptolemy had been) that planetary motions must ultimately be understood in terms of the motions of real spherical bodies in which the planets were embedded,[3] he saw that the equant hypothesis would make it necessary to attribute a nonuniform motion to the deferent sphere that carried the planet's epicycle around. According to Ibn al-Haytham this was unacceptable because it contradicted Ptolemy's position as exhibited jointly in the *Almagest* (where the uniformity principle is repeatedly asserted) and in the *Planetary Hypotheses* (where a system of the universe is outlined in terms of nested spheres that produced the motions already described in the *Almagest*). It is to Ibn al-Haytham's credit that he had the courage to draw and boldly state what he believed to be an inevitable conclusion: that the arrangements proposed for planetary motions in the *Almagest* were "*false*" (his own word) and that the true arrangements were yet to be discovered.[4] It is now known that Ibn al-Haytham's criticisms played an important part in stimulating the research at thirteenth-century Marāgha that led to the construction of "non-Ptolemaic" models the purpose of which was to preserve the uniformity principle *without* sacrificing the mathematical effect of the equant.[5] This research may be characterized as an attempt to save the principles of Ptolemaic astronomy (uniformity and circularity of motion, eccentrics, and epicycles) as well as the phenomena the models of the *Almagest* had been designed to account for. In other words, the Marāgha astronomers were not aiming to overthrow Ptolemaic astronomy but only to reform it. And I would therefore venture to say that the results reached at Marāgha (and later at Damascus), insofar as they were successful, would have been perfectly acceptable to Ptolemy himself (and to Ibn al-Haytham).

II

It is important to distinguish clearly between this line of research and what took place in Andalusia in the century separating Ibn al-Haytham and the Marāgha astronomers. The final outcome of the Andalusian endeavor was a book on *The Principles of Astronomy* in which its author – al Biṭrūjī (or Alpetragius) – gave sample illustrations of how the apparent motions of the planets could in his opinion be produced by means of concentric spheres, without the use of eccenters or epicycles.[6] Thus whereas the Marāgha astronomers of a later century would aim to straighten out Ptolemaic astronomy by bringing it into line with its own principles, the goal of al-Biṭrūjī was to get rid of two of Ptolomy's basic principles altogether. Al-Biṭrūjī wrote his book toward the end of the twelfth century (probably around 1200 A.D.), but he tells us that he received his inspiration from the philosopher and court physician Ibn Ṭufayl (d. 1185), author of

the well-known philosophical narrative of *Ḥayy ibn Yaqẓān*, the only written work of his that has survived. According to al-Biṭrūjī, Ibn Ṭufayl claimed that he had "come upon" (or found) an arrangement that brings about the motions of the planets without assuming eccenters or epicycles and that he had promised to write on this subject.[7] Apparently that promise was never made good, but it was these remarks that set al-Biṭrūjī on the long path that finally led him to write his own book. As we shall see, the writings of Averroes (who died in 1198) and Maimonides (who died in 1204) exhibit a similar, negative attitude toward Ptolemaic astronomy. Since Averroes was very close to Ibn Ṭufayl (who had introduced him as a young man to the Almohad ruler, probably in 1168 or 1169), we may assume that he, too, was acquainted with Ibn Ṭufayl's views.

There is nothing in the available sources to indicate a direct connection between Averroes and al-Biṭrūjī, but at least their common philosophical parentage in the person of Ibn Ṭufayl is beyond doubt. Intellectual relations between Averroes and Maimonides are more difficult to ascertain. Maimonides was about thirty years old when he left the Maghrib (northwest Africa) for Egypt where, in his mature age, he wrote the *Guide for the Perplexed*, the book in which he deprecated the Ptolemaic system.[8] It is known that he received copies of Averroes's commentaries on the Aristotelian corpus before he completed the *Guide*.[9] But whether or not he was directly influenced by Averroes's arguments against Ptolemy, it is at least clear that the two thinkers shared a common culture and a common philosophical background. Despite his long sojourn in Egypt Maimonides was in matters of philosophy much closer to the Spanish brand of Aristotelianism than to the Eastern version that had been forged by Avicenna. To quote the words of Shlomo Pines, "When . . . [Maimonides] wrote the *Guide*, he was, in the domain of philosophy and philosophic theology, still almost exclusively involved with the problems with which he must have been familiar in his youth in Spain and the Maghrib."[10]

From this brief account it is clear that when we consider the movement of thought that culminated in the formulation of a new astronomical theory by al-Biṭrūjī, we are dealing with a compact situation in which a small number of individuals were bound together by a distinctive intellectual milieu and, in some decisive cases, by direct personal ties. Sometimes, the ideas of these individuals, particularly their commitment to certain Aristotelian doctrines, are linked to a slightly earlier philosopher, Ibn Bājja (d. 1138), frequently called by modern scholars "the founder of Spanish Aristotelianism," and who is known, on the authority of Maimonides, to have criticized the use of epicycles in planetary theory.[11] But although it is fairly certain that such a link existed, a line must be drawn between partial criticisms such as those of Ibn Bājja (and Jābir ibn Aflaḥ) and the radical rejectionism associated with Ibn Ṭufayl, Averroes, al-Biṭrūjī, and Maimonides.[12]

According to al-Biṭrūjī the universe consists of nested spheres (or spherical shells) that have the earth as their common center. Each of these spheres transfers to the one immediately below it a portion of the motion it receives from the one immediately above, the ultimate source of motion being a *primum mobile* that

lies above the sphere of the fixed stars and that rotates with a constant motion from east to west. As a result of this mode of transference of energy, velocities diminish and variations of motion increase as one proceeds from the outermost sphere toward the center.

The planets and the fixed stars, being rigidly fixed in their appropriate spheres, maintain a constant distance from the earth, which, being at the center, is itself at rest. With only constant and concentric motions available, the apparent irregularities of the sun, the moon, and the planets, and the movement of precession, were to be explained in terms of a spiral motion, a rotation of poles around poles; and by imposing polar motions upon one another al-Biṭrūjī hoped to explain planetary stations and retrogressions as well as motions in longitude, latitude, and anomaly. All this was presented as conformable to the solid principles of Aristotelian physics.

To give just one or two examples. The annual motion of the sun is governed by the motion of what is called the pole of the sun, a point intended to maintain a quadrant's distance from the sun. The pole of the sun moves on a small circle in the sphere of the sun about the pole of the celestial equator. Taking the radius of this small circle to be equal to the inclination of the ecliptic, and assuming the pole of the sun to move on this circle (from west to east) twice as fast as the mean motion of the sun, al-Biṭrūjī manages to show that the pole will be a quadrant away from the sun at the equinoctial and solsticial points. (Unfortunately, however, the sun will not move on the ecliptic between these points although not departing too far from it.)

The case of the planets is more difficult and even less successfully dealt with. Again, the motion of the planets in the vicinity of the ecliptic is governed by the rotation of poles near the poles of the equator. With a radius equal to the inclination of the ecliptic, a circle is drawn about the north pole of the equator. Call this circle the polar deferent, the name given to it by B. Goldstein, whose interpretation of the text I am following here. Then, with a radius equal to the maximum latitude of the planet, another circle (smaller than the first) is drawn about a point on the polar deferent. Call this circle "polar epicycle," a name al-Biṭrūjī would probably have objected to.[13] It represents the path of a point called the pole of the planet. The planet itself, which initially may lie on the ecliptic, is supposed to be a quadrant's distance from its pole. The idea is that the motion of the planet is governed by the rotation of its pole about the center of the "polar epicycle" as this center slides on the polar deferent. The two rotations combine to produce the motion of the planet in longitude and in latitude.

Recent analysis of al-Biṭrūjī's system has revealed many shortcomings and many unanswered questions. For example, the planetary model as it stands would require Saturn to depart sometimes from the ecliptic by more than 26°, while a divergence of only 3°3′ (the mean of the Ptolemaic extreme latitude) is intended (Kennedy).[14] The planet's pole will not always be a quadrant's distance from the planet (Kennedy and Goldstein), again contrary to what is intended. And al-Biṭrūjī makes no accommodation for Ptolemy's equant hypothesis, nor does he refer to it (Goldstein).

The solar model is equally inadequate. Even on the hypothesis of a polar motion twice that of the sun, the latter will not always be found on the ecliptic, as already pointed out; nor is it clear why the hypothesis itself should be accepted (Kennedy). Then, in the attempt to arrange for this model to exhibit an annual irregularity, al-Biṭrūjī "heaps chaos upon confusion" (in the words of Kennedy) by introducing elements required by the incompatible system of Ptolemy.[15]

Thus, though ingenious and inventive, al-Biṭrūjī's system does not stand up to close astronomical examination, and its appeal to some medieval minds contrasts greatly with the judgment passed on it by modern scholars. Dreyer calls it "quaint,"[16] and Carmody describes it as a "delusion."[17] Goldstein concludes his analysis of the planetary model with these words: "The attempt to find satisfactory philosophical principles for the description of planetary motion was perhaps noble, but its success in this instance was quite meager."[18] Kennedy concludes a review of Goldstein's study of al-Biṭrūjī with the statement that "serious planetary theory was beyond al-Biṭrūjī."[19] And, in an earlier review of Carmody's edition of Michael Scot's translation, he wrote:

> [al-Biṭrūjī's] basic device, that of rotating poles on concentric spheres, was highly ingenious and he exploited it to the full. But the resulting system represented an improvement only to minds more firmly attached to the concept of concentric spheres than they were influenced by observable facts. As an interpretation of the real universe it is vastly inferior to the Ptolemaic one. Perhaps the situation is best summed up by the statement that al-Biṭrūjī was a philosopher.[20]

There is no doubt that al-Biṭrūjī deserves to be flunked. Like Humpty Dumpty he had a great fall, and no one can put the pieces together again. We may simply bury his remains and forget about the poor fellow, or we may choose to meditate further on the nature and circumstances of his failure. In what follows I shall choose the latter course, on the advice of those historians who believe that failures as well as successes can be instructive in the study of scientific endeavor. And, first, let us look at how al-Biṭrūjī viewed his own task and achievement. This is what he wrote at the end of his discussion of planetary motions:

> Neither time nor good fortune has enabled me to complete the inquiry into the details of planetary motions, or to pursue all that belongs to the planets in respect of their risings and settings and the times of their visibility, or to learn the conditions of their conjunctions and eclipses and all that is contained in the *Almagest* regarding them. These are matters that require a long time and the collaboration of those who are expert in them. Indeed the remainder of [our] life would not be enough if the ability is not there. Our aim was merely to draw attention to the way in which the ascertained [east to west] motion brings about the different and diversified (*al-mukhtalifa al-mutafannina*) motions, and to make known (reading "*wa al-ta'rīf*" in place of "*waal-ta'rīd*") a possible arrangement for the heavens.[21]

Al-Biṭrūjī, it is clear, believed in the truth of a program he knew he had not accomplished. He did not intend the models presented in his book as constituting final solutions to the problems involved but rather as illustrations of the *kind*

of solution that must be sought. He doubted his own ability to fulfill the task in hand, but his successes, limited though they were, would have enhanced his confidence in and hope for the general program. Such an attitude is not unheard of in the history of science. Al-Biṭrūjī calls to mind another philosopher, Descartes, who offered a plan for the whole of physics with nothing to support its validity but a number of a priori arguments and a few problematic applications. It will be remembered that the Cartesian models for gravitational, optical, and other phenomena were not only inadequate, but sometimes contradictory. They, too, were meant as illustrations of a type of explanation, not as final results. The comparison with Descartes may, however, be objected to on the grounds that Descartes lived before Newton whereas al-Biṭrūjī ventured on his project a thousand years after Ptolemy, whose powerful system of the universe would have been well known to him. So, probably, it would be more to the point to compare al-Biṭrūjī with the Bernoullis, Johan and Daniel, who, in the eighteenth century, were diligently trying to reconcile Newton's law of gravitation with the Cartesian theory of vortices.[22] The fact that al-Biṭrūjī was not mathematically strong is, I think, irrelevant to this comparison. He and his eighteenth-century counterparts were motivated by the same desire, which was to harmonize some empirically confirmed results with a rationally satisfying idea, and both his program and theirs were doomed to failure.

It may be noted that both al-Biṭrūjī and the Marāgha astronomers were driven by the same sort of theoretical concerns. But again it must be emphasized that their respective commitments and aims were not identical. The Marāgha astronomers were committed to Ptolemaic astronomy, and theirs was the limited aim of reconciling certain features of the Ptolemaic system with Ptolemaic principles. Al-Biṭrūjī, on the other hand, had inherited a stronger and much more rigid program that, as it turned out, was impossible to execute. It is this rigid character of his program that I should now like to consider.

III

It is often asserted that al-Biṭrūjī belonged to an Aristotelian school of philosophy that had been initiated in Spain by Ibn Bājja and of which Averroes was the strongest representative. The assertion is justified inasmuch as the authority and views of Aristotle are expressly invoked and defended in the writing of so-called members of this school, including al-Biṭrūjī. What we have to do with here, however, is a certain attitude to Aristotle and a certain interpretation of his doctrines that I find quite puzzling. Aristotle had always been an authority, indeed the foremost authority, for Islamic philosophers. But it was only Averroes in twelfth-century Spain who raised Aristotle to the status of a perfect human being, an individual in whom the intellectual faculty had reached its highest possible degree of human perfection.[23] Again, it would not be difficult to point to examples of an earlier critical attitude on the part of Islamic scientists toward certain aspects of Ptolemaic astronomy, but it was only in twelfth-century Spain

that Islamic thinkers in the Greek philosophical tradition went as far as to reject Ptolemaic astronomy in toto. Al-Biṭrūjī's position may have been simply a consequence of such a belief in Aristotle's infallibility, but this belief itself needs some explanation.

In order to persuade the reader that the attitude of Averroes to Ptolemaic astronomy is something of a puzzle, I shall quote a few passages from two of his works that were concerned with Aristotle's *Metaphysics*. The first of these works is a paraphrase (*talkhīṣ*) of the *Metaphysics* that Averroes is supposed to have written in 1174, when he was forty-eight years old.[24] The second and better known work, composed at a later date (after 1186), is the large commentary (*tafsīr*) on the *Metaphysics*.

The following is what Averroes wrote in the *Talkhīṣ* regarding the question of number of celestial motions:

As for the number of these motions, it must be received from the mathematical science of astronomy. Let us lay down what is most widely held regarding them here [in al-Andalus] and in our own time, and what has not been subject to dispute among practitioners of this science [*ṣinā'a*] from Ptolemy to the present time; and let us leave out disputable matters to the experts in that science. Moreover, it is not possible to determine many of these motions without using the generally accepted premises [*muqaddamāt mashhūra*], given that a period of many life-times would be required to determine many of these motions; and generally accepted premises in a given science are those which practitioners of that science do not dispute. For this reason, then, we have here adopted such premises; and we therefore say: that the motions agreed upon for the heavenly bodies are thirty-eight [*sic*]: five for each of the three superior planets – Saturn, Jupiter and Mars, five for the moon, eight for Mercury, seven for Venus, one for the sun (provided that it is imagined to move in an eccentric sphere only and not in an epicyclic sphere), and one for the all-embracing sphere, i.e., the sphere of the fixed stars.[25]

Averroes departs here from the Aristotelian theory, derived from Eudoxus and Callippus, which had led Aristotle to posit fifty-five or forty-seven movements.[26] It is to be noted that he justifies this departure by appealing to Aristotle's view that the metaphysician must accept the number of such motions from the astronomer who may introduce modifications in the light of new observations.[27] It is as if Averroes were saying that just as Aristotle had been obliged to adopt the number of motions determined by the astronomy of his day, our duty requires us to adopt the motions established by later astronomers, including those of our own time. In other words, Ptolemy and his successors were to be preferred to Eudoxus, Callippus, and, consequently, Aristotle.[28]

After briefly discussing and rejecting the assumption of a ninth sphere as a debatable and doubtful doctrine, Averroes goes on to argue that the number of movers must then be the same as those accepted movements, provided it is assumed that a single mover is responsible for the daily motion of all the planets – an opinion Averroes accepts, following (as he implies) Alexander. The idea of regarding the daily motion of the planets as essentially one and therefore proceeding from one mover, leads Averroes to adopt a view Ptolemy had argued for in the *Planetary Hypotheses*. This is the view in which the whole celestial

sphere is regarded as a unique celestial living being. These are Averroes's words:

If all that is as we described (or assumed), then a motion that is essentially one must be attached to a single thing in motion. But a single thing in motion must be moved by a single mover. We therefore ought to think of the whole [celestial] sphere as a single spherical animal whose convex [surface] is that of the sphere of the fixed stars and whose concave [surface] touches the sphere of fire; and that its motion is one and universal; and that the motions that exist in it for each one of the planets are particular motions; and that the great motion resembles the locomotion of an animal while the particular motions resemble those of the members of an animal.[29]

Averroes goes on:

And it is for this reason that these motions do not require centers about which they move, like the earth in relation to the great motion, for most of these motions have been shown in [the science of] mathematics to have centers other than the center of the world, and that their distances from the earth are not the same. Accordingly, we do not need to imagine a multiplicity of spheres whose centers are the same as the center of the world and whose poles are those of the world but which are distinct from one another. Let us rather imagine that between the spheres proper to each of the planets there exist certain bodies [which behave], not separately from one another, nor as [individually] endowed with a proper motion, but which [move] by virtue of being parts of the whole, and that the planets perform their daily motion on these bodies.[30]

These words clearly show that it was not only with regard to the number of motions that Averroes was willing (at the time of writing the *Talkhīṣ*) to differ from Aristotle. As he argues for the validity of the picture that he derives from the *Planetary Hypotheses* he makes it quite clear that he adopts the Ptolemaic principle of nonconcentric spheres. For in that picture, he says, such spheres do not require the existence of a stationary body at the center of their motions. One may thus adopt the Ptolemaic assumption of eccentric and epicyclic spheres without abandoning a fundamental feature of Aristotelian cosmology, a unique earth at rest representing the absolute center of the world. As we shall immediately see, Averroes's position here is in contradiction with the view he later took in the *Tafsīr.* But before we turn to that view let us insist again that, for the Averroes of 1174, to be an Aristotelian did not mean that one was shut up in an absolutely closed system, at least as far as the science of astronomy was concerned.

IV

In the large commentary on Aristotle's *Metaphysics* (the *Tafsīr*), Averroes comments on the same passage with which the words quoted from the *Talkhīṣ* were concerned. Here he compares the doctrines derived by Aristotle from the mathematicians of his own age with those developed by later mathematicians, especially Ptolemy. He repeats and endorses the Aristotelian view that the metaphysician must accept the number of such motions from the astronomer, whose job it is to add (if necessary) new motions in the light of new observations. As an example he mentions two new motions ascribed by Ptolemy to the moon.[31] He draws the lesson that in such matters, which by their difficulty preclude the possibility of indubitable premises, one is obliged to rely on generally accepted propositions (*al-mashhūrāt*) as long as they are not disputed (?by astronomers). He then reports the opinion of Eudoxus, who attributed three spheres each to the sun and moon: one for the daily east-to-west motion, a second for the motion in longitude, and a third for the motion in latitude. "As for the mathematicians of our own time," Averroes adds, "they have assumed only one motion" (in place of the last two), "namely that of the planet in its inclined sphere, thus giving rise to a motion in longitude and another in latitude with respect to the ecliptic."[32] Thus, he concludes, "the sun will have only two motions, *unless it is necessary to introduce a third on account of its observed acceleration.*"[33]

It is clear from these last words that Averroes was not thinking of a second *eccentric* motion (or sphere) for the sun. He explains his reason in the following passage, which I shall translate in full because of its historical importance.

[a] For to assert the existence of an eccentric sphere or an epicyclic sphere is contrary to nature. As for the epicyclic sphere, this is not at all possible; for a body that moves in a circle must move about the center of the universe (*al-kull*), not aside from it; for it is the revolving thing itself (*al-mutaḥarriku dawran*) that produces (*yafʿal*) the center. Thus if a revolution about a center other than this center were to take place, then a center would exist other than this center, and there would exist an earth other than this earth. But all this has been shown to be impossible in natural science.

[b] It is similarly the case with the eccentric sphere proposed (*yaḍaʿuhu*) by Ptolemy. For if many centers existed, we should have a multitude of heavy bodies outside the place of the earth, and the center would cease to be unique, and it would be extended and divisible. But all this is not possible.

[c] Moreover, if eccentric spheres existed, then certain parts of the heavenly bodies would be redundant, their use being restricted to filling in an empty space, as is thought to be so in the case of the bodies of animals. But nothing of what appears of the motions of these planets makes it necessary to assume the existence of an epicyclic or an eccentric sphere.

[d] It may be possible to replace these two things by the spiral motions (*al-ḥarakāt al-lawlabiyya*) assumed by Aristotle in this astronomy (*hādhihi l-hayʾa*) in imitation of (*ḥikāyatan ʿan*) those who came before him. For it appears that astronomers before Hipparchus and Ptolemy assumed no epicyclic or eccentric spheres. Ptolemy stated this in his book on *Planetary Hypotheses*, and he claimed that Aristotle and his predecessors had assumed instead spiral motions, thereby increasing, as he claimed, the number of motions. Those who came after them, however, found a simpler way – that is, they were able to account for the phenomena (*amkanahum an yaḍaʿū mā yaẓharu*) by reference to fewer bodies, by which he meant the epicyclic and the eccentric sphere. He also claimed that this way was better inasmuch as it was accepted that nature does nothing redundant and therefore would not employ more means than it needs to bring about the motion of something.

[e] Ptolemy was not, however, aware of what had obliged the ancients to appeal to

spiral motions, namely the impossibility of epicyclic and eccentric spheres. But when people came to believe that this [new] astronomy was simpler and easier for [explaining] the revolutions (*'awd al-ḥarakāt*) now recorded in Ptolemy's book, they neglected the ancient astronomy until it became so obsolete that people are not now able to understand what Aristotle says in this place [in the *Metaphysics*] about those [ancient] people. This has been admitted by Alexander and Themistius but without their being aware of the reason we have mentioned.

[f] We should therefore embark on a new search for this ancient astronomy, for it is the true astronomy that is possible from the standpoint of physical principles. It is in my view based on the motion of one and the same sphere about one center and different poles, which may be two or more in accordance with the phenomena. For such motions can give rise to the acceleration, retardation, accession, and recession [*iqbāl wa idbār*] of a planet, and other motions for which Ptolemy failed to produce an arrangement [*hay'a*]. Such motions would also be the approaching and receding of a planet, as in the case of the moon. In my youth [*fī shabābī*] I had hoped to accomplish this investigation, but now in my old age [*fī shaykhūkhatī*] I have despaired of that, having been impeded by obstacles. But let this discourse spur someone else to inquire into these matters [further]. For nothing of the [true] science of astronomy exists in our time, the astronomy of our time being only in agreement with calculations [*al-ḥusbān*] and not with what exists.[34]

This long and rich passage raises more questions than can be examined here. What is interesting from the point of view of our present problem is that in it Averroes reveals an attitude that is entirely different from that which he had expressed at the time of writing the *Talkhīṣ*. Averroes still maintains that in matters relating to celestial motions the philosopher or metaphysician must rely on the results of mathematical astronomy. He clearly recognizes the explanatory power of the Ptolemaic system by his unequivocal admission that the astronomy of his own time "agrees with calculations." And yet he finds himself obliged to reject the whole basis on which this system stands. In doing so he completely ignores the arguments he had himself produced in the *Talkhīṣ* and simply puts in their place an a priori argument that eliminates eccenters and epicycles as a matter of principle. Can we convincingly explain Averroes's new attitude by merely describing it as Aristotelian? Did he not write the *Talkhīṣ* also as an Aristotelian? Did he not once think that he was following Aristotelian methodology when he preferred the Ptolemaic system to the earlier one of Eudoxus? But now, instead of characterizing the later theory as more informed, he postulates a perfect astronomy that antedated Aristotle, that Aristotle only hinted at in his writings, and that perished as a result of the triumph of Ptolemaic astronomy. That triumph, Averroes explains, was due to a misguided application of the principle of economy – misguided because it ignored the demonstrable requirements of natural philosophy; and therefore, according to him, it is the duty of the astronomer who is interested in truth as well as computation to try to discover anew that ancient and forgotten astronomy. That is the task al-Biṭrūjī set for himself a little later, and we may accordingly look at his and Averroes's deliberations as a chapter in the history of the myth of ancient wisdom that is known to have continued to play a part in the later development of Western thought.[35]

V

The "Aristotelianism" revealed in the passage from the *Tafsīr* requires explanation because of the unexpectedly extreme position it exhibits. An explanation must be sought outside of the text itself and, as it turns out, in cultural and, perhaps, even psychological rather than strictly cognitive and "rational" terms. Here I can only outline such an explanation briefly and somewhat dogmatically. It consists in looking at Averroes's (and al-Biṭrūjī's) position as part of a more general phenomenon, an intellectual trend that prevailed in Andalusia under the Almohads among scholars working in such diverse fields as law, grammar, medicine, and philosophy. And this trend may itself be related to a noticeable and often expressed Andalusian self-assertiveness vis-à-vis the rest of the Islamic world. On one level this attitude shows itself in the many essays composed by Andalusian scholars at different times on the distinctive and superior virtues of their land (*faḍā'il al-Andalus*). But the Andalusian sense of identity went further than self-praise and actually expressed itself in the creation of systems of ideas that were distinctly Andalusian and consciously directed against intellectual authorities in the Eastern part of Islam. Already in the eleventh century one of the most original thinkers of Muslim Spain, Ibn Ḥazm of Cordova (d. 1064), developed a literalist doctrine of law that he set against all other recognized *sunnī* doctrines. By equating religion (*dīn*) exclusively with what can be found explicitly stated in the Qur'ān and in the *Sunna* (Traditions of the Prophet), and by denying religious merit to all efforts on the part of legists to form inferences or opinions or preferences of any kind he was undermining the authority of the legal schools and their followers. It is to be remembered that the Almohad rule in North Africa and Spain came into being as a result of the forceful implementation of an articulated ideology. The founder of the Almohad movement, Ibn Tūmart (d. 1130), may not have been a thoroughgoing literalist, but his writings seem to exhibit literalist tendencies.[36] In any case such tendencies have been attributed to the second Almohad ruler Abū Ya'qūb Yūsuf (r. 1163–84) by Spanish historians of the period, and it is known that his son Ya'qūb al-Manṣūr (r. 1184–99) openly called for a literalist approach in the practice of law. He went as far as to order the burning of Mālikite books on *furū'*, or recipes for detailed applications of religious law as developed by the followers of Mālik who dominated the theological field in Andalusia and the Maghrib. Averroes, who served both Yūsuf and Ya'qūb as court physician and as a judge, appears to have had some such approach to law and theology in mind when, at the end of his famous treatise on the harmony of religion and philosophy, he wrote that through the "triumphant rule" of the Almohads the masses had been summoned to a "middle way . . . which is raised above the low level of the followers of authority but is below the turbulence of the theologians."[37] It was also during the reign of Ya'qūb al-Manṣūr that Ibn Maḍā' of Cordova, another protégé of the Almohad court who had been appointed chief judge by al-Manṣūr's father, wrote his attack on the theory of the agent, until then the almost universally accepted basis of Arabic grammar. This attack shared certain features with Ibn

Ḥazm's literalism: It rejected the grammatical equivalent of legal reasons (*'ilal*) and it was aimed at freeing the study of language from the clutches of the established authorities of Arabic grammar.[38]

As for Averroes, his negative attitude toward the Muslim philosophers of the East and the pride he took in his own country and culture should be well known and can be easily documented. It was he who wrote the most vigorous and most detailed reply to al-Ghazālī's resounding attack on philosophy, some seventy years after al-Ghazālī's death.[39] Anyone who looks at this reply will see that it is a criticism not only of al-Ghazālī, but also of Avicenna, the leading philosophical authority in Islam up to the time of Averroes. One of al-Ghazālī's mistakes, Averroes argued, was to attribute to the ancient philosophers doctrines that Avicenna had invented or borrowed from the discredited Mutakallimūn (dialectical theologians).[40] Time and again Averroes castigates Avicenna, and also al-Fārābī,[41] for having corrupted or at least departed from the true teachings of the first, and indeed perfect, philosopher. The reply to al-Ghazālī is only one example. When Averroes finds occasion in his other writings to compare the views of Aristotle with opposing views of later Islamic thinkers, such as al-Kindī[42] and Ibn al-Haytham,[43] it is Aristotle that he judges to be right and the others wrong. It is difficult not to regard this attitude of Averroes's and the commentatorial style he adopted in most of his philosophical writings as a literalism in philosophy that paralleled the theological literalism of Ibn Ḥazm. Thus Averroes's glorification of Aristotle and his rejection of the authority of Muslim peripatetics in the East can be seen as two aspects of the same attitude.

If Averroes considers himself closer to Aristotle than any of his Islamic predecessors, then it should not come as a surprise that he should be concerned to find an explanation of that privileged position. The explanation he develops in several of his writings is interesting as it relates to that Andalusian self-consciousness to which I referred. In the *Kulliyyāt,* Averroes has a few words about climatology, a theory that accounted not only for the physical properties of the various regions of the earth but also for the intellectual as well as physical features of their inhabitants. Averroes reports Galen's view that the most temperate climate is the fifth, where Greece, the country of Hippocrates, is located, and pointedly adds that "this land of ours, namely al-Andalus," lies at the beginning of the fifth climate.[44] But whereas in his commentary on the *Republic,* Averroes is content to remark, against Plato, that "individuals" may excel in the sciences outside of Greece, as is found, for example, in Iraq, Egypt, and al-Andalus,[45] he takes an exclusive position in the middle commentary on Aristotle's *Meteorology.*[46] Here he repeats Galen's opinion that it is the fifth climate, not the fourth (as others think), that is the most temperate, and, having decided that geographical latitude (and proximity to the sea) is the crucial factor, he groups Andalusia together with Greece and ignores Iraq. Of course, the Arabs and the Berbers came to Andalusia from elsewhere, but they had been acclimatized over the centuries, a process that, we are led to assume, had reached maturity in the time of Averroes. That, Averroes tells us, is the reason why the sciences have flourished among the Andalusians.

NOTES

1 The idea of levels or orders of achievement in writing or composition (*marātib al-sharaf fī al-tawālief*) is mentioned, for example, by Ibn Ḥazm of Cordova (eleventh century A.D.), who lists seven such orders headed by that in which an author puts forward something that has not been previously discovered (*shay'lam nusbaq ilā istikhrājih fa-nastakhrijuh*); see his *al-Taqrīb li-ḥadd al-manṭiq . . .,* ed. Iḥsān 'Abbās, Beirut: Dār al-Ḥayāh, ?1959. The same idea already occurs in the opening sentences of al-Khwārizmī's *Kitāb al-Jabr wa al-muqābala,* also using the term *istikhrāj* for unprecedented discovery. A marginal note in the Bodleian MS of this work gives some examples of authors in this highest category: Archimedes, Abū Ḥanīfa (founder of a Muslim legal school), and al-Khwārizmī himself, see the edition by 'A. M. Musharrafa and M. M. Aḥmad, *Kitāb al-J. wa al-m.,* Cairo: The Egyptian University, 1939, p. 15 and the facing facsimile. *

2 An edition of Ibn al-Haytham's *al-Shukūk 'alā Baṭlamyūs* (*Dubitationes in Ptolemaeum*) has been published by A. I. Sabra and N. Shehaby, Cairo: Dār al-Kutub, 1971. An English translation of Ibn al-Haytham's criticisms of Ptolemy's equant hypothesis is included in A. I. Sabra, "An eleventh-century refutation of Ptolemy's planetary theory," in *Science and History: Studies in Honor of Edward Rosen* (Studia Copernicana XVI), Wrocław etc.: Ossolineum (The Polish Academy of Sciences Press), 1978, pp. 117–31.

3 It is sometimes forgotten or ignored that, in view of the cosmology that prevailed in ancient and medieval times, such a conviction was in fact inevitable. The stars are physical bodies that can be seen to move across the sky. They did not of course make their journey in empty space (for the void did not exist), and of course they did not swim like fish in some lowly matter that opened up before them and closed in behind them (for the substance of the heavens was impassable). They must therefore be embedded in immense, solid, and transparent spheres or spherical shells that rotate inside one another about their own centers, each unimpeded by its neighbor. That was the conclusion reached in classical antiquity and universally accepted by medieval astronomers. The history of theoretical astronomy from Ptolemy to Copernicus cannot be understood without realizing that astronomers during that period were not merely interested in saving the appearances by means of purely mathematical devices. There was no such thing as a mathematical astronomy consciously conceived as a discipline entirely independent of physics or natural philosophy. It is true that Ptolemy's *Almagest,* like Newton's *Principia,* was not itself a work on natural philosophy, but rather laid down certain mathematical principles that govern celestial phenomena. But Ptolemy himself had been no more positivistic in his outlook than was Newton. Like Newton he was interested in questions of nature and causes, and his *Planetary Hypotheses* may in some respects be seen as having somewhat the same relationship to the earlier *Almagest* as Newton's *Scholia* and *Quaeries* had to the *Principia.* The two works, *Almagest* and *Planetary Hypotheses,* differed greatly in character but they were parts of one and the same enterprise, which was to save the principles as well as the phenomena of the heavens. Some of these principles were mathematical, but others were physical and had to do with the nature of celestial bodies. In the thirteenth century, Naṣīr al-Dīn al-Ṭūsī expressed the general understanding of the astronomers of his time when he wrote the following in his famous and influential *Tadhkira:* "Every science must possess the following: [a] an object which this science investigates, [b] principles which are either self-evident or need to be proved in another science but are taken for granted in this science, and [c] problems (*masā'il*) which are proved in this science. Now the objects of astronomy (*hay'a*) are the simple bodies, both

superior and inferior, in respect of their quantities, positions and inseparable motions. The principles of astronomy that need proof are demonstrated in three sciences: metaphysics, geometry, and physics. The problems of astronomy are aimed at gaining knowledge of these bodies themselves, and their quantities and distances and the reasons (*ʿilal*) for their varying positions" (*Tadhkira*, MS. Leiden Or. 905, fols. 1b–2a). The physical principles of astronomy, as al-Ṭūsī also explained, included statements that asserted the spherical shape of the universe and of all bodies in the celestial part of it, the circular and uniform motion of all such bodies, the impossibility of the void and incorruptibility of the heavens, the impossibility for any celestial body of combining contrary motions or tendencies toward such motions (Ibid., fols. 4b–5a). These, and other principles, constituted boundary conditions that a wholly successful solution of a theoretical problem in astronomy was desired to satisfy. Sometimes we find them dogmatically repeated in philosophical writings whose authors were not active or successful in astronomical research. In the case of al-Ṭūsī, however, they formed the basis of inquiries that went beyond paying lip service to a dominant natural philosophy. And similar statements to those of al-Ṭūsī are also found in the *Nihāyat al-sūl* of Ibn al-Shāṭir in the fourteenth century.

4 See the article by the author referred to in n. 2, pp. 121–2. Also idem, "Ibn al-Haytham's treatise: Solution of difficulties concerning the movement of *iltifāf*," *Journal for the History of Arabic Science*, 3 (1979), Arabic text, pp. 183–210, esp. pp. 206–7; English summary, pp. 388–92.

5 On the Marāgha astronomers and Ibn al-Shāṭir see E. S. Kennedy, "Late medieval planetary theory," *Isis*, 57 (1966), pp. 365–75. In addition to the articles listed by Kennedy in this article (p. 365, n. 1 and p. 369, n. 9), see Willy Hartner, "Naṣīr al-Dīn al-Ṭūsī's lunar theory," *Physis*, 11 (1969), pp. 287–304; idem, "Copernicus, the man, the work, and its history," *Proceedings of the American Philosophical Society*, 117 (1973), pp. 413–22; idem, "Ptolemy, Azarquiel, Ibn al-Shāṭir, and Copernicus on Mercury: A study of parameters," *Archives Internationales d'Histoire des Sciences*, 24 (1974), pp. 5–25; Bernard R. Goldstein, "Remarks on Ptolemy's equant model in Islamic astronomy," in *Prismata* (Festschrift für Willy Hartner), ed. Y. Maeyama and W. G. Saltzer, Wiesbaden: Franz Steiner Verlag, 1977, pp. 165–8; George Saliba, "The original source of Quṭb al-Dīn al-Shīrāzī's planetary model," *Journal for the History of Arabic Science*, 3 (1979), pp. 3–18 (includes an extract from the Arabic text of a work by al-ʿUrḍī); idem, "The first non-Ptolemaic astronomy at the Marāghah School," *Isis*, 70 (1979), pp. 571–6; idem, "A Damascene astronomer proposes a non-Ptolemaic astronomy," *Journal for the History of Arabic Science*, 4 (1980), Arabic text pp. 3–17; English summary, pp. 97–8. A volume edited by E. S. Kennedy and ʿImād Ghanem, *The Life and Works of Ibn al-Shāṭir*, Aleppo: Institute for the History of Arabic Science, 1976, includes many of these articles. To this must be added "Les sphères célestes selon Nasîr-Eddîn Attûsî" [which includes a French translation of chap. 11 of al-Ṭūsī's *Tadhkira*, made from the Paris MS Bibl. Nat. ar. 2509], published as Appendix VI in P. Tannery, *Recherches sur l'histoire de l'astronomie ancienne*, Paris: Gautier-Villars & Fils, 1893, pp. 337–61.

6 See Bernard R. Goldstein, *Al-Biṭrūjī: On the Principles of Astronomy*, an edition of the Arabic and Hebrew versions with translation, analysis, and an Arabic-Hebrew-English glossary, vol. I (analysis and translation), vol. II (the Arabic and Hebrew versions), New Haven, Conn.: Yale University Press, 1971. Also Francis J. Carmody, Al-Biṭrûjî: *De motibus celorum*, Critical edition of the Latin translation of Michael Scot, Berkeley: University of California Press, 1952.

7 These are al-Biṭrūjī's words: "As you know, brother, Abū Bakr ibn al-Ṭufayl, may

the exalted God have mercy on him, used to tell us that he had come upon (*ʿathara ʿalā*) an arrangement (*hayʾa*) and principles (*uṣūl*) for these [planetary] motions other than those two principles which Ptolemy had laid down, that is without in any way assuming an eccentric or an epicyclic sphere, while maintaining at the same time all these motions without entailing anything impossible. He promised to write on this matter, and his rank in learning is such that ignorance cannot be attributed to him [reading: *lā yujahhal*]. I have continued to think about this from the time I heard it from him, searching the statements of those who came before us, but without finding anything that relates to it other than a few hints, such as the statement of the Philosopher in the Second Treatise of the book *On the Heavens* – [namely:] We say further that two motions [properly] belong to the spherical body (*al-jism al-mustadīr*), one circular [?or rotational] [*mustadīra*) and the other a turning round in the manner of a spiral (*idāra lawlabiyya*) [*kulisis kai dinēsis*] (my translation: see Goldstein's edition referred to in n. 6, vol. II, p. 49, and Aristotle's *De caelo*, II, 8, 290ᵃ10). The Arabic translation of *De caelo* (?by Yūḥannā al-Biṭrīq) published by ʿA. Badawī has *mutaqalliba* (rolling) for *mustadīra* in al-Biṭrūjī's quotation (Aristotelis *De Caelo et Meteorologica*, Cairo: Maktabat al-Nahḍa, 1961, pp. 259–60). In *De caelo*, II, 8, Aristotle is concerned to argue that the stars and the planets have neither a rolling (*kulisis*). nor a rotational motion (*dinēsis*). Al-Biṭrīq's translation refers to the second of these motions by a number of confusing expressions: *idāra lawlabiyya* (turning in the manner of a spiral, or whirling), *dawriyya* (circular or rotational), *multawiya mudawwara* (twisting and rotating), *multawiya mutalaffifa* (twisting and winding). Rolling motion (*kulisis*) is, however, happily rendered by *mutaqalliba* and *mudaḥraja*.

Al-Biṭrūjī quotes only the opening lines in Aristotle's passage, but his text calls one of the two proper motions for a sphere *mustadira* (circular) and the other *idāra lawlabiyya* (turning in the manner of a spiral). *If* we assume that the two motions are referred to in the same order in all three texts (i.e., Aristotle's Greek text, al-Biṭrīq's Arabic translation, and the translation as quoted by al-Biṭrūjī), then the same motion would be called rolling (*mutaqalliba*) in al-Biṭrīq's text and circular or rotational (*mustadīra*) in al-Biṭrūjī's quotation. Both texts, however, would call the second motion (*dinēsis*) a "turning in the manner of a spiral." In spite of what has been written on this subject (I have in mind particularly the article by F. J. Carmody, "The planetary theory of Ibn Rushd," *Osiris*, 10 [1952], pp. 556–86), the confusion surrounding the transmission of Aristótle's passage to al-Biṭrūjī and Averroes has yet to be sorted out. The confusion appears to have set in already at the first stages of the transmission, in the process of translating the passage into Arabic, and it is there that further research must begin.

8 Maimonides' critique and ultimate rejection of Ptolemaic astronomy *as a representation of the true arrangement of the heavenly spheres and of their motions* are set forth in chap. 24 of book II of his *Guide for the Perplexed* (see translation by S. Pines, University of Chicago Press, 1963, pp. 322–7). The opening two paragraphs, puzzling though they are at some points, outline some of the main reasons for this rejection: "You know of astronomical matters what you have read under my guidance and understood from the contents of the 'Almagest.' But there was not enough time to begin another speculative study with you. What you know already is that as far as the action of ordering the motions and making the course of the stars conform to what is seen is concerned, everything depends on two principles: either that of the epicycles or that of the eccentric spheres or on both of them. Now I shall draw your attention to the fact that both those principles are entirely outside the bounds of reasoning and opposed to all that has been made clear in natural science. In the first place, if one affirms as true the existence of an epicycle revolving round a certain sphere, positing at the same time that that revolution is not around the center of

the sphere carrying the epicycles – and this has been supposed with regard to the moon and to the five planets – it follows necessarily that there is rolling, that is, that the epicycle rolls and changes its place completely. Now this is the impossibility that was to be avoided, namely, the assumption that there should be something in the heavens that changes its place. For this reason Abū Bakr Ibn al-Ṣā'igh [Ibn Bājja] states in his extant discourse on astronomy that the existence of epicycles is impossible. He points out the necessary inference already mentioned. In addition to this impossibility necessarily following from the assumption of the existence of epicycles, he sets forth there other impossibilities that also follow from that assumption. I shall explain them to you now.

The revolution of the epicycles is not around the center of the world. Now it is a fundamental principle of this world that there are three motions: a motion from the midmost point of the world, a motion toward that point, and a motion around that point. But if an epicycle existed, its motion would be neither from that point nor toward it nor around it" (ibid., pp. 322–3).

Maimonides (as the first lines of this quotation show) was not unaware of the predictive power of Ptolemaic astronomy, but in his view the Ptolemaic system did not conform with the accepted principles of natural philosophy. But, unlike Ibn al-Haytham, Averroes, the Marāgha astronomers, and Ibn al-Shāṭir, he did not think it incumbent upon (or even possible for) the astronomer to discover *the true* arrangement: "However, I have already explained to you by word of mouth that all this does not affect the astronomer. For his purpose is not to tell us in which way the spheres truly are, but to posit an astronomical system in which it would be possible for the motions to be circular and uniform and to correspond to what is apprehended through sight, regardless of whether or not things are thus in fact . . . However, regarding all that is in the heavens, man grasps nothing but a small measure of what is mathematical; and you know what is in it. I shall accordingly say in the manner of poetical preciousness: *The heavens are the heavens of the Lord, but the earth hath He given to the sons of man.* I mean thereby that the deity alone fully knows the true reality, the nature, the substance, the form, the motions, and the causes of the heavens. But He has enabled man to have knowledge of what is beneath the heavens, for that is his world and his dwelling-place in which he has been placed and of which he himself is a part. This is the truth. For it is impossible for us to accede to the points starting from which conclusions may be drawn about the heavens; for the latter are too far away from us and too high in place and in rank" (ibid., pp. 326–7). It is seen that Maimonides' position here has much in common with that of Osiander, who, in a well-known preface to Copernicus's *De revolutionibus*, maintained that truth was beyond the reach of the astronomer and should be of no concern to him, the only valid aim of the science of astronomy being to provide tools for accurate predictions.

9 See Pines's introduction to his translation of the *Guide*, (n. 8), p. cviii.

10 Ibid., p. cix.

11 Maimonides' words: "I have heard that Abū Bakr [Ibn Bājja] has stated that he had invented [*awjada*] an astronomical system in which no epicycles figured, but only eccentric circles. However, I have not heard this from his pupils. And even if this were truly accomplished by him, he would not gain much thereby. For eccentricity also necessitates going outside the limits posed by the principles established by Aristotle, those principles to which nothing can be added" (ibid., p. 323).

12 Jābir ibn Aflaḥ (who flourished at Seville in the first half of the twelfth century) wrote an *Iṣlāḥ* or emendation of the *Almagest* that was known to Maimonides and to al-Biṭrūjī and was epitomized by Quṭb al-Dīn al-Shīrāzī in the thirteenth century (see the article on Jābir by R. P. Lorch in *Dictionary of Scientific Biography*, VII [1973], pp. 37–9). Ibn al-Shāṭir in the fourteenth century also knew some of Jābir's views (such as placing Venus above the sun – contrary to Ptolemy) either directly through being acquainted with the *Iṣlāḥ* or indirectly through the writings of Mu'ayyad al-Dīn al-'Urḍī and al-Shīrāzī (see Ibn al-Shāṭir's *Nihāyat al-sūl*, Bodleian MS Marsh 139 [dated A.H. 768], fol. 4ᵛ). Jābir's criticisms of Ptolemy did not, however, extend to the principles of eccenters and epicycles as such.

13 See E. S. Kennedy's review of Goldstein's edition and analysis of al-Biṭrūjī's *Principles*; in *Journal for the History of Astronomy*, 4 (1973), pp. 134–6, esp. p. 134: "The reviewer would occasionally have preferred a more literal terminology. For instance, the translation frequently cites the 'polar epicycle' . . . However, the text has *dā'irat mamarr*, literally 'passage circle.' Here the translation, to some degree, begs the question of interpretation."

14 E. S. Kennedy, review of Francis J. Carmody's edition of Michael Scot's translation of al-Biṭrūjī's work; in *Speculum*, 29 (1954), pp. 246–51, esp. p. 248.

15 Review cited in n. 13, p. 136.

16 J. L. E. Dreyer, *A History of Astronomy from Thales to Kepler*, Dover, 1953, p. 267.

17 See introduction to Carmody's edition cited in n. 6, p. 11.

18 Goldstein, *al-Biṭrūjī*, I, p. 14.

19 Kennedy, *Jour. Hist. Astronomy*, 4 (1973), p. 136.

20 *Speculum*, 29 (1954), p. 251

21 Goldstein, *al-Biṭrūjī* pp. 427–9 (my translation).

22 See A. J. Aiton, *The Vortex Theory of Planetary Motions*, London: Macdonald, and New York: American Elsevier, 1972, esp. chap. 9.

23 Some of the strongest expressions of admiration for Aristotle occur in the *Middle Commentary* on the *Meteorology*, where Averroes lavishes on the Greek philosopher such descriptions as infallible, singled out by God for perfection, always correct regardless of the occasional criticism of his commentators. Aristotle is also said here to be aided by divine power and to have abrogated the philosophy of his predecessors. This attitude found good reception among Jewish philosophers, some of whom went one step further and claimed that Aristotelian science had been derived from the ancient Hebrews. See Irving Maurice Levey, "The Middle Commentary of Averroes on Aristotle's Meteorologica (Ph.D. diss. Harvard University, 1947), pp. 36ff. See later remarks on ancient wisdom.

24 Cf. S. Munk, *Mélanges de philosophie juive et arabe*, Paris: J. Vrin, 1955, p. 423. The *Middle Commentary on the Meteorology* was written after 566/1170–1 (ibid., p. 422).

25 *Talkhīṣ Mā Ba'd al-Ṭabī'a*, ed. 'Uthmān Amīn, Cairo: Al-Bābī al-Ḥalabī, 1958, pp. 130–1. The numbers add up to 37, not 38. The larger number would be obtained if an epicyclic model for the sun were adopted. There is a German translation of the *Talkhīṣ* by Simon van den Bergh, *Die Epitome der Metaphysik des Averroes*, Leiden: Brill, 1924; and an earlier Spanish translation by Carlos Quirós Rodríguez: Averroes, *Compendio de metaphysica*, texto arabe con traducción y notas, Madrid, 1919.

26 Regarding Aristotle's calculation (or miscalculation) of the number of agent and reagent spheres, see article by G. E. L. Owen, "Aristotle: Method, Physics and Cosmology," in *Dictionary of Scientific Biography*, I (1970), esp. pp. 257–8.

27 Aristotle's text, as quoted by Averroes in his larger and later Commentary (*Tafsīr*), may be translated as follows: "As for the number of motions (*kathrat al-ḥarakāt*), we ought to infer it (*nastadillu 'alayhā*) from what is said regarding the motions of the planets in [that part of] philosophy that is especially concerned with the mathematical sciences" (Bouyges's edition, vol. III, p. 1646). And again, "As for the number of these [motions]

we shall now state [it] ourselves in accordance with what some mathematicians have said, so that it may be imagined in some way (*yutawahhamu bi-naw'in mā*) and in order that our mind (*fikrunā*) may receive a determinate number (*kathra mahdūda*). As for what remains of that [inquiry], we may either search for it ourselves or base our investigation on those who searched for [those motions] if something other than what has just been said becomes manifest to those who deal with these matters. We must, however, appreciate (*nuhibb*) both parties but follow those whose investigation is more thorough" (ibid., vol. III, pp. 1657–8; compare *Metaphysics* XII, viii, 7–8, Loeb edition, vol. 2, London, 1962, pp. 155–7).

28 It may be said that a departure from Aristotle with regard to the number of celestial motions is not the same thing as a departure from Aristotelian cosmology. True. But it should be observed that in the passage quoted from the *Talkhīṣ* Averroes shows no qualms about eccenters or epicycles: The motion associated with the sun may be one (if the sun moves in an eccentric circle or sphere), or two (if the sun moves in an epicycle carried around by a deferent).

29 *Talkhīṣ* (n. 25), p. 133. The psychobiological view of the universe Averroes adopts here from Ptolemy's *Planetary Hypotheses* is so little known and so frequently overlooked that it deserves to be quoted at some length. Ptolemy wrote in the second part (lost in Greek but extant in Arabic) of that work: "Physical reasoning (*al-qiyās al-ṭabī'ī*) leads us to say that the ethereal bodies do not admit of being acted upon nor suffer change, though they vary [their positions] throughout time, inasmuch as this befits their wonderful substance and accords with the power of the stars (*al-kawākib*) which are [embedded] in them, [stars] whose light clearly goes through all these things that spread around them (*al-mabthūtha ḥawlahā*) without hindrance or affection – in the same way that [those bodies] are penetrated by what is in us that is homogeneous with them (*mimmā yujānisuhā*), such as sight and understanding (*al-fahm*). We are also led to maintain the unchangeability of ethereal bodies by our doctrine that their shapes are circular and that their actions are the actions of things whose parts are uniform. Now for each one of these movements that differ in respect of quantity or kind there exists a body that moves about poles in its proper place with a volitional movement (*ḥaraka irādiyya*) in accordance with the power of each one of the planets from which there originates the movement which proceeds from the governing faculties (*al-quwā al-ra'īsa* = ?*ta hēgemonika*) that resemble the faculties residing in us, and which [movement] imparts as much motion to the bodies which are homogeneous with them [i.e., the planets] and which are like parts of the universal animal, as is proportionately suitable to each of them." (B. Goldstein, ed., *The Arabic Version of Ptolemy's Planetary Hypotheses, Transactions of the American Philosophical Society*, new ser., vol. 57, pt. 4, 1967, p. 36).

Ptolemy continues a few pages later: "Now if someone imagines that the earth is at the center and that the air and fire revolve along with the revolution of what envelops them and forces them to move; further, if he takes the observable behavior of birds as an analogy (*mithāl*) for the motion of what exists in the heavens; then such an analogy (*mā maththala* [or *muththila*] *min hālik*) would not be objectionable. For the proper motion of birds proceeds first from the psychic faculty (*al-quwwa al-nafsāniyya*) that resides in them, then the impulsion (*inbi'āth*: ? *hormē*) produced by this faculty goes forth to the nerve, then – in [our] example – to the legs or hands or wings, and there the process of being passed on from one thing to another comes to a stop. . . . We should similarly imagine the situation with respect to the celestial animal (*al-ḥayawān al-falakī*). We should, that is, consider that each of the planets has the rank of a governing power (*quwwa siyāsiyya*), and that it possesses a psychic faculty and that it moves itself and naturally confers motion upon the bodies adjacent to it: first to that which lies close to it, then to the next; for example, it conveys motion first to the epicyclic sphere, then to the eccentric sphere, then to the sphere whose center is the center of the world. This motion conferred by the [planet] varies in the several places, just as in our [human] case the motion of the muscle (*al'aḍal*) is not similar to that of the impulsion (*inbi'āth*) nor the latter to that of the nerve nor this to that of the leg; but [all] vary somewhat as they proceed outward" (ibid., pp. 40–1).

30 *Talkhīṣ* (n. 25), pp. 133–4.

31 A reference to double elongation and prosneusis, which Averroes calls, respectively, *al-ḥaraka al-muḍā'afa* and *ḥarakat al-muḥādhāh*. *Tafsīr* (n. 27), vol. III, p. 1659.

32 *Tafsīr* (n. 27), vol. III, p. 1661.

33 Ibid. Emphasis added.

34 Ibid., vol. III, pp. 1661–4. I have divided the text into paragraphs and supplied them with letters in square brackets. In connection with Averroes's reference to *Planetary Hypotheses* in [d] see my article cited in n. 4.

35 See F. A. Yates, *Giordano Bruno and the Hermetic Tradition*, London: Routledge & Kegan Paul, 1964; D. P. Walker, *The Ancient Theology*, Ithaca, N. Y.: Cornell University Press, 1972; J. E. McGuire and P. M. Rattansi, "Newton and the 'pipes of Pan,'" *Notes and Records of the Royal Society of London*, 21 (1966), pp. 108–43.

36 I. Goldziher, *Le livre de Mohammed ibn Toumart, mahdi des Almohades. Texte arabe accompagné de notices biographiques et d'une introduction.* Alger: Imprimerie Orientale Pierre Fontana, 1903. See introduction, pp. 1–101.

37 G. E. Hourani (trans.), *Averroes on the Harmony of Religion and Philosophy*, London: Luzac & Co., 1976 (first printed in 1961), pp 70–71 and n. 197, pp. 116–117. In the same passage Averroes also acknowledges his rulers' "many benefits, especially to the class of persons who have trodden the path of study and sought to know the truth" – obviously a reference to such scholars as Ibn Ṭufayl and Averroes himself. Averroes was here envisaging nothing less than an alliance between himself and the ruling authorities against the "turbulent" dialectical theologians (or'practitioners of *kalām*) and the slavish followers of legal authorities. All this, of course, was in keeping with his well-known elitist approach to the study of philosophy.

38 An edition by Shawqī Ḍayf of Ibn Maḍā"s "Refutation of the Grammarians," *Kitāb al-Radd 'alā al-nuḥāh*, was published in 1947 (Cairo: Dār al-Fikr al-'Arabī). See editor's introduction.

39 Ibn Rushd, *Tahafot al-Tahafot*, texte arabe établi par M. Bouyges, S. J., Beirut: Imprimerie Catholique, 1930. English translation and extensive notes by Simon van den Bergh as Averroes's *Tahafut al-Tahafut* (*The Incoherence of the Incoherence*), 2 vols., London: Luzac & Co., 1969.

40 Averroes's criticisms of the earlier Islamic philosophers are not, of course, confined to the *Incoherence of the Incoherence*. Reference may be made here to his *Sermo de substantia orbis* (dated 1178 – Munk, *Mélanges*, n. 24, p. 423) where, in the words of Arthur Hyman, he "refutes what he considers the erroneous opinions of earlier philosophers, especially those of Avicenna, and reestablishes what in his opinion are the pure Aristotelian doctrines" (Arthur Hyman, Ph.D. diss., Harvard University, 1953, p. iii). The following passage from the same work (Hyman's translation from the Hebrew version) reveals the exaggerated view of Aristotle's position in the history of thought that was mentioned earlier (see n. 23): "The starting point of the investigation is what we have gathered from Aristotle concerning these matters. For concerning existent things no opinion has reached us from the ancients which is truer than his, or less subject to doubt

or presented in better order. Therefore we take his opinion to be that human opinion which man can attain by nature, that is it is the most advanced of those opinions which man, insofar as he is man, man by his own knowledge and intellect obtain. Thus, as Alexander puts it, "It is Aristotle on whom we are to rely in the sciences' " (Hyman, p. 146).

41 Of the seven references to al-Fārābī in the *Incoherence of the Incoherence*, five are critical. In the last two, Averroes draws support from al-Fārābī against Avicenna, but in general the two Eastern philosophers are bundled together as misguided interpreters of Aristotle who had misled al-Ghazālī from the true doctrines of the ancients. See indexes of Bouyges's edition of the *Tahafot* and van den Bergh's translation, both cited in n. 39.

42 In *Kitāb al-Kulliyyāt* (Latin *Colliget*), a compendium on the generalities of medicine composed before 1162, Averroes accused "the man known as al-Kindī" of having misled people by having introduced into medicine (a natural science) considerations that properly belonged to arithmetic and harmonics (*ṣinā'at al-'adad wa ṣinā'at al-mūsīqā; artem alhabachi vel algorismi et musice*), a reference to al-Kindī's treatise on compound medicines. See Abu el Ualid . . . ben Roxd . . . (Averroës), *Quitab el Culiat* (*Libro de las Generalidades*, Larache (Morocco): Publicationes de Instituto General Franco para la Investigación Hispano-Árabe, 1939, p. 168 (this publication being a facsimile of the Arabic text from a MS dated 583/1187, not 1186). For the Latin translation of the *Colliget*, see Averrois Cordubensis *Colliget Libri VII*, in *Aristotelis Opera cum Averrois commentariis*, supp. I (Venice, 1562; repr. Frankfurt am Main, 1962). For a new edition of the passage cited here, see M. R. McVaugh, *Arnaldi de Villanova Opera Medica Omnia II: Aphorismi de Gradibus*, Granada-Barcelona, 1975, esp. p. 323, lines 30ff. The Arabic text and a French translation of al-Kindī's treatise have been published by Léon Gauthier, *Antécédents greco-arabes de la psycho-physique*, Beirut, 1939.

43 Averroes's criticism of Ibn al-Haytham, like his criticism of al-Kindī, concerns the question of the relation of physics to mathematics. In the *Middle Commentary* (*Expositio Media*) *on Aristotle's Meteorology*, Averroes contrasts Aristotle's idea of subordination of sciences (when the conclusions of one science are taken as hypotheses in another) with Ibn al-Haytham's view of optical inquiry as "composed of" physics and mathematics, which view Averroes takes to be a wrong departure from Aristotle. See A. I. Sabra, "The physical and the mathematical in Ibn al-Haytham's theory of light and vision," in *Commemoration Volume of Bīrūnī International Congress in Tehran*, Tehran: High Council of Culture and Art, 1976, pp. 439–78, esp. pp. 448–50.

44 "The most temperate lands (*al-bilād*) are those in which autumn is short and springtime is long, and these are the lands that lie in the fifth climate, especially those among them that are close to the sea. Now autumn in these lands of ours, namely the lands of al-Andalus, is about two months and they lie at the beginning of the fifth climate. There is no temperate time (*zamān mu'tadil*) below the equator, as is claimed by many people, and we have shown this in what we have written elsewhere; nor is [any] part of the fourth climate better in any way than the fifth (*wa lā ayḍan yafḍdulu* [ms. *yfḍl*] *min ba'ḍ al-iqlīm al-rābi' a'alā al-khāmis bi-shay'*). And Galen is of the opinion (*yarā*) that the most temperate places are the lands of the Greeks (*bilād yūnān*) among which is the land of Hippocrates." (*K. al-Kulliyyāt*, n. 42, p. 42; *Colliget*, Venice ed. cited. supp. I, fol. 32ᵛ.)

45 However, even if we accept that they [the Greeks] are the most disposed by nature to receive wisdom, we cannot disregard [the fact] that individuals like these – i.e. those disposed to wisdom – are frequently to be found. You find in this the land of the Greeks and its vicinity, such as this land of ours, namely Andalus, and Syria and Iraq and Egypt, albeit this existed more frequently in the land of the Greeks" (*Averroes on Plato's "Republic"*, trans. Ralph Lerner, Ithaca, N.Y.: Cornell University Press, 1974, p. 13; first bracket added by the present writer).

46 "Et ideo nos videmus hanc terram nostram magis propinquam in sua natura terris Graecorum quam terrae Babyloniae. Clima autem aequale seu contemperatum est quintum, ut inquit Galenus, non quartum, ut crediderunt multi homines. Signum autem huius est quod illic inveniuntur complexiones aequales magis. Signum autem forte super complexiones est color et capilli. Color autem aequalis est albus et clarus; capillus autem aequalis est magis propinquus ad illum qui est quasi medius inter planitiem et crispitudinem seu ad planum quam ad crispitudinem. Esse autem huius coloris et capilli parum invenitur in terra Arabum, et ideo vocant album rubeum; terrae autem Babyloniae sunt mediae ad terras Arabum, scilicet quod color brunus dominatur super eos homines, sicut est dispositio Arabum; iste autem color in capillis invenitur naturaliter, scilicet magis in hominibus climatis quinti, quando non coniunguntur cum gentibus extraneis, sed cum habitantibus extraneis quae habitant illic de propinquo. Sed, cum prolongaverit tempus in istis hominibus, tunc redit natura eorum ad naturam illorum hominum illius climatis, sicut accidit filiis Arabum et Barbarorum in terra Andalosiae, scilicet quod ipsi conversi sunt ad naturam gentis propriae illi terrae, ideo multiplicatae sunt in eis scientiae . . ." (*Aristotelis Opera cum Averrois Commentariis*, n. 42, V. fols. 435ᵛ–436ʳ).

XVI

A TWELFTH-CENTURY DEFENCE OF THE FOURTH FIGURE OF THE SYLLOGISM*

It was widely believed until recently that the fourth figure, omitted by Aristotle in his systematic treatment of the syllogisms, was invented by Galen in the second century A.D. This view is known to have been first circulated in the West through the work of Averroes.[1] It is, however, contradicted by Galen himself in the *Introduction to Logic*, the only complete logical work of his that has survived in Greek. Galen says in this work that there cannot be more than three figures for the categorical syllogisms, adding that he has shown this in his book *On Demonstration.*[2] A new light was thrown on this question when J. Lukasiewicz drew attention to a published Greek scholium which may be taken to explain how the doctrine of four figures came to be attributed to Galen. The unknown scholiast divides categorical syllogisms into simple and compound. In accordance with Aristotle, he divides simple syllogisms consisting of three terms into three figures. Then he continues: 'Galen, however, says in his *Apodeictic* that there are four figures, because he looks at the compound syllogisms consisting of four terms, as he has found many such syllogisms in Plato's dialogues.'[3] It would thus appear that someone who misunderstood the relevant passage in Galen's book, or who had simply heard about 'Galen's four figures', assumed that Galen's doctrine applied to simple, not compound syllogisms.[4]

It is known that Galen's *Demonstration* was translated into Arabic in the second half of the ninth century.[5] Although this Arabic version eventually

* I wish to thank the authorities of the Süleymaniye Library in Istanbul for their kindness in providing a microfilm of the Aya Sofya MS. 4830 on which this article is based.

[1] Averroes ascribes the fourth figure to Galen both in his *Expositio media* (Bk. I, Chaps. 8 and 23) and *Epitome* (Ch. 1) of the *Prior Analytics*; he does not indicate his source in either place. His reason for rejecting the fourth figure is not unlike that of Avicenna (see Part III of the present article), namely that the intellect does not grasp this figure '*secundum naturam*'. See the Juntine edition of Aristotelis *Opera cum* Averrois *Commentariis* (Venice, 1562/74 — photographic reprint, Frankfurt-am-Main, 1962), vol. i, part 1, fols. 24r, 69v; vol. i, part 2b, fol. 45v. Carl Prantl quotes the relevant passages from Averroes in *Geschichte der Logik*, Leipzig, 1927, i, p. 571, n.99; ii, p. 390, n.322.

[2] Galeni *Institutio Logica* (ed. C. Kalbfleisch), Leipzig, 1896, p. 26; German translation in Jürgen Mau, *Galen: Einführung in die Logik. Kritisch-exegetischer Kommentar mit deutscher Übersetzung*, Berlin, 1960, p. 14 (of the separately bound translation). See William and Martha Kneale, *The Development of Logic*, Oxford, 1962, p. 183.

[3] Jan Lukasiewicz, *Aristotle's Syllogistic*, 2nd enlarged ed., Oxford, 1957, p. 39. (1st ed., 1951.)

[4] *Ibid.*, pp. 41–42.

[5] Not all of the 15 books (*maqālāt*), of which the Greek text of Galen's *Demonstration* consisted, were available to the Arabs. Jibrīl ibn Bakhtīshū' (d. 828–29) found some maqālas which were then translated for him by Job of Edessa. In spite of his extensive search, Ḥunayn ibn Isḥāq could only obtain 'about half' of the whole work. The books which he found, and which he later (before 856) translated into Syriac, comprised the following: a small part of Bk. II, most of Bk. III, about the first half of Bk. IV, Bk. IX except for some part at the beginning, Bks. X–XIV, and Bk. XV wanting a part at the end. All that Ḥunayn had found was later translated into Arabic, Bks. II–XI by his pupil 'Īsā ibn Yaḥyā, and Bks. XII–XV by Isḥāq ibn Ḥunayn. Cf. G. Bergsträsser, *Ḥunain ibn Isḥāq über die syrischen und arabischen Galen-Übersetzungen*, Leipzig, 1925, pp. 47–48 (Arabic

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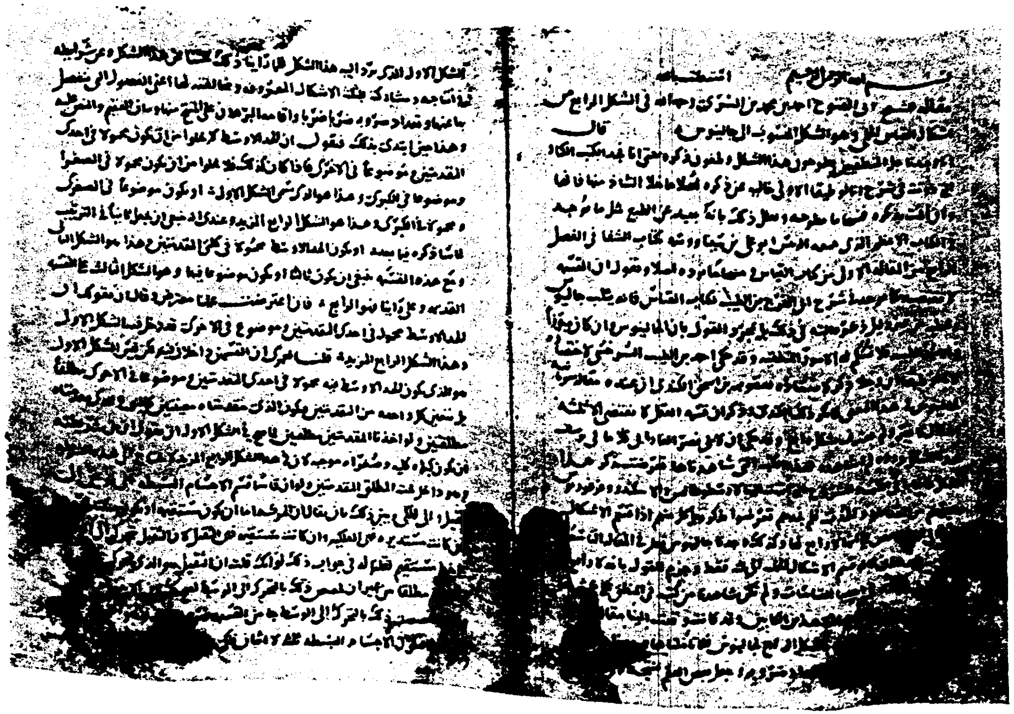
fol. 123r

fol. 122v



MS. Aya Sofya 4830

2

fol. 124r fol. 123v

fol. 125r fol. 124v

3

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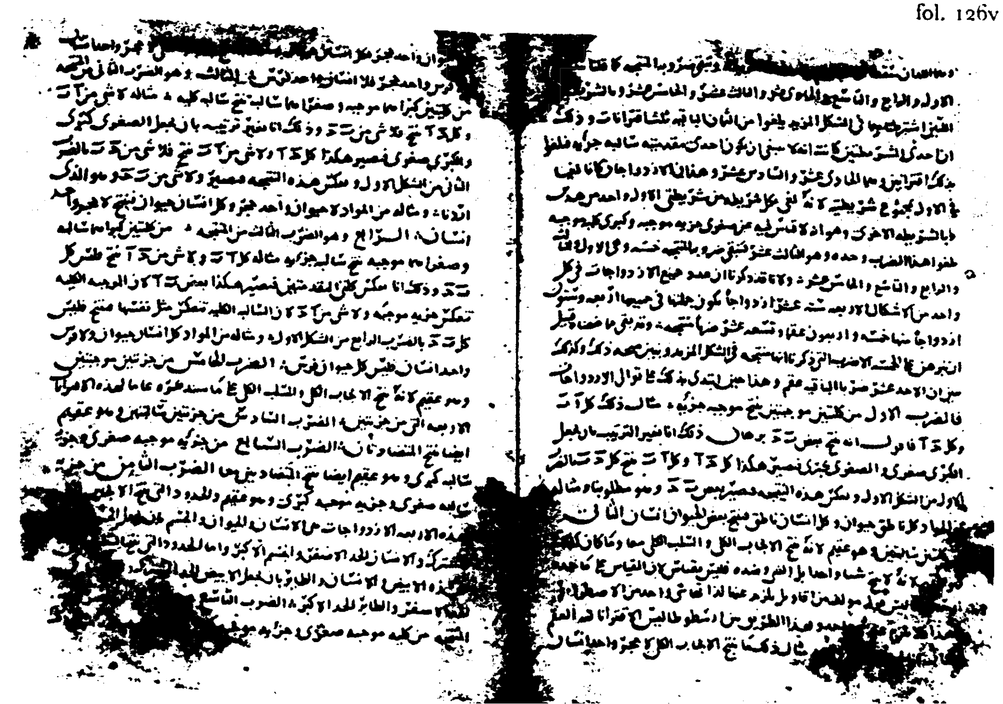

XVI

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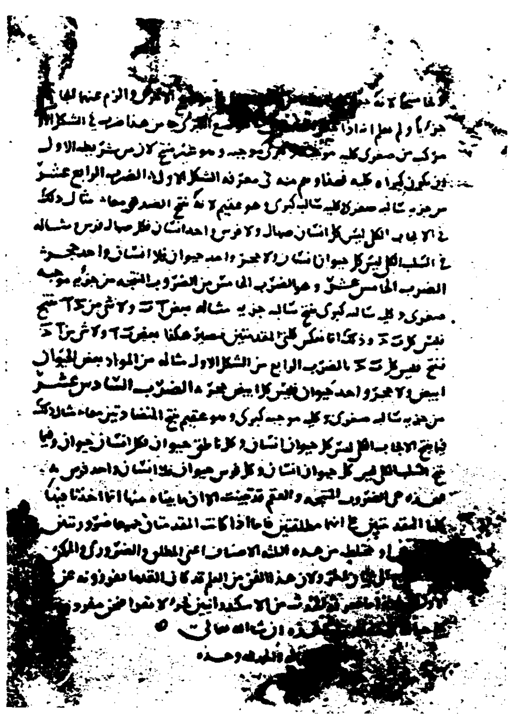

suffered the same fate as the Greek original,[6] or so it seems from the imperfect knowledge we have of the contents of the many oriental libraries, we now know that it was obtainable until at least three centuries later.[7] It is, therefore, only natural to turn to the Arabic sources for more light on the historical problems connected with the fourth figure. The present article is a first attempt in this direction. It is based on a twelfth-century Arabic work, entitled 'On the fourth figure of the categorical syllogism, which is the figure attributed to Galen'. The author, Najm al-Dīn Abū-l-Futūḥ Aḥmad ibn Muḥammad ibn al-Sarī, also known as Ibn al-Ṣalāḥ, was educated at Baghdād where he studied logic, mathematics and medicine; he later settled in Damascus where he died in A.H. 548 = A.D. 1153–54.[8] Several of his works (all short treatises) on problems of mathematics, astronomy and physics have survived in manuscripts mainly at Istanbul, Leiden and Oxford;[9] none has been edited, translated or adequately studied.[10] The treatise on the fourth figure exists in the

text), pp. 38–39 (German translation). See also Max Meyerhof, 'New light on Ḥunain ibn Isḥāq and his period', *Isis*, VIII, 1926, pp. 687, 690, 700, 707 and 718.

We learn from Ibn al-Sarī's treatise that the discussion of the fourth figure in Galen's *Demonstration* occurred in Bk. IX (see Part II of this article).

[6] A manuscript copy of Abū Bakr al-Rāzī's *al-Shukūk 'alā Jālīnūs* (*Dubitationes in Galenum*), discovered by Paul Kraus, has been reported to contain extensive quotations from Galen's *Demonstration* (see S. Pines, 'Rāzī Critique de Galien', *Actes du Septième Congrès International d'Histoire des Sciences*, 1953, pp. 485–86). An edition is being prepared by Professor Pines.

[7] Ibn al-Qifṭī (d. 1248) noted that only a part of Galen's *Demonstration* was extant: '*almawjūdu ba'ḍuhu*' (*Ta'rikh*, p. 131). This does not necessarily mean that the extant part survived until his time; he may have been simply reporting the statement of Ḥunayn ibn Isḥāq (see note 5 above) that he could not obtain a complete copy of the Greek text.

[8] Cf. Ibn al-Qifṭī, *Ta'rikh* (ed. A. Müller and J. Lippert), Leipzig, 1903, p. 428; Ibn Abī Uṣaybi'a, *'Uyūn* (ed. A. Müller), Cairo-Königsberg, ii, 1882, pp. 164–67. H. Suter, *Die Mathematiker und Astronomen der Araber und ihre Werke*, Leipzig, 1900, No. 287, p. 120; C. Brockelmann, *Geschichte der arabischen Litteratur* (GAL) i, Leiden, 1943, p. 621, Supplementband i, Leiden, 1937, p. 857.

According to Ibn al-Qifṭī, Ibn al-Sarī was a Sumaysāṭī, that is from Sumaysāṭ in northern Mesopotamia (see Yācūt, *Mu'jam al-buldān* (ed. F. Wüstenfeld, iii, Leipzig, 1868, pp. 151–52). Ibn Abī Uṣaybi'a asserts, however, that he was a Persian who came from Hamadān. Our author's name has been vocalized Sarī, Surā and Surrī. Suter has all three forms in his *Mathematiker* (1900), but he later suggested that perhaps the last form should be preferred to the other two (cf. H. Suter, 'Einige geometrische Aufgaben bei arabischen Mathematikern', *Bibliotheca Mathematica*, 3 Folge, viii, 1907, p. 24, n.2). On this suggestion, the origin of our author would be Surr, a village near Rayy (see al-Sam'ānī, *Kitāb al-ansāb*, ed. D. S. Margoliouth, Leiden, 1912, p. 298. Suter refers to As-Sojutii *De Nominibus relativis*, ed. P. J. Veth, Leiden, 1840, p. 136). I have followed the vocalization in the Leipzig edition of Ibn al-Qifṭī's *Ta'rikh* (1903), but the matter is not settled.

[9] For manuscripts of Ibn al-Sarī's writings see, in addition to the works of Suter and Brockelmann mentioned in the previous note: M. Krause, 'Stambuler Handschriften islamischer Mathematiker', *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abteilung B: Studien, Vol. 3, Berlin, 1936, pp. 485–87; P. Voorhoeve, *Handlist of Arabic Manuscripts in the Library of the University of Leiden and Other Collections in the Netherlands* (Bibliotheca Universitatis Leidensis: Codices Manuscripti VII), Leiden, 1957, pp. 121 and 431. The Bodleian MS. Thurston 3 contains more items by Ibn al-Sarī than is indicated in J. Uri's *Catalogus* . . . , Oxford, 1787, Arab. Moh. 913, and, hence in Suter's *Mathematiker*; Brockelmann (*op. cit.*) does not refer to it.

[10] There exists, however, a brief account by Suter of three treatises of Ibn al-Sarī's which are preserved in a Leiden MS.; see H. Suter, 'Einige geometrische Aufgaben . . .', *loc. cit.*, pp. 30–33.

Istanbul MS. Aya Sofya 4830, folios 122v–128v. It is the first in a group of seven treatises by Ibn al-Sarī. At the end of the seventh treatise (fol. 160v) the copyist informs us that he transcribed this group in the month of Ramaḍān A.H. 626 (=A.D. 1229) at Damascus 'from a sound copy'.[11]

The aim of Ibn al-Sarī in his treatise is to establish the identity of the fourth figure, to enumerate and prove the valid moods in it and to disprove the invalid ones. Hence the importance of this work; written in the first half of the twelfth century, it is, as far as is known, the earliest defence of the fourth figure that has come down to us.[12] Furthermore, Ibn al-Sarī made an extensive study of the discussions about the fourth figure in the works of Greek and Arabic writers available to him, and he reports his findings in a prologue to his treatise. This report, in spite of its brevity, furnishes valuable information. Of particular interest is Ibn al-Sarī's reference to the Arabic versions of Galen's *Demonstration* and another of his logical works, which has also been lost, *On the Number of the Syllogisms*. In both of these works, we are told by Ibn al-Sarī, Galen definitely rejected a fourfold division of the figures. This is further evidence against the view that Galen was responsible for the addition of a fourth figure. Verification of a reference of Ibn al-Sarī's to Avicenna's *al-Shifā'* further reveals a statement by the latter to the effect that Galen rejected the fourth figure.[13]

It transpires from Ibn al-Sarī's prologue that the attribution of the doctrine of four figures to Galen was asserted in the Arabic period as early as the ninth century, in the time of al-Kindī (d. *c.* 873). Nevertheless, we learn that both al-Kindī and al-Fārābī (d. 950/51) rejected this doctrine; Avicenna (d. 1037) later took the same view. But there were those who assumed the fourth figure to have come from Galen; Ibn al-Sarī mentions two: the Nestorian philosopher Abū-l-Faraj ibn al-Ṭayyib (d. 1043/4) and 'Dinḥā the priest'. The latter was the author of the only defence of the fourth figure which Ibn al-Sarī had seen, but which, as we shall see, he found defective.

It is worth noting that when Ibn al-Sarī set out to defend and expound the theory of the fourth figure, he was fully aware of being in opposition to the views of all authorities known to him, from Aristotle to Avicenna. This he made no attempt to conceal; on the contrary, it is clear that he wrote his prologue to bring it out and to emphasize it.

In Part II of this article I shall give a translation of the prologue to Ibn al-Sarī's treatise, followed in Part III by a commentary on the translated text. Part IV will be devoted to an analysis of Ibn al-Sarī's arguments contained in the body of his treatise.

II

TRANSLATION OF THE PROLOGUE TO IBN AL-SARĪ'S TREATISE

(In the translation of this and other passages, parentheses include additions implied by the text, transliterations of Arabic words (in italics), and references to the manuscript

[11] The treatise on the fourth figure is one of two works ascribed to Ibn al-Sarī by Ibn Abī Uṣaybi'a (*'Uyūn*, ii, p. 167).

[12] Thus William and Martha Kneale note (*op. cit.*, p. 183) that 'we have no trace of anyone who defended the doctrine of four figures before the end of the Middle Ages.'

[13] See Part III below.

by numbers of folios and lines; brackets include additions by the translator. The same practice will be followed even when paraphrasing the text in Part III.)

'A treatise by al-Shaykh Abū-l-Futūḥ Aḥmad ibn Muḥammad ibn al-Sarī —may God be merciful to him—on the fourth figure of the categorical syllogism, which is the figure attributed (*mansūb*) to Galen.

> He said: We have found most logicians omit (*yaṭrahūna*) this figure and fail to mention it (*yalghūna dhikrah*), so that the large commentaries on the *Prior Analytics* do not refer to it at all. The exceptions which do mention it either discard it (*yaṭrahūnahu*) for the reason that it is remote from nature (*ba'īd 'an al-ṭab'*)—as we find in the greatest book which the Ra'īs Abū 'Alī ibn Sīnā (Avicenna) has collected (*jama'ahu*) and called *al-Shifā'*, Maqāla I, Faṣl IV of Kitāb al-Qiyās—or completely reject it (*yarudduhu aṣlā*), saying that it is not required by the division [of the figures], as we find in the *Commentary* of Abū-l-Faraj ibn al-Ṭayyib *on kitāb al-qiyās*. Thus he criticizes (*yathlibu*) Galen and accuses him of error, not by producing any evidence whatsoever, but by merely saying that while Galen distinguished himself in medical matters, he is not to be trusted in matters of logic. Aḥmad ibn al-Ṭayyib al-Sarakhsī said in his *Abridgement of the Analytics* that a man mentioned to his master Ya'qūb ibn Isḥāq al-Kindī that he possessed a Syriac treatise of Galen's expressing the same opinion (*fī hādhā-l-ma'nā*). But al-Kindī denied (*ankara*) that, and said that logical division (*qismat al-'aql*: mental division) requires but three figures, no more, and he did not recognize a fourth figure. It has also been related that Abū Naṣr al-Fārābī has a discourse on the invalidation (*tazyīf*) and rejection (*radd*) of this figure which I have not seen.
>
> These, then, are the books we have found which refer to this figure. As for the other books and commentaries which have come down to us from Aristotle, Alexander, Porphyry and others among the ancients and moderns, we have found that they do not refer to it. Rather, they all divide the figures into three and state that there is no fourth. We have found that Galen did the same in the ninth Maqāla of (his) *Book of Demonstration*. For he divided the categorical figures into three only, and asserted (*jazama-l-qawl*) that they have no fourth; he did the same in his book *On the Enumeration of the Syllogisms*.[14] These are the only two of his logical books that we have seen, in spite of their large number indicated in the *Fihrist*.[15]
>
> Now there came our way a treatise by a man called Dinḥā the priest (*al-qass*), entitled *The Fourth Figure of Galen*. When we looked into it we

[14] *Fī iḥṣā al-qiyāsāt*. This is the *De syllogismorum numero* (Galeni *Opera*, ed. C. G. Kühn, vol. xix, p. 43). The Arabic title quoted by Ḥunayn and later by Ibn al-Nadīm and Ibn al-Qifṭī is *Fī 'adad al-maqāyīs* (*On the number of the syllogisms*). Ḥunayn translated this work into Syriac; his son Isḥāq later made an Arabic translation which Ḥunayn improved for 'Alī ibn Yaḥyā (Bergsträsser, *op. cit.*, p. 51 (Arabic), p. 42 (German). Another Arabic translation by Iṣṭifan (Stefanos) ibn Basīl is mentioned by Ibn al-Nadīm (*Fihrist*, I, p. 291) and Ibn al-Qifṭī (*Ta'rikh*, p. 132). See Meyerhof, *op. cit.*, pp. 705–6.

[15] This is Galen's *Pinax* (*De libris propriis*); the Arabic translation was made by Ḥunayn ibn Isḥāq. Cf. Bergsträsser, *op. cit.*, pp. 2, 3–4 (Arabic), pp. 2–3 (German); Meyerhof, *op. cit.*, p. 690.

found it defective . . . [in the conditions (he states) for (this) figure],[16] in the enumeration of its moods, and in (his) regarding some sterile (moods) as productive and in (his) badly understanding the first figure to which this figure is reduced. When we noted that, we inquired into this figure and into the conditions for its being productive and what it shares with the three known figures and what distinguishes it from them, I mean the *differentiae* (*al-fuṣūl*) which separate it from them. (We have also inquired into) the enumeration of its moods one by one, and we have proved those that are productive and shown those that are sterile and pointed them out. Now it is time for me to begin.' (Fols. 122b–123a 4.)

III

COMMENTARY

Ibn al-Sarī divides logicians with respect to their attitude towards the fourth figure into two main groups. The first group, forming the majority, simply ignored that figure altogether. The second group, consisting of only a few, he further divides into two. First, there were those who allowed that a complete enumeration of the figures required four divisions, but then argued for rejecting the fourth division omitted by Aristotle; the representative of this group was Avicenna. Second, there were those who maintained that the division of figures required no more than three. According to Ibn al-Sarī, this group included Abū-l-Faraj ibn al-Ṭayyib[17] as well as Galen himself. Note, however, that Ibn al-Ṭayyib must have ascribed to Galen either the doctrine of four figures or the half-way doctrine of a fourfold division; hence his 'criticism' of Galen reported by Ibn al-Sarī.

The statement about the 'Syriac treatise of Galen's' suffers from the vagueness of the expression '*fī hādhā-l-maʿnā*'; it is not clear which form of the doctrine of four figures this treatise maintained. That al-Kindī denied a fourfold division is unambiguously asserted; but whether his denial also related to the attribution of the rejected doctrine to Galen is not clear. Al-Sarakhsī's *Abridgement of the Analytics*, from which Ibn al-Sarī derived his information regarding the 'Syriac treatise' and al-Kindī's denial is not known to have survived.[18]

There are no known manuscript copies of al-Fārābī's 'discourse' on the

[16] The dots indicate two words which I could not read in the photocopy; the words included in the brackets represent what appears to be a plausible reconstruction of the text rather than a straightforward reading.

[17] According to Ibn al-Qifṭī (*Ta'rikh*, p. 223), Ibn al-Ṭayyib lived beyond A.H. 420 and probably died in A.H. 435 = A.D. 1043/44. He is there reported to have written commentaries on 'ancient logical books'; but no copies of the work mentioned by Ibn al-Sarī are known to have survived. On Ibn al-Ṭayyib see S. M. Stern, 'Ibn al-Ṭayyib's commentary on the Isagoge', *Bull. of Sch. of Oriental and African Studies*, xix, 1957, p. 422.

[18] Al-Sarakhsī died in 899. Ibn al-Qifṭī (*Ta'rikh*) attributes to him various 'abridgements' of the *Categoriae* (p. 35), an abridgement of *De interpretatione* (p. 36), and a book entitled *Kitāb anūlūṭiqā* (p. 78). The last work may be the one mentioned by Ibn al-Sarī; it is not, however, described by Ibn al-Qifṭī as an abridgement. See Brockelmann, GAL, i, Leiden, 1943, pp. 231–2; SI, Leiden, 1937, p. 375.

fourth figure. But Ibn al-Sarī's statement that al-Fārābī rejected this figure is borne out by another work which has survived; in his *Kitāb al-qiyās al-ṣaghīr*, al-Fārābī enumerates fourteen valid moods in the first three figures, and says 'these are all the categorical syllogisms'.[19]

Of all the Arabic works specifically mentioned by Ibn al-Sarī, only one, the *Shifā'* of Avicenna, has survived. The part of this work devoted to syllogistic, namely Fann IV of Jumla I, entitled 'al-Qiyās', has not yet been printed. For consulting the passage referred to by Ibn al-Sarī, I have relied on two manuscripts, the Bodleian MS. Pococke 122 and the India Office MS. Loth 475; Mr. Saʿīd Zāyid, who is currently editing this part of Avicenna's work in Cairo, has also kindly provided me with a transcription of this passage based on the manuscripts used for his edition. In the absence of a printed text, I shall refer here to the two manuscripts mentioned.

Avicenna first remarks that a simple categorical syllogism has two premisses and three terms of which one is common to both premisses.

> 'This term [he continues] is either a predicate of one [of the two other terms] and a subject to the other, or a predicate of both, or a subject to both. When it is a subject to one and predicate of the other, it is either a predicate of the conclusion's subject and a subject to the conclusion's predicate—and this is called the first figure—or it is a predicate of the conclusion's predicate and a subject to the conclusion's subject, which is the figure that has been cancelled (*ulghiya*)—for the reason we shall mention—after it has been necessitated by division.
>
> For when they divided the figures according to the threefold division we have mentioned, and found them to be three, they specified one of them as the first figure, and took it to be that whose middle (term) is a subject to one (of the other terms) and a predicate of the other. Then when they looked at it with respect to what may be gathered from it (*min ḥaythu yajtamiʿu minhu mā yajtamiʿu*), they took it to be only that which preserves the subject to the middle as subject [in the conclusion] and the predicate of (the middle) as predicate [in the conclusion]. Now this is a more specific consideration than that for which they [formerly] regarded it as a first figure.
>
> And when they regard it as a first figure, not merely because the middle is predicate (of one of the other terms) and subject (to the other), but because the middle is a predicate of the conclusion's subject and a subject to the conclusion's predicate, they thus cancel (*alghaw*) a fourth division. The most excellent of physicians (*fāḍil al-aṭibbā'*) mentions (*yadhkuru*) this, but not in this way. Rather, this cancellation is by reason of the fact that it [the fourth division] is something unnatural and inadmissible and not in agreement with the usual (way) of thinking (*ghayr mulā'im li-ʿādat al-naẓar wa-l-rawiyya*). Further, it can be dispensed with by virtue of the conversion of the conclusion of that which is a first figure, as we shall make clear in another place.' (Poc. 122, fols. 114a–115a; Loth 475, fol. 124a.)

[19] See Mubahat Türker (ed.), 'Fārābī' nin bazi mantik eserleri', *Revue de la Faculté de Langues, d'Histoire et de Géographie de l'Université d'Ankara*, xvi, 1958, p. 257.

Although it is certain that the phrase 'the most excellent of physicians' could only refer to Galen, one regrets that Galen's doctrine is described only negatively: 'not in this way'. One thing, however, is obviously asserted here by Avicenna, namely that Galen was among those who 'cancelled' the 'fourth division' in which the middle term is a predicate of the conclusion's predicate and a subject to the conclusion's subject.

Avicenna explains his reasons for rejecting the fourth figure a little later in the text. He asserts that this figure (which, in this place, he considers to be the second of the *four* divisions)

> 'has been cancelled (*ulghiya*) only because it is very remote from nature (*ba'īd 'an al-ṭab' jiddā*). For the second figure is remote from nature with respect to the composition (*naẓm*) of only one premiss, viz. the major; and the third is remote from (nature) with respect to the composition of only one premiss, viz. the minor. When remoteness concerns one respect (only), the mind may tolerate it and may grasp the intention (*faṭina li-l-gharaḍ*). The remaining division, however, requires for its reduction to the natural condition (*al-amr al-ṭabī'ī*) that it should be wholly changed (*yaḥtāju ilā taghayyur yalḥaqu jamī'ah*); but it can be dispensed with; therefore, it, and whatever is like it, must be cancelled.' (Loth 475, fols. 124b–125a.)

Thus, according to Avicenna, syllogisms of the fourth figure are farther removed from the 'natural' syllogisms of the first figure than those of the second and third figures. For, he maintains, to reduce a syllogism of the second figure to the first, only one premiss needs to be converted. Here he seems to imply, however, that this premiss must always be the major. This would be a mistake; Camestres requires the conversion of its *minor* premiss; and Baroco is not reducible by conversion of either premiss. Similarly, says Avicenna, the reduction of syllogisms in the third figure calls for a conversion of only one premiss, the minor. But again, we note, Disamis is an exception; and Bocardo cannot be reduced by conversion. Does Avicenna then mean to imply that to reduce a syllogism in the fourth figure to the first, *both* premisses should be converted ('wholly changed')? Is it this need to convert both premisses (or, perhaps, the premisses and the conclusion as well) that is the measure of 'remoteness' of a syllogism in the fourth figure from the 'natural condition'?

We shall see that Ibn al-Sarī, who had mainly Avicenna's objections in mind when he wrote his defence, carefully examined these possibilities.[20]

The identity of 'Dinḥā the priest' is not established. Was he the same as Abū Zakariyyā Dinkhā (or Dinḥā or Dīnḥā), a tenth-century Jacobite who wrote a book on Roman and Greek rulers and philosophers, and who was 'given to philosophy and dialectics' (*wa-kāna mutafalsifā jadilā*) with whom the historian al-Mas'ūdī (d. *c.* 957) had 'many discussions . . . about the trinity and other (matters)' in Baghdād and in Takrīt in the church called *al-Khaḍrā'* in the year A.H. 313 (A.D. 925/6)?[21]

[20] See p. 23.
[21] Cf. 'Alī ibn al-Ḥusayn al-Mas'ūdī, *al-Tanbīh wa-l-ishrāf* (*Bibliotheca Geographorum Arabicorum*, ed. M. J. de Goeje, viii, Leiden, 1894), p. 155. Referred to by G. Graf in *Geschichte der christlichen arabischen Literatur*, ii, Vatican City, 1947, pp. 250 f.; see under 'Denḥā'.

IV

ANALYSIS OF IBN AL-SARĪ'S DEFENCE

1. *Identity and Position of the Fourth Figure*

Ibn al-Sarī characterizes the figures according to the position of the middle term. There are four possibilities. The middle term may be a predicate in one premiss and a subject in the other; this comprises two cases: either (1) it is predicate in the minor premiss and subject in the major, 'and this is called the first figure', or (2) it is subject in the minor and predicate in the major, 'and this is the added fourth figure, and in my view it should be second in order—for the reason I shall mention later'. (3) In the second [traditional] figure the middle occurs as predicate in both premisses, and (4) in the third figure it is subject in both. (Fol. 123a 4–10.)

According to Ibn al-Sarī the last two figures should be considered third and fourth; but to avoid confusion I shall continue to assign to them the orders they have in the Aristotelian scheme and I shall always call the added figure 'fourth'.

Our author then begins his defence as follows:

> 'Suppose that someone were to raise the following objection, your saying that the middle term is predicate in one premiss and subject in the other includes the first figure and this added fourth figure. We would say, indeed the two divisions are included in it. But the first figure is not that in which the middle term is predicate in one of the premisses and subject in the other—unqualifiedly. Rather, each premiss is determined, and (the figure) whose premisses are determined is as a species with respect to that (figure) whose premisses are unqualified. If we were to take the premisses unqualifiedly, it would not be true to state as a condition for the first figure that its major be universal and its minor be affirmative, for this condition does not hold for this added fourth figure. Suppose that someone were to divide simple bodies into two kinds, heavy and celestial. And (suppose) he were to explain this saying: motion is either straight or circular; if circular, it is celestial, and if straight, it is (the motion of) heavy bodies—for a heavy body moves towards the centre in a straight line. We would answer him as follows: had you said that a heavy body is that which moves on a straight line, unqualifiedly and without specifying this by the direction towards the centre, it would have been granted you that bodies are of two kinds. But since you specified the motion by the direction towards the centre, there emerges a third division, namely, that of the motion *from* the centre. Thus, simple bodies would be three, not two: i.e. celestial, light and heavy—even though light and heavy bodies are two species of the body which has a straight motion (and) which is the counterpart (*qasīm*) of celestial body.
>
> It is also in the same way that we say of the moods of the figures that they are four in respect to quantity. For either [1] the two premisses are universal or [2] particular, or [3] the major (premiss) is universal and the minor particular, or [4] the minor is universal and the major particular.

It would not be permissible for us to say that these four moods are three, as division requires them to be [three] by taking them as follows: the premisses are either [1] (both) universal or [2] (both) particular, or [3] one of them universal and the other particular; for here we speak unqualifiedly. But when we begin to enumerate the moods we determine that (mood) which we need and neglect the remaining (mood) because it is subsumed under a genus one of whose two species we have specified.

This then is what differentiates this (fourth) figure as a syllogism from the other three figures.' (Fols. 123a 10–123b 8.)

* * *

As we have seen, Ibn al-Sarī considers that the added figure should be given priority over the second and third figures of the Aristotelian scheme. Although this question of priority is of no consequence in itself, Ibn al-Sarī's remarks in support of his view form an important part of his defence of the new figure, and they should be reported in full.

He gives two reasons for placing the new figure immediately after the first: (1) This added figure has one property in common with the first, namely that in both the middle term is predicate in one premiss and subject in the other. But the first (in which the middle is predicate in the minor premiss and subject in the major) should be given priority because it is nearest to nature and does not require proof. Proof of the added figure, however, presupposes the first. (2) The new figure produces a greater variety of conclusions than the second or third figure of the Aristotelian division, and for this reason it should be placed before them. Three types are produced by the new figure: the universal negative [E], the particular affirmative [I] and the particular negative [O]. Only E and O are produced in the second figure, and only I and O in the third. Since 'the philosopher' [Aristotle] put the first before the other two because it produces the four conclusions, we must follow him and give priority to the added figure since it produces three conclusions.[22] (Fols. 124a 14–124b 5.)

Ibn al-Sarī then considers, and answers, three possible objections against his suggestion.[23]

Objection 1: The added figure should be considered fourth because it is 'remote from nature'; to prove some of its moods [Fesapo, Fresison] both premisses must be converted, whereas no mood of the other two figures requires the conversion of two premisses. (Fol. 124b 6–8.)

[22] In *Anal. Pr.* (i. 4, 26b 29 ff.) Aristotle says: 'It is evident also that all the syllogisms in this figure are perfect (for they are all completed by means of the premisses originally taken) and that all conclusions are proved by this figure, viz. universal and particular, affirmative and negative. Such a figure I call first' (*The Works of Aristotle*, ed. W. D. Ross, Oxford, 1950, i). Cf. also *ibid.*, 28a f. and 29a 15, 30. Avicenna counted three distinctions of the first figure: 'The first figure was called "first" because the deduction it performs (*intājahu*) is self-evident, because it produces all conclusions—whereas the second produces only the negative and the third only the particular—and because it produces the strongest (*afḍal*) conclusion, viz. the universal affirmative.' *Al-Shifā'*, India Office MS., Loth 475, fols. 124a–124b.

[23] For these objections, compare the passages quoted from Avicenna's *al-Shifā'* in Part III above.

Answer: Two conversions are also required for the reduction of moods in the other two figures, one conversion applied to one premiss and another to the conclusion. As examples, he mentions Camestres from the second figure, and a mood of the third figure 'whose major is a universal affirmative and whose minor is a particular affirmative'. Here he errs; the correct example from the third figure is Disamis. He goes on to say, obviously against Avicenna, that the second and third are 'farther removed from nature' than the added figure, for, as distinguished from the latter, they each contain one mood [Baroco in the second and Bocardo in the third] which needs to be demonstrated by a *reductio ad absurdum* process. 'But it is obvious that the indirect proof (*burhān al-khulf*) is more foreign to (*aghrab*), and farther removed from nature than the proof by conversion.' (Fol. 124b 8–19.)

Objection 2: Some moods of the added figure require the conversion of both premisses and the conclusion. This seems to be Ibn al-Sarī's interpretation of Avicenna's statement that the reduction of the fourth figure to the first requires that it be 'wholly changed'. (Fol. 124b 19–21.)

Answer: No mood in this figure calls for three conversions. Two of its moods require two conversions each (namely Fesapo and Fresison), but do not require the conclusion to be converted. In cases where the conclusion needs to be converted, no premiss is converted at all. (Fol. 124b 21–23.)

The answer is correct in every point.

Objection 3: The added figure may be dispensed with by virtue of the first. (Fol. 125a 3–4.)

Answer: The same is true of the second and third figures 'as is shown in the *Prior Analytics*'. If the fact that a figure may be dispensed with necessitates the cancellation (*ilghā'*) of that figure, then we ought to cancel the second and third figures as well.[24] But this is not permissible, for we must make our enumerations in all matters as complete as possible. (Fol. 125a 4–6.)

2. *Syllogistic Conditions*

A syllogism, by which word Ibn al-Sarī means a valid combination of premisses and conclusion, must satisfy certain conditions (*sharā'iṭ*). Some conditions are common to syllogisms in all four figures; they are three—negatively stated as follows:

C1 There is no syllogism from two negative premisses.
C2 There is no syllogism from two particular premisses.
C3 There is no syllogism from a negative minor premiss and a particular major premiss. (Fol. 123b 10–13; fol. 125a 8–10.)

The following are conditions which hold for one or two figures:

C1FI The minor premiss must be affirmative. (Fol. 123b 13; fol. 125a 13.)
C2FI The major premiss must be universal. (Fol. 123b 13; fol. 125a 13.)
C1FII = C2FI. (Fol. 125a 15–16.)
C2FII The premisses must differ in quality. (Fol. 124a 4–5.)
C1FIII = C1FI. (Fol. 125a 16–17.)
C1FIV One premiss must be affirmative, and the other universal = No premiss may be a particular negative. (Fol. 123b 14–15.)

[24] Note that *ilghā'* is the word used by Avicenna; see p. 19.

C2FIV A particular affirmative minor cannot be combined with a universal affirmative major = there is no syllogism from a particular affirmative minor and a universal affirmative major. (Fol. 123b 15–16; compare fol. 125a 20–21.)

Ibn al-Sarī is particularly concerned to show that conditions C1FIV and C2FIV set the fourth figure apart from the other three, and, therefore, they establish its independent identity. Thus, he notes, the fourth figure agrees with the first in that one premiss must be affirmative and the other universal; in the first figure, however, it is further specified which premiss should be affirmative (viz. the minor) and which should be universal (viz. the major). Furthermore, condition C1FIV is not satisfied by Baroco in the second figure or Bocardo in the third; while condition C2FIV applies to the second figure but not to the first or third. (Fols. 123b 16–124a 4.)

3. *Enumeration of productive and sterile moods*

The word 'mood' (*ḍarb*) is used by Ibn al-Sarī to refer primarily to a combination (*iqtirān*, *izdiwāj*) of premisses which is said to be either productive (*muntij*) or sterile (*'aqīm*) according to whether it syllogistically yields a conclusion or not. There are four kinds of determinate (*maḥṣura*, bounded) premisses: the universal affirmative [A], the universal negative [E], the particular affirmative [I], and the particular negative [O]. There result sixteen combinations of premisses which he enumerates in the following order—*always putting the minor premiss first:*

1 = AA, 2 = EE, 3 = AE, 4 = EA, 5 = II, 6 = OO, 7 = IO, 8 = OI, 9 = AI, 10 = EO, 11 = AO, 12 = EI, 13 = IA, 14 = OE, 15 = IE, 16 = OA.

These are all the moods to be considered in all the four figures. If we were to admit indeterminate (*muhmala*) premisses, there would be 36 combinations; but this is unnecessary because an indeterminate premiss is of the same force as a particular. (Fols. 125b 7–126a 3.)

Ibn al-Sarī then enumerates the productive moods in all figures by a process of elimination of the sterile moods, in which he applies the conditions previously stated. Eight moods are eliminated from all figures by applying the common conditions C1–C3. These are: 2, 5, 6, 7, 8, 10, 12 and 14. (Fol. 126a 3–11.)

The productive moods in the four figures are then arrived at as follows:

Figure I. By C1FI, eliminate 3 [*sic*] and 16. In fact this condition eliminates 4, not 3. Later on, however, Ibn al-Sarī himself lists 3 among the productive moods in this figure. By C2FI, eliminate 9 and 11. The remaining productive moods are four in this order: 1 = AA, 3 [*sic*] = AE, 13 = IA and 15 = IE. [Corresponding to Barbara, Celarant, Darii and Ferio.] (Fol. 126a 11–15.)

Figure II. By C1FII, eliminate 9 and 11. By C2FII eliminate 1 and 13. The remaining productive moods are four: 3 = AE, 4 = EA, 15 = IE, and 16 = OA. [Corresponding to Cesare, Camestre, Festino and Baroco.] (Fol. 126a 15–21.)

Figure III. By C1FIII, eliminate 3 [*sic*], . . . The remaining productive moods are six: 1 = AA, 4 = EA [*sic*], 9 = AI, 11 = AO, 13 = IA and 15 = IE. [Correcting 4 = EA to 3 = AE, these six moods correspond to Darapti, Felapton, Disamis, Bocardo, Datisi, and Ferison.] (Fols. 126a 21–126b 2.)

Figure IV. By C1FIV, eliminate 11 and 16. By C2FIV, eliminate 13. The remaining productive moods are 5: 1 = AA, 3 = AE, 4 = EA, 9 = AI, and 15 = IE. [Corresponding to Bramantip, Fesapo, Camenes, Dimaris and Fresison.] (Fol. 126b 2–9.)

Thus out of 64 moods in the four figures, 45 are sterile and 19 are productive (fol. 126b 9–11). The so-called 'subaltern moods' are nowhere mentioned.

4. *Demonstration of the productive moods of the fourth figure*

A mood in the second, third, or fourth figure is productive when a certain conclusion can be proved to follow from it by reduction to the first figure. The premisses together with the conclusion thus proved form a syllogism. Ibn al-Sarī formulates the syllogisms of the fourth figure both in variables and in concrete terms; both formulations are called by him 'examples'. As distinguished from Aristotle's usual practice, he formulates the syllogisms as inferences, not conditional sentences. Thus his syllogisms begin with a conjunction of the premisses, followed by the words 'I say that . . .', or 'it (i.e. the combination of premisses) produces . . .'.

As we have seen, Ibn al-Sarī counts five productive moods in the fourth figure yielding the same number of syllogisms. He proves them as follows:

1 = AA, produces I [Bramantip]

Example: Every A is B and every C is A, I say it produces 'some B is C'.
Proof: By changing the order of the premisses we get 'Every C is A and every A is B', which produces 'every C is B' by the first (productive) mood of the first figure [Barbara]. By conversion, this consequence becomes 'some B is C', which was to be proved. (Fol. 126b 14–18.)

2 = EA, produces E [Camenes]

Example: 'No A is B and every C is A' produces 'no B is C'.
Proof: By changing the order of the premisses we obtain 'no C is B' by the second (productive) mood of the first figure [Celarant], which by conversion gives 'no B is C'. (Fol. 127a 2–7.)

3 = AE, produces O [Fesapo]

Example: 'Every A is B and no C is A' produces 'not every B is C'.
Proof: By converting both premisses we get 'some B is A and no A is C', which produces 'not every B is C', by the fourth (productive) mood of the first figure [Ferio]. (Fol. 127a 8–12.)

4 = AI, produces I [Dimaris]

Example: Every A is B and some C is A, I say it produces 'some B is C'.

Proof: By changing the order of the premisses we obtain 'some C is B' by the third (productive) mood in the first figure [Darii]. By conversion this consequence becomes 'some B is C'. (Fols. 127a 22–127b 5.)

5 = IE, produces O [Fresison]

Example: 'Some A is B and no C is A' produces 'not every B is C'.
Proof: By converting both premisses we get 'some B is A and no A is C', which produces 'not every B is C' by the fourth mood of the first figure [Ferio]. (Fol. 128b 8–12.)

5. *Rejection of the sterile moods*

Ibn al-Sarī explains the principle of his method of rejecting what he calls 'sterile moods' in connection with a discussion of the mood EE in the fourth figure. This mood, he says, is sterile

> 'because it produces both the universal affirmative and the universal negative, and whatever (mood) is of this description is sterile, because it does not produce one thing but the thing and its contrary (*ḍiddah*) and thus it is not a syllogism; for the syllogism as defined by Aristotle is a sentence (*qawl*)[25] composed of sentences (*aqāwīl*) which by themselves necessarily imply one thing (*shay' wāḥid*), whereas this (mood) does not imply one thing. It was in this way that Aristotle showed the sterile combinations (*al-iqtirānāt al-'uqm*) in the *Prior Analytics*. Example of what produces the universal affirmative: no stone is a man and no animal is a stone, therefore (*fa-*) every man is an animal. Example of what produces the universal negative: no stone is a man and no horse is a stone, therefore (*fa-*) no man is a horse.' (Fols. 126b 19–127a 2.)

This is typical of Ibn al-Sarī's treatment of the sterile moods in the fourth figure. They are all described as 'producing the two contraries [A and E] together', even when the premisses are both particular (fol. 127a 13–22). And his method is always the same: first, he points out three concrete terms which verify the premisses and a universal affirmative conclusion; this 'shows' that the mood 'produces' an A-conclusion. Then he points out another set of three terms verifying the premisses and a universal negative conclusion, 'showing' that the same mood 'produces' an E-conclusion.

It is clear that Ibn al-Sarī uses the verb 'to produce' in the sense of 'to imply'. He is therefore mistaken in asserting that a 'sterile' combination of premisses 'produces' both an A-conclusion and an E-conclusion. For if the combination of premisses has no syllogistic force (which is certainly what he means by 'sterile'), then nothing at all follows from them. Ibn al-Sarī here misunderstands Aristotle's method of rejection of asyllogistic forms, a method in which the exemplification through concrete terms is intended to verify the premisses without verifying an alleged conclusion. This misunderstanding

[25] '*qawl*' is as loose in Arabic as the corresponding '*logos*'. In translating '*qawl*' here by 'sentence' I have followed the meaning given to it in *De interpretatione*, iv; see Isḥāq ibn Ḥunayn's translation of this work in 'Abdurraḥmān Badawī (ed.), *Manṭiq Arisṭū*, i, Cairo, 1948, p. 63.

Ibn al-Sarī shared with Alexander,[26] and it may have been from Alexander that he derived it, directly or indirectly.

The source of the definition of syllogism which Ibn al-Sarī here ascribes to Aristotle is not known to me. Elsewhere in his treatise (fol. 127b 17–20) he quotes *Anal. pr.* i, 4, 26b 5–9 in the Arabic translation of Tadhārī (Theodore).[27] But the definition as rendered in this translation (which is quite faithful to Aristotle's text) differs from that quoted by Ibn al-Sarī; it reads: 'the syllogism is a sentence (*qawl*) in which more than one thing being posited, something else (*shay' mā ākhar*) follows of necessity from the existence of those things themselves which have been posited'.[28] The crucial difference is the substitution of *shay' wāḥid* in Ibn al-Sarī's version for *shay' mā ākhar*; for his argument rests on interpreting *shay' wāḥid* to mean: one thing, not more.

6. *Dinḥā's mistakes*

Ibn al-Sarī tells us at the end of the prologue to his treatise that he had seen only one work purporting to defend the fourth figure, namely that of 'Dinḥā the priest'. According to Ibn al-Sarī this attempt was fraught with various mistakes which he indicates in the prologue. Since he returns to these mistakes later in the treatise, it may be useful to quote his discussion, as it allows us a glimpse into a work that is otherwise unknown.

The first mistake has to do with the mood in the fourth figure with a minor premiss E and a major premiss I. Ibn al-Sarī correctly remarks that this mood is 'sterile'. But Dinḥā held the opposite view:

> 'Dinḥā claimed that this mood is productive. He gave the following example, "no A is B and some C is A", and claimed that this produces "not every B is C". For he changes the order, putting the premisses in place of one another, thus obtaining "some C is A and no A is B", which produces "not [every] C is B" by the fourth (productive) mood of the first figure [Ferio]. But in our view this is not productive; for what is required (*al-maṭlūb, qaesitum*) is to produce "not every B is C", C being the major term; while it produced the opposite (*'aks*) of this, (namely) "not every C is B". But this conclusion cannot be converted, because it is a particular negative. This (mood), therefore, has produced the opposite of the conclusion (*'aks al-maṭlūb*), not the conclusion. If, therefore, Dinḥā regards this mood as productive, then let him regard two other moods in the first figure as productive, namely the mood which has a universal negative minor and a universal affirmative major, and the mood which has a universal negative minor and a particular affirmative major. Example: "no B is A and every A is C or some A is C", producing "not every C is B"—for if we convert both premisses, these moods become "some C is A and no A is B", which produces by the fourth (productive) mood of the first figure: "not every C is B". But the desired conclusion was "some B is C", not the converse (*lā 'aksah*). It is for this (reason) that Aristotle did not regard these two moods as productive. And if some of the ancients have regarded them as

[26] Cf. Lukasiewicz, *op. cit.*, pp. 67–68.

[27] Cf. 'Abdurraḥmān Badawī (ed.), *loc. cit.*, p. 117, lines 4–7; *mithāl dhālik . . . 'alā shay' min dhālik*. Ibn al-Sarī has *al-abyaḍ, walyakun, quqnus, walḥayy* for *al-bayāḍ, falyakun, quqnūs, falḥayy* in Badawī's text.

[28] *Ibid.*, p. 108.

productive, the same objection may be raised against them.' (Fol. 128a 7–19.)

Dinḥā's second mistake noted by Ibn al-Sarī relates to the mood in the fourth figure with a minor I and a major A:

'This mood also was considered productive by Dinḥā. For he interchanged the premisses and deduced from them (*alzama 'anhumā*) a particular affirmative—not knowing that by putting the minor in place of the major we obtain a mood in the first figure consisting of a universal affirmative minor and a particular affirmative major. But this mood is not productive, for one condition of the first (figure) is that its major should be universal. This, then, is an error (*waham*) in his (Dinḥā's) knowledge of the first figure.' (Fols. 128a 23–128b 4.)

* * *

Concluding his treatise, Ibn al-Sarī remarks that in the demonstration of the productive moods in the fourth figure, he has taken the premisses to be assertoric (*muṭlaqa*). 'But if they were both necessary, or possible or compounded of these three kinds, I mean the assertoric, the necessary and the possible, they would require another demonstration (*bayān ākhar*). But since the ancients used to separate this branch of the science from the first—and for this (reason) the modern Alexandrians define it as a part that is not read[29]—we shall deal with it separately in a treatise following this one—the Exalted God willing.' (Fol. 128b 17–21.)

I have not been able to trace a treatise of Ibn al-Sarī's on the subject of modal syllogisms.

[29] '*wa-lihādhā mā yu'arrifuhu al-ḥādithu mina 'l-iskandarāniyyīn bijuz' lā yuqra'*.' In the translation of this sentence I have regarded '*mā*' as *maṣdariyya* or simply redundant. This unusual, but by no means rare use of '*mā*' occurs elsewhere in the same treatise: fol. 124b, lines 5 and 7. At the end of Bk. I, Ch. 7 of the Arabic translation of the *Prior Analytics*, a gloss, possibly added by the translator, Tadhārī, reads as follows: 'Ends the third figure. The modern Alexandrians (*al-ḥadathu mina 'l-iskandarāniyyīn*) read up to this place in the Book of Syllogism; and they call what comes after it in this book the part that is not read (*al-juz' ghayr al-maqrū'*), i.e. the discourse on the syllogisms composed of modal premisses' (*Manṭiq Arisṭū*, ed. Badawī, i, p. 132). This may quite well have been the source of Ibn al-Sarī's similar statement above. For the tradition derived from al-Fārābī, to the effect that the teaching in the 'Alexandrian Schools' was limited to the assertoric syllogisms, see Ibn Abī Uṣaybi'a, *'Uyūn*, ii, p. 135; Max Meyerhof, 'Von Alexandrien nach Bagdad: Ein Beitrag zur Geschichte des philosophischen und medizinischen Unterrichts bei den Arabern', *Sitzungsberichte der preussischen Akademie der Wissenschaften* (Philosophisch-historische Klasse), 1930, xxiii, pp. 393–94.

AVICENNA ON THE SUBJECT MATTER OF LOGIC *

I THINK it is true to say that modern logicians have no great interest in the ancient debate about whether logic was a part or an instrument of philosophy. They are of the opinion that the debate, at least in the form it took in the ancient schools of Greek philosophy, raised a question the solution of which was largely a matter of convention. Avicenna would readily agree, and for the same reason; in one place at least he characterized the question as nothing more than a quibble about the meaning of words. But in both ancient and medieval discussions the question was often linked with another concerning the subject matter of logic. If, as the Platonists and the Stoics maintained, logic is a part of philosophy and the various parts of philosophy are studies of various portions or aspects of being, then what portion or aspect of being should be assigned to logic? *This* was not a verbal question. And since Avicenna decided to come down on the side of the Academy and not on the side of his "friends" the Peripatetics who maintained that logic was only an instrument, it is not surprising that he should take the trouble in his *Kitāb al-Shifā'* to expound his views on these two interrelated questions.

* To be presented in an APA symposium on Avicenna, December 30, 1980. Calvin Normore and Arthur Hyman will comment; their papers are not available at this time.

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The dispute as to whether logic was a theory or an instrument has a further significance for the historian of Islamic thought: it became part of a continuing struggle of far-reaching consequences between the champions of Arabic and Islamic learning and the followers of an imported Hellenistic tradition. It should be remembered that in Islam the *trivium* was not an accepted category: logic and grammar stood on opposite sides of the fence, supported by rival groups. The word chosen by the translators of Aristotle for the art of logic, *manṭiq* (speech), could naturally be used, and was sometimes used, as a title of works on grammar; and this alone was bound to impose the question as to which group, the grammarians or the logicians, was to be regarded as the true custodian of sound discourse. The ensuing controversies may have been motivated in part by religious or nationalistic impulses, but they were not devoid of philosophical interest—being ultimately concerned with the relation between language and thought. The documents that have survived from the early period of this debate appear to indicate that it was the grammarians who had the better of the argument. Logicians were content at first to claim that they were concerned with meanings whereas grammar was competent to deal only with words. This was too simple a view of the task that the grammarians had set for themselves. In the famous debate that took place in the tenth century between the logician Mattā ibn Yūnus and the grammarian Abū Sa'īd al-Sīrāfī, both recognized leaders in their respective disciplines, the case for logic appears to be vulnerable and lacking in sophistication. With regard to the logicians' view of grammar, Sīrāfī cites example after example of how subtle shades of meaning are reflected in linguistic features which it was the business of the grammarian to identify. It is true that Mattā—whose translations of Greek philosophical works into Arabic were done not directly from the Greek but from Syriac—is being repeatedly put on the defensive by being reminded of his ignorance of Greek, the original language of the logic he now claims to explicate in another language (Arabic) which he has not mastered. The audience is clearly unsympathetic to him, and he is rarely given the chance to answer the questions put to him. But even with the admission that this was perhaps not a fair trial, one comes away from reading it with the clear impression that it was Sīrāfī who had a deeper appreciation than his adversary of the incongruences generated by the "creation of a language within a language," as he aptly described the logicians' enterprise.

Abū Naṣr al-Fārābī, at one time a student of Mattā, was the first Arabic logician to take seriously the questions of the relation of logic to grammar and of language to thought. His *Enumeration of the Sciences* briefly formulates the idea of logic as universal grammar (grammar furnishes the rules proper to the utterances of a given language; logic furnishes the rules common to the utterances of all languages), and states some of the implications of the connection between "inner" and "outer" speech. His book on *The Utterances Employed in Logic* can be viewed as an attempt to face the problems of introducing an artificial mode of speaking into natural Arabic. In his so-called *Short Commentary on Aristotle's* Prior Analytics, he tried to convince other Muslims of the usefulness of Greek logic by illustrating Aristotelian forms of inference in terms taken from Islamic theology and jurisprudence. He thus preceded al-Ghazālī in formulating the kind of argument that finally succeeded in securing for Aristotelian logic a permanent place in Muslim education.

It is one of the paradoxes of Islamic intellectual life that the man most responsible for admitting Aristotelian logic into the scheme of traditional learning was an opponent of Greek philosophy who wrote a powerful book in its refutation. The great religious thinker Abū Ḥāmid al-Ghazālī (d. 1111) not only rejected the metaphysical doctrines of Peripateticism and Neoplatonism but also warned against the dangers of studying astronomy and mathematics. Nevertheless he wrote several books on Aristotelian logic and, what was more crucial for the history of logic in Islam, prefaced his influential work on Islamic jurisprudence (*al-Mustaṣfā*) with a lengthy introduction in which he went so far as to say that without logic one could not be sure of any part of knowledge (whether secular or religious, theoretical or practical). Al-Ghazālī was able to do this because he understood logic as a mere instrument, a kind of "balance" for weighing arguments which did not commit its user to any doctrine or belief.

Ghazālī's attitude was in great contrast to that of another religious thinker who lived some two hundred years after him. Ibn Taymiyya was a strict jurisconsult who had no use for mystical or speculative approaches to religion, let alone an unbridled habit of thought such as philosophy. He could not tolerate the fact that Greek logic had appeared to gain a firm foothold in the field of religious learning, and he singled it out for a concentrated and persistent attack. Unlike Ghazālī, Ibn Taymiyya believed that Aristotelian logic was committed to certain metaphysical doctrines from

which it could not be detached. One could not adopt Aristotelian logic without being contaminated by its false presuppositions. One of these was a pervasive belief in universals or essences. According to Ibn Taymiyya, only individuals exist, and what is called "essence" is nothing but a conventional device for grouping individuals together for practical or theoretical purposes of our own. A universal term like 'man' does not refer to something shared by individuals, for no such thing exists. The inevitable conclusion was a total rejection of Aristotle's theories of definition and syllogism.

Ibn Taymiyya's criticisms, often acute and original, constituted the most radical critique of Aristotelian logic in the Arabic language. But they were not the work of a concerned logician; their aim was not to reform logic but to destroy it. They came too late in history to achieve that aim on a wide scale; the trend initiated by Ghazālī had taken root and was already well established. But the logic that Ghazālī had made acceptable was not a part of philosophical inquiry, a search for new truths, but an instrumental discipline consisting of a fairly fixed set of rules that one learned in order to apply them in other disciplines.

Avicenna, too, appreciated the instrumental character of logic, but his perspective differed from that of any of the thinkers I have mentioned. In broad terms he belonged to the same philosophical tradition to which al-Fārābī and al-Kindī before him also belonged. But he was more independent of mind than either of his two predecessors, and he spoke more often than they with an individual voice of his own. He was frequently critical or skeptical of the Aristotelianism he embraced and modified, and openly dissociated himself from the Peripatetic school of Baghdad, feeling himself the equal of the ancient commentators whom he read of course in translation. In matters of logic, as in other parts of his philosophy, he helped himself more freely than members of that school to Platonic (and Stoic) doctrines which had already been fused together in the late Greek writings that became the common heritage of Islamic philosophers. He did not always do this in the spirit of eclecticism, but often as the work of an independent thinker who felt able and obliged to make up his own mind.

Logic occupies a major part of *Kitāb al-Shifā'*, the huge philosophical *summa* which Avicenna completed just before he reached the age of forty. It is known from Avicenna's own account and from a supplementary account provided by his pupil al-Jūzjānī, that the book was composed in varying and sometimes difficult circumstances which had their effect on the character of its various parts. Al-

Jūzjānī says, for example, that Avicenna dictated most of the Physics and the Metaphysics during a period of only twenty days without referring to other writings. When he came to write the Logic, however, he was able to consult the books of others "whose order [of treatment] he followed and whose objectionable views he discussed"—a fact which the historian of Avicenna's logic must bear in mind. The logical part of the *Shifā'* is divided into sections corresponding generally to the parts of Aristotle's *Organon*. Only the first section, the Introduction paralleling Porphyry's *Isagoge*, was translated into Latin in the middle ages. It happens to be the section in which Avicenna directly addresses the question of the subject matter of logic. Avicenna's discussions thus continue a tradition that goes back to the Greek commentators, and his own treatment of these questions entered into the stream of philosophical thought in the West. In this paper I shall not in general be concerned to reconstruct the complex process of transmission of Greek ideas into Arabic, or follow Avicenna's discussions into the writings of medieval thinkers; my main object will be to identify and clarify Avicenna's views. I hope that the following remarks, despite their preliminary character, will not fail to show that Avicenna's style of thinking and writing does lend itself profitably to the kind of analytical approach attempted here.[1]

[1] Following are some references for the above remarks: The most comprehensive study of the history of the opposition to ancient learning (including logic) in Islam remains that of Ignaz Goldziher, "Stellung der alten islamischen Orthodoxie zu den antiken Wissenschaften," *Abhandlungen der königlich preussischen Akademie der Wissenschaften*, Jahrgang 1915, philosophisch-historische Klasse, Nr. 8, Berlin, 1916. There is an English translation by D. S. Margoliouth of the debate between Mattā and al-Sīrāfī in *Journal of the Royal Asiatic Society*, London, 1905, pp. 79–129. Muhsin Mahdi studied the debate in "Language and Logic in Classical Islam," in G. E. von Grunebaum, ed., *Logic in Classical Islamic Culture* (Wiesbaden, 1970), pp. 51–83. Gerhard Endress, "The Debate between Arabic Grammar and Greek Logic" [in Arabic with English summary], *Journal for the History of Arabic Science*, I (1977): 106–339 (Arabic), 320–322 (English), includes a bibliography of Arabic sources and of studies in European languages; see also the text of Yaḥyā ibn 'Adī "On the Difference between Philosophical Logic and Arabic Grammar" edited by Endress in *Journal for the History of Arabic Science*, II (1978): 38–50 (Arabic). I have used 'Uthmān Amīn's edition of Fārābī's *Enumeration of the Sciences* (*Iḥṣā' al-'ulūm*) (Cairo, 1931). Fārābī's text together with two Latin versions (one of which is a translation by Gerard of Cremona) were published by A. G. Palencia, *Al-Farabi: Catálogo de las ciencias* (Madrid, 1932; 2nd ed., Madrid, 1953). For the transmission of Fārābī's text to the west see M. Bouyges, "Sur le *De scientiis* d'Alfarabi," in *Mélanges de l'Université St. Joseph*, IX (1921): 49–70. Fārābī's views on logic, language, and grammar are analyzed by M. Mahdi in "Science, Philosophy, and Religion in al-Fārābī's *Enumeration of the Sciences*," in J. E. Murdoch and E. D. Sylla, eds., *The Cultural Context of Medieval Learning* (Boston: Reidel, 1975): 113–147, esp. pp. 118–124. The text of Al-Fārābī's *Utterances Employed in Logic* was edited

To avoid confusion I shall refer to sections of Avicenna's Logic in the *Shifā'* by English titles, such as "*Introduction*" or "*Interpretation*," reserving for Greek works their commonly used titles in Latin or Greek.

I

Chapter 2 of the *Introduction*, on the chief divisions of the sciences, ends with a longish passage giving Avicenna's first statement in the *Shifā'* on the nature of logic and its relation to the other sciences.[2] The essences of things (*māhiyyāt al-ashyā'*—*māhiyya*: τὸ τί ἦν εἶναι may exist in the actual things (*a'yān al-ashyā'*) or in thought (*fī al-taṣawwur*). Certain accidents (*a'rāḍ*) attach themselves to the essences when these possess one or the other of the two modes of existence. We may therefore examine the essences in themselves, without reference to their existence in individuals or in thought, or our examination of them may involve those adventitious properties which accrue to them in consequence of their external or mental existence. Avicenna here gives some examples of the kind of accidents that may attach to essences as mental entities: being subject or predicate, universality or particularity of predication, essential or accidental predication. He explains his examples by briefly remarking that "in external things there is no essential or accidental predication, nor is a thing a subject or a predicate, a premiss or a syllogism or the like" (*al-Madkhal*, 15: 7/8).

These opening sentences are misleading in that they give the impression that the accidents exemplified here come into being simply as a result of bringing the essences into one's mind. But

by M. Madhi (Beirut: Dar El-Mashreq, 1968). N. Rescher published an English translation of Fārābī's *Kitāb al-Qiyās al-Ṣaghīr* as *Al-Fārābī's Short Commentary on Aristotle's* Prior Analytics (Pittsburgh: University Press, 1963) [reviewed in *Journal of the American Oriental Society*, LXXXV (1965): pp. 241–243]. For references on al-Ghazālī and Ibn Taymiyya and their writings in Arabic and European translations, see the articles devoted to them in the second edition of the *Encyclopaedia of Islam*. Also C. A. Qadir, "An Early Islamic Critic of Aristotelian Logic: Ibn Taimiyyah," *International Philosophical Quarterly*, VIII (1968): 498–512; and S. A. Kamali, *Types of Islamic Thought* (Aligarh: Institute of Islamic Studies, ?1967); H. K. Shaddad: *Ibn Taymiyya's Critique of Aristotelian Logic*, unpublished London Ph.D. thesis, 1972. An English translation of Avicenna's autobiography is in W. E. Gohlman, *The Life of Ibn Sīnā* (Albany: SUNY Press, 1974). A discussion of the sources of Avicenna's logic is in N. Shehaby, *The Propositional Logic of Avicenna* (Boston: Reidel, 1973), Introduction. Various aspects of Avicenna's logic are discussed in S. Nuseibeh, *The Foundations of Avicenna's Philosophy*, Harvard University Ph.D. thesis, 1978.

[2] Ibn Sina, *al-Shifā'*, *al-Manṭiq*: 1. *al-Madkhal*, G. Anawāti, M. El-Kohdeiri and F. El-Ahwānī, eds. (Cairo: al-Maṭba'a al-Amīriyya, 1952), pp. 15/6. Chapter 2 corresponds to the first chapter in the edition of the "Logyca" in: Avicenna, *Opera Philosophica* (Venice, 1508), réimpression en fac-similé agrandi (Louvain: Édition de la bibliothèque S.J., 1961).

Avicenna seems to be struggling to dispel this misunderstanding in the following words:

> If we wish to investigate things and gain knowledge of them we must conceive them; thus they necessarily acquire certain states (*aḥwāl*) that come to be in conception: we must therefore consider those states which belong to them in conception, *especially as we seek by thought to arrive at things unknown from those that are known*. Now things can be unknown or known only in relation to a mind: and it is as concepts that they acquire what they do acquire in order that we move from what is known to what is unknown regarding them, without however losing what belongs to them in themselves; we ought, therefore, to have knowledge of these states and of their quantity and quality and of how they may be examined in this new circumstance (*al-Madkhal*, 15: 9–17; emphasis added).

I take this to mean that, although the properties of being a subject or a predicate or the like can attach only to concepts and not to external things, they do so only when the concepts are manipulated for the purpose of arriving at (or conveying) a piece of knowledge. Thus, in addition to the two varieties of investigation whose aim is to gain knowledge of external and mental things as such, there exists an inquiry whose aim is to be of use in carrying out the other two investigations. Such an inquiry is called "logic."

Avicenna is thus arguing that logic has its own subject matter which it does not share with any other science. But because of the very nature of this subject matter (properties acquired by concepts when organized for the purpose of attaining or transmitting knowledge), he maintains at the same time that the goal of logical investigation is to help in other investigations. He concludes this passage by saying that if philosophy is understood as the investigation of external and conceptual things as such, then logic is not a part of philosophy, but, as an aid in other investigations, it is an instrument of philosophy. If, however, the term 'philosophy' is applied to "all manner of theoretical investigation," then logic is a part of philosophy and a tool for the other parts. To Avicenna's mind, the question whether logic is a part or an instrument of philosophy is both false and futile—false because it presupposes a nonexistent contradiction between the two roles and futile because "to busy oneself with such matters serves no purpose." But this brief discussion at least allows him to offer something like a definition of logic: it is an inquiry into concepts, and into their properties, insofar as they can be made to lead to knowledge of the unknown (16: 10–12).

II

In the *Introduction* Avicenna has no name for those concepts which, on account of certain properties that attach to them in the context of proof, he sets apart as the proper object of logic. He does, however, provide such a name in his *Metaphysics*, in a passage which is believed to be "the origin of that discussion of first and second intentions which continued until the end of medieval logic":[3]

> As you have known, the object of logic was the secondary intelligible concepts (*al-ma'ānī al-ma'qūla al-thāniya*)—those that depend upon (*tastanid ilā*) the primary intelligible concepts—insofar as they may be of use in arriving at the unknown from the known, and not insofar as they are thoughts (*ma'qūla*) having an intellectual existence that is not attached to matter at all or attached to non-corporeal matter.[4]

As has been noted more than once,[5] Avicenna's doctrine had a precursor in the Porphyrian distinction between terms in first position (πρώτη θέσις) and terms in second position (δευτέρα θέσις), a distinction which we do find in Arabic writers before and contemporary with Avicenna. A look at some of these writers will show the wider scope of Avicenna's remarks, brief though they are.

Al-Fārābī's *Commentary on Aristotle's* De interpretatione makes the standard observation: 'name' and 'verb' are terms in second position, whereas, he implies, the categories are terms in first position.[6] The notes (*ta'līqāt*) that Ibn Bājja (Avempace, d. 1138) wrote on Fārābī's account (?the same as the just-mentioned *Commentary* or rather a separate paraphrase of Aristotle's *De int.*) furnish a longer list of terms in second position including 'particle', 'definite'

[3] William and Martha Kneale, *The Development of Logic* (New York: Oxford, 1962), p. 230. See also David Knowles, *The Evolution of Medieval Thought* (London: Longmans, 1965), p. 197.

[4] *Al-Shifā', al-Ilāhiyyāt*, G. Anawātī and Sa'īd Zāyed, eds., I (Cairo: al-Hay'a al-'Āmma li-Shu'ūn al-Maṭābi' al-Amīriyya, 1960), pp. 10/1.—"Subiectum vero logicae, sicut scisti, sunt intentiones intellectae secundo, quae apponuntur intentionibus intellectis primo, secundum hoc quod per eas pervenitur de cognito ad incognitum, non inquantum ipsae sunt intellectae et habent esse intelligibile, quod esse nullo modo pendet ex materia, vel pendet ex materia, sed non corporea" [*Avicenna Latinus. Liber de philosophia prima I–IV*, édition critique de la traduction Latine médiévale par S. Van Riet (Louvain: E. Peeters, and Leiden: E. J. Brill, 1977), p. 10: 73–79. K. Gyekye exposed some of the confusions connected with the terms 'prima intentio' and 'secunda intentio' in *Speculum*, XLVI (1971): 32–38. Note also Avicenna's understanding of *ma'ānī* (πράγματα, in the sense of actual things) as *maqāṣid* (*intentions*)—fn 19 below.

[5] See the article on *Manṭiq* (logic) by S. Van den Bergh in the first edition of the *Encyclopaedia of Islam*. Also, Kneales, *op. cit.*, p. 229. See *Porphyrii Isagoge et In Aristotelis Categorias Commentarium*, A. Busse, ed. (Berlin, 1887), Index.

[6] *Alfārābī's Commentary on Aristotle's* De interpretatione, Wilhelm Kutsch and Stanley Marrow, eds. (Beyrouth: Imprimerie Catholique, 1960), p. 20: 22—p. 21: 3.

(ὁριστός), 'indefinite' (ἀόριστος), 'straight' (εὐθύς), 'oblique' (πτῶσις), 'derivative' (παρώνυμος), as well as 'name' and 'verb'.[7]

Let us look next at the relevant sentences in the notes that a late tenth-century translator and commentator of Aristotle, the Syrian Christian al-Ḥasan ibn Suwār,[8] has written on the *Categories*. He states first that Aristotle's aim in the *Categories* is to discuss those "single utterances in first position (*fī al-waḍ' al-awwal*) which signify the highest genera of things (*al-umūr*) by means of the affections (*āthār*) [produced] by them in the soul, and [to discuss] things insofar as they are signified by the utterance."[9] In regard to the expression 'first position' al-Ḥasan explains:

> We say utterances in the first position in order to distinguish them from utterances in second position; for utterances in first position are those names and labels(?) that are first applied to things as signs (*simāt*, '*alāmāt*) that signify them in a general way (*dalāla mujmala*), such as calling this "silver" and this "copper" and this "gold," while utterances in second position are those that signify what we have set apart as utterances in first position, such as calling ["name"] every utterance signifying something definite, without time, as "Zayd" and "'Amr," and calling "verb" everything that additionally signifies time, as "stood up" and "will stand up." These are utterances in second position because we have posited them subsequent to the existence of the others (*ibid.*, 362: 8/9).

Finally, here are two examples of what a leading logician and teacher of logic in eleventh-century Baghdad, Abū al-Faraj ibn al-Ṭayyib (d. 1043), had to say about the two expressions in question. In his *Commentary on Porphyry's* Isagoge, Abū al-Faraj wrote:

> . . . utterances are investigated in two ways, as utterances in first position and as utterances in second position. Utterances in first position are those that signify things (*al-umūr*), such as "Zayd," "'Amr," and "has struck." Utterances in second position are those

[7] Ibn Bājja, *Ta'līqāt fī Kitāb Bārī Armīnyās wa min Kitāb al-'Ibāra li-Abī Naṣr al-Fārābī*, M. S. Sālim, ed. (Cairo: Maṭba'at Dār al-Kutub, 1976), p. 12: 13–13: 9.

[8] On Ibn Suwār, see Rescher, *The Development of Arabic Logic* (Pittsburgh: University Press, 1964), pp. 140/1. The notes are published by Khalil Georr in *Les catégories d'Aristote dans leurs versions syro-arabes* (Beyrouth: Imprimerie Catholique, 1948), pp. 361–386.

[9] *Ibid.*, 361: 1–4. In this passage 'utterance' (*lafẓ*), 'affections' (*āthār*), and 'things' (*umūr*) coincide with Aristotle's φωνή, παθήματα, and πράγματα as they are introduced at the beginning of the *De interpretatione*. While commenting some lines later on this part of his own passage, Ibn Suwār uses *ma'ānī* (concepts) and *āthār* (affections) as synonyms and describes them as forms (images?) (*ṣuwar*) of the actual things (*umūr*); p. 362: 8/9.

> that signify utterances in first position. . . . And [Porphyry's] concern here is with utterances in first position.[10]

The only examples given by Abū al-Faraj of words in second position are 'name' and 'verb', which, as he also observes, are discussed at the beginning of Aristotle's *De interpretatione*.[11]

What is lacking in all these examples is any statement to the effect that terms (or concepts) in second position constitute the specific subject matter of logic.

III

Al-Fārābī in his *Commentary* on Aristotle's *De interpretatione* does employ the phrase "secondary concepts" (*al-ma'qūlāt al-thawānī*), the very same expression which we encountered in Avicenna's *Metaphysics*. I shall here paraphrase Fārābī's text without attempting to do full justice to his rather difficult arguments, my aim being simply to indicate the context in which he introduces that phrase. The occasion is Aristotle's statement[12] at 16^b19 ff which prompts Fārābī to speculate about the combinative function of "existential verbs" (is, exists). An existential verb, he says (45:1–46:4), in-

[10] *Ibn al-Ṭayyib's Commentary on Porphyry's Eisagoge*, Arabic text edited with introduction and a glossary of Greek-Arabic logical terms by Kwame Gyekye (Beyrouth: Dār al-Machreq, 1975), p. 35: 11–17. An English translation of the first four "lessons" in Ibn al-Ṭayyib's *Commentary* was first published by D. M. Dunlop with the mistaken attribution to al-Fārābī: D. M. Dunlop, "The Existence and Definition of Philosophy, from an Arabic text ascribed to al-Fārābī," *Iraq*, XIII, 2 (1951): 76–94. The mistaken attribution was corrected by Samuel Stern in *Bulletin of the School of Oriental and African Studies*, XIX (1957): 419–425. Gyekye's English translation [*Arabic Logic: Ibn al-Ṭayyib's Commentary on Porphyry's Eisagoge* (Albany: SUNY Press, 1979)] begins with the "sixth lesson," and we therefore still lack a translation of the "fifth lesson" (pp. 27–40 in Gyekye's edition of the Arabic text) from which the above quotations are taken.

[11] *Ibn al-Ṭayyib's Commentary* . . . (Arabic text), p. 38: 12–14. The terms 'first position' and 'second position' are used several times in Avicenna's *Introduction*, but not in the sense of the texts referred to above and not in the context of his discussion of the subject matter of logic. For example, at the beginning of the chapter on genus, Avicenna writes, "We say that the word which in the language of the Greeks signified the concept genus used first (*bi-ḥasab al-waḍ' al-awwal*) to mean to them something else, then it was transferred (*nuqilat*) by second imposition (*bi-al-waḍ' al-thānī*) to the concept called "genus" by the logicians (*Intro.*, p. 47: 3–5) = "Dicemus quod verbum significans intentionem generis prius apud eos secundum primam impositionem significabat aliud, et deinde per impositionem secundam translatum est ad significandum intentionem que apud logicos vocatur genus" (fol. 6^rA). See also a little later in the same chapter (Arabic, p. 37: 15–19; Latin, fol. 6^r A: 58–65), and again, at the beginning of chapter 10 (Arabic, p. 54: 8 ff: Latin, fol. 7^rA: 7 ff).

[12] "When uttered just by itself a verb is a name and signifies something—the speaker arrests his thought and the hearer pauses—but it does not yet signify whether it is or not. For not even 'to be' or 'not to be' is a sign of the actual thing (nor if you say simply 'that which is'); for by itself it is nothing, but it additionally signifies some combination, which cannot be thought of without the components." J. L. Ackrill's translation in the Clarendon Aristotle Series.

dicates three things: a time, a combination or connection, and an unspecified subject. A question may arise as to how an existential verb, whether used existentially or copulatively, can perform the combination. The problem (as presented by Fārābī) is that a non-existential verb like 'walks' is analyzable into 'is walking', so that "man walks" is equivalent to "man is walking," where 'is' performs the combinative function. Should we then say that "man is (exists)" is also analyzable into "man is existing," where existence would occur twice—once as a connector and again as a predicate?

Fārābī answers that in the case of the existential 'is' ("when 'is' is predicated by itself") no absurdity would result from such a repetition. But no repetition would need to be involved in the case of the copulative 'is' (when the latter is "predicated for the sake of something else"). In this last case 'is' signifies only time, an (unspecified) subject, and "the notion of a copulative existence," the predicate (say, white) being something apart from that.

Fārābī then goes on to pose the unexpected question of whether the copulative 'is' did not itself require a connector which in turn required a connector and so on to infinity—to which question he gives the following enigmatic answer in terms of "secondary concepts":

> There would be nothing impossible or absurd in this consequence [of an infinite series of connectors], for the notion of connector is here one of the secondary concepts, and it is neither impossible nor absurd for secondary concepts to go on to infinity, as you have heard me say many times and as I have set down in writing.

He finally adds that repetition of one and the same secondary concept is not necessary, but does no harm if it occurs.

All this is rather baffling. But the character of Fārābī's arguments, the sudden but surprisingly brief appearance of the idea of an infinite chain of connectors regarded as secondary concepts, and the reference to his previous teachings and writings, all this is clear indication that we do not yet possess all we need to have to penetrate the thoughts of early Arabic logicians on the subject that has concerned us. Avicenna's remarks in the *Introduction* and in the *Metaphysics* thus remain the clearest and fullest statement on the topic of the subject matter of logic which has come down to us from the period between the translation of Aristotle's logical works into Arabic and the middle of the eleventh century. It is remarkable that they also seem to have become henceforward the standard doctrine to which later Arabic logicians turned for a ready answer to the question of what logic was about. To illustrate this last point

I shall quote a passage that describes the situation as it appeared to an erudit living in the eighteenth century:

> The authorities are of the opinion that the subject matter [of logic] comprises the secondary concepts (*al-ma'qūlāt al-thāniya*), not in respect of what they are in themselves, nor insofar as they exist in the mind (for this [inquiry] is a function of philosophy), but insofar as they lead or can be of use in leading to the unknown. Thus a universal concept in the mind, when compared to the particulars under it, will be considered essential or accidental to them according as it enters into or lies outside their essences, and it will be considered a species if it coincides with those essences. . . . Now for a universal concept to be essential, accidental or a species or the like, is not something external but something that arises in universal natures when they exist in the mind. It is so with a proposition's being predicative or conditional and with an argument's being a syllogism, an induction or an example. . . . The logician [also] investigates tertiary and higher-level concepts, for these are essential attributes of secondary concepts. "Proposition," for example, is a secondary concept which may be investigated in regard to its division, conversion, or conclusiveness when combined with other propositions. Thus "conversion," "conclusiveness," "division," "contradiction" are concepts on the third level of thought; and if, in a logical inquiry, something is judged to be one of the divided parts or contradictories, then that thing will belong to the fourth level of thought, and so on.[13]

The gist of all this had already been said by Avicenna; only the idea of a multi-level hierarchy of concepts is lacking in Avicenna's writings. When and in what context did that idea become articulated; and with what consequences, if any? These are questions that must await further search of the enormous bulk of logical writings that relentlessly piled up in the centuries separating Avicenna from the author of the above passage.

IV

Avicenna further develops his views in chapters 3 and 4 of the *Introduction*, on the utility and the subject matter of logic, respectively. His discussion in chapter 3 begins with the famous distinction between *taṣawwur* and *taṣdīq* which we find in almost every Arabic writer on logic after Avicenna. The same terms have been found in the logical writings of al-Fārābī,[14] but their ultimate provenance

[13] 'Alā' ibn 'Alī al-Tahānawī, *Kashshāf iṣṭilāḥāt al-funūn*, A. Sprenger and W. Nassau Lee, eds., repr. (Beirut: Khayat, I, 1966), pp. 34/5; also IV (no date), p. 1035.

[14] See: Ibrahim Madkour, *L'Organon d'Aristote dans le monde arabe*, 2nd ed. (Paris: Vrin, 1969), pp. 53–56. (1st ed., 1934); Harry A. Wolfson, "The Terms *taṣawwur* and *taṣdīq* in Arabic Philosophy and Their Greek, Latin, and Hebrew

remains somewhat uncertain. Paul Kraus[15] suggested in 1936 that they translated the Stoic terms φαντασία and συγκατάθεσις, and, on *
the basis of this suggestion, M.-D. Chenu[16] has argued for a Stoic infiltration of medieval Latin thought by way of translating into Latin certain Arabic texts containing the words *taṣawwur* and *taṣdīq*.

Literally, *taṣawwur* is the act of grasping or receiving a form (*ṣūra*, εἶδος) in the mind, and *taṣdīq* is the act of taking or believing something to be true (*ṣādiq*). Both are described by Arabic logicians as acts of knowledge (*'ilm*), but truth and falsity are said to have to do only with the second. Often the two words are also applied, respectively, to the form received or the proposition believed to be true. 'Concept' and 'conception' usually do well as translations of '*taṣawwur*', ('thought' is also appropriate in some contexts); for rendering "*taṣdīq*" one has to oscillate between a number of words such as 'assertion', 'belief', 'judgment', 'proposition'. The medieval Latin translation of Avicenna's *Introduction* has *intellectus* for *taṣawwur* and *credulitas* for *taṣdīq*.

Whatever the origin of these two terms, Stoic or otherwise, a similar distinction to what they are meant to convey can be easily found in Aristotle, and it appears that Avicenna at least conflated the two distinctions. We read in Aristotle's *De interpretatione*, 16ᵃ9 ff:

> Just as some thoughts in the soul are neither true nor false while some are necessarily one or the other, so also with spoken sounds. For falsity and truth have to do with combination and separation. Thus names and verbs by themselves—for instance 'man' or 'white' when nothing further is added—are like the thoughts that are without combination and separation; for so far they are neither true nor false.[17]

Equivalents," *The Moslem World*, XXXIII (1943): 114–128; Wolfson refers to al-Fārābī's '*Uyūn al-masā'il* in F. Dieterici, *Alfārābī's philosophische Abhandlungen* (Leiden, 1890–1892), Arabic, p. 56; German, pp. 92–93; F. Jadaane, *L'Influence du stoïcisme sur la pensée musulmane* (Beyrouth: Dar El-Machreq, 1968), pp. 106–113, reviewed by F. Rosenthal in *Der Islam*, XLVI (1970): 89–91.

[15] *Recherches Philosophiques*, V (1935/1936): 498–500.

[16] "Un Vestige du stoïcisme," *Revue des sciences philosophiques et théologiques*, XXVII (1938): 63–68.

[17] Ackrill's translation in the Clarendon Aristotle Series. Compare also Aristotle's *De anima*, III. 9, 432ᵃ10: "But imagination [φαντασία] is different from assertion [φάσις] and denial; for truth and falsity involve a combination of thoughts [νοήματα]" (Hamlyn's translation in the Clarendon Aristotle Series). The Arabic version by Isḥāq ibn Ḥunayn has *tawahhum*, *ithbāt*, and *ma'ānī* for the bracketed Greek words; see 'Abd al-Raḥmān Badawī, *Arisṭūṭālis, Fī al-Nafs*, etc. (Cairo: Maktabat al-Nahḍa, 1954), p. 79. For the terminology of these distinctions in Averroes and the Latin medieval philosophers, see Chenut, *op. cit.*

The ninth-century Arabic translation of this passage (by Isḥāq ibn Ḥunayn) does not use the words *taṣawwur* and *taṣdīq*.[18] But in the corresponding chapter in Avicenna's *Interpretation* the plural *taṣawwurāt* is used interchangeably with *āthār*, the equivalent in Isḥāq's translation of Aristotle's παθήματα (ἐν τῇ ψυχῇ)[19]; and in a passage of the same chapter that parallels the lines just quoted from Aristotle, *ma'qūl* (= νόεμα) and *i'tiqād* (belief) are made to stand for *taṣawwur* and *taṣdīq*: a single thought (*ma'qūl*), says Avicenna, is neither true nor false; only the belief (*i'tiqād*) associated with relating one thought to another affirmatively or negatively is true or false (*al-'Ibāra*, 6: 1–4). Now there is no term in Aristotle's text that corresponds to Avicenna's *i'tiqād* (a word which usually rendered

* the Greek πίστις); only combination and separation of thoughts are said by Aristotle to have truth or falsity. But Avicenna clearly understands Aristotle's remarks in terms of a distinction between acts of conceiving single thoughts and acts of belief applied to the conceived relations between thoughts. How Avicenna himself understood the distinction is made abundantly clear in chapter 3 of the *Introduction:*

> . . . a thing is knowable in two ways: one of them is for the thing to be merely conceived (*yutaṣawwar: intelligatur*) so that when the name of the thing is uttered, its meaning (*ma'nā: intentio*) becomes present in the mind without there being truth or falsity, as when someone says 'man', or 'do this!' For when you understand the meaning of what has been said to you, you will have conceived it. The second is for the conception to be [accompanied] with belief (*taṣdīq: credulitas*), so that if someone says to you, for example, "every whiteness is an accident," you do not only have a conception (*taṣawwur*) of the meaning of this statement, but [also] believe it (*ṣaddaqta*) to be so. If, however, you doubt whether it is so or not, then you have conceived what is said, for you cannot doubt what you do not conceive or understand, but what you have gained through conception in this [latter] case is that the form of this composition and what it is composed of, such as "whiteness" and "accident," have been produced in the mind. Assertion (*taṣdīq*), however, occurs when there takes place in the mind a relating (*nisba: comparatio*) of this form to the things themselves as being in accordance with them: denial (*takdhīb: mentiri*) is the opposite of that (*al-Madkhal*, 17: 7–17. Latin trans., *ed. cit.*, fol. 2ᵛA, lines 27–46).

[18] See the edition by 'Abd al-Raḥmān Badawī in *Manṭiq Arisṭū*, I (Cairo: Maṭba'at Dār al-Kutub, 1948), p. 59.

[19] The title of the first chapter in Avicenna's *Interpretation* reads: "On the relations between things (*al-umūr*), concepts (*al-taṣawwurāt*), and spoken and written words. . . ," where '*taṣawwurāt*' clearly corresponds to Aristotle's "affections in the soul." See *al-Shifā'*, *al-Manṭiq*: 3. *al-'Ibāra* [M. El-Khodeiri, ed. (Cairo: Dār al-Kātib al-'Arabī, 1970), p. 1: 6]. The chapter then begins with the sentence: "Man is endowed with a sensitive faculty in which the forms (*ṣuwar*) of external things are impressed, and from which they proceed to the soul where they are impressed again permanently" (*ibid.*, p. 1: 8–9). Almost immediately Avicenna's language reverts to that of Isḥāq's version of the *De interpretatione*: "spoken sounds signify what are called affections (*āthār*) in the soul, and these signify the things (*al-umūr*) which are called meanings (*ma'ānī*), i.e. intentions (*maqāṣid*) of the soul, just as the affections also are meanings in relation to spoken words; and written words signify spoken words for they parallel the construction of the latter" (*ibid.*, p. 2: 15–p. 3: 3). It is interesting to note Avicenna's interpretation of '*ma'ānī*' which in Isḥāq's translation stood for πράγματα, real things. Cf. Badawī, ed., *Manṭiq Arisṭū*, I, *ed. cit.*, p. 59: 8.

It is clear from this text that *taṣdīq* is *not* the relation between subject and predicate in a predicative proposition. Such a relation is here called "form of composition" which (as in the case of doubting) can be entertained in the mind without truth or falsity being applied to it; that is, it can be the subject of mere conception (although, of course, unlike the conception of a single thought, it is *capable* of being described as true or false). *Taṣdīq* is the attribution of this relation or form to the things themselves.

The role of belief or assertion is again emphasized by Avicenna in his *Interpretation*, in an account of what a predicative statement is made of. "A predicative proposition (*qaḍiyya ḥamliyya*), he says, consists of three things, a subject-concept, a predicate-concept, and a relation (*nisba*) between the two. Concepts (*ma'ānī*) do not, however, become subjects and predicates by being gathered together in the mind; in addition to this the mind must believe (*ya'taqid*) affirmatively or negatively the relation between the two concepts" (*al-'Ibāra*, 37:15–38:3). He goes on to insist that mere concatenation (*tatālī*) of terms does not make up a statement. To be a complete expression of a predicative preposition a sentence must therefore contain, in addition to the terms indicating the subject and predicate, a sign that indicates the relation or connection between these. Such a sign is of course the copula (*rābiṭa*: connector) which, he says, may take the form of a verb (as in Greek or Persian or, sometimes, Arabic), or a noun (the Arabic pronoun *huwa*), or a vowel change (modifying the predicate term or both subject and predicate terms). In any case, three linguistic elements are needed to correspond, one to one, with the three essential components of a predicative proposition (38:4–39:3).

But if assertion is something apart from the relation to which it is applied in predicative propositions, should not such propositions be analyzed into four, rather than three, components, and should not their complete verbal expressions contain four, rather than three, elements? As far as I can see, the question is nowhere broached by

Avicenna himself, but it was raised by later Arabic logicians, no doubt led to do so by Avicenna's own remarks. Some argued that since a "relation of judgment" (*nisba ḥukmiyya*) is found equally in the affirmation and negation of that relation, it must be clearly distinguished from both; four components must therefore be recognized in the make-up of a predicative proposition. Others maintained that the copula would not be able to perform its function as a connector unless it signified both the judgment relation and its affirmation or negation. The copula would thus give expression to both the assertion (*taṣdīq*) made and the concept (*taṣawwur*) to which the assertion is applied, and there would thus be no need for a separate assertion sign. This is not the occasion to pursue this discussion in the various writers, but it seems that it was the latter view that finally prevailed. It was also the latter view that very likely expressed Avicenna's own implicit opinion.[20]

I have dwelt at some length on the distinction between *taṣawwur* and *taṣdīq* because it became the accepted doctrine of all Arabic logicians. As pointed out earlier, *taṣawwur* and *taṣdīq* divided between them the whole sphere of knowledge, the first being attainable by definition, the second by argument. Logic, being concerned with the appropriate means of acquiring knowledge, therefore divided into two parts: a theory of definition (*mabḥath al-taṣawwurāt*) and a theory of proof (*mabḥath al-taṣdīqāt*). The following passage succinctly expresses this pervasive doctrine. It comes from Avicenna's *Kitāb al-Najāh*,[21] a summary account of *Kitāb al-Shifā'*:

> Every knowledge is either conception (*taṣawwur*) or belief (*taṣdiq*). Conception is the prior knowledge (*al-'ilm al-awwal*) and it is acquired by means of definition or the like. . . . Belief is acquired only by means of syllogism or the like. . . . Thus definition and syllogism are two instruments by means of which knowledge of unknown things is acquired through discursive thought (*al-rawiyya*). . . . Now every syllogism and every definition is made up by bringing intelligible notions (*ma'ānī ma'qūla*) into a definite composition so that each would have a matter from which it is composed and by means of which the composition is effected. And just as a house or seat cannot properly be made from any chance matter or in any chance form, but rather every thing has its own matter and form which are proper to it, so also there belong to every object of knowledge (*ma'lūm*) which is knowable by means of discursive thought a proper matter and form by means of which that object may be grasped (*taḥaqquq*). And just as a house may be improperly built because of deficient matter or form or both, so also discursive thought may be vitiated (*fasād*) on account of its matter even if the form is valid (*ṣāliḥa*), or on account of the form even if the matter is appropriate (*ṣāliḥa*), or on account of both (3/4).

[20] A somewhat detailed picture of the discussions referred to here can be had from the commentaries assembled around the widely used compendium of logic, called *al-Risāla al-shamsiyya*, which Najm al-Dīn al-Kātibī composed in the thirteenth century (Rescher, *op. cit.*, pp. 203/4). See the edition of the *Risāla* (together with eight commentaries) published in Cairo in two volumes (I: al-Maṭba'a al-Amiriyya, 1323/1905; II: Maṭba'at Kurdistān al-'Ilmiyya, 1327), II, pp. 15 ff.

[21] Cairo: al Kurdi, 1938.

V

Fārābī had said in his *Enumeration of the Sciences* that the objects with which the rules of logic are concerned are "the thoughts (*ma'qūlāt*) in so far as they are indicated by the utterances, and the utterances in so far as they indicate the thoughts." We establish an opinion in ourselves by setting up in our minds those thoughts which are apt to verify it. This process is called by the ancients "inner speech" (*al-nuṭq al-dākhil*). To impart the truth of an opinion to someone else we employ the forms of speech (*aqāwīl*) suitable for achieving that purpose. This is called "outer speech" (*nuṭq khārij bi-al-ṣawt*). It is the function of logic to provide the rules (*qawānīn*) that guide us toward the proper conduct (called "*qiyās*," reasoning) of both kinds of speech.[22]

Avicenna seems to have had some such statements in mind when he wrote in chapter 3 of the *Introduction* (=chapter 4 in the Latin edition) that "there is no value in the doctrine of those who say that the subject of logic is to investigate utterances in so far as they indicate notions (*al-ma'ānī*)."[23] It is noticeable that in the two previous chapters the question of language and its relation to logic is nowhere brought into the discussion.[24] Now that Avicenna has put forward in those chapters his own view of the subject and use of logic, he feels he can settle that question without much belaboring

[22] Al-Fārābī, *Iḥṣā' al-'ulūm*, 'Uthmān Amīn, ed. (Cairo: Maktabat al-Khānjī, 1931), pp. 17/8. The corresponding text in Gerard's Latin translation occurs on pp. 133/4 of Palencia's edition cited in fn 1 above.

[23] *Al-Madkhal*, 23: 5/6: "non valet quod ille dixit, scilicet quod logyca instituta est ad considerandum dictionem secundum hoc quod significant intellecta" (*ed. cit.*, fol. 3rB: 19–21).

[24] Except that chapter 3 concludes with the statement that "this art [of logic] is related to internal reasoning (*al-rawiyya al-bāṭina*), which is called inner speech, in the same way as grammar is related to the outward expression called outer speech" (*al-Madkhal*, p. 20: 14/15). This is also strongly reminiscent of some of Fārābī's statements and language in the *Enumeration*, e.g., "the relation of the art of logic to reason and thoughts (*al-'aql wa al-ma'qūlāt*) is as the relation of grammar to language and utterances" (*ed. cit.*, p. 12: 5/6).

of words. Unfortunately his remarks are much too brief. The logician, he says, would have been able to dispense with utterances only if it were possible to learn logic by means of "pure thought." But we are forced to use utterances,

> . . . especially as it is not possible (*muta'adhdhir: non potest*) for the reasoning faculty (*al-rawiyya: ratio*) to arrange notions (*al-ma'ānī: intellecta*) without imagining the utterances corresponding to them, reasoning being rather a dialogue with oneself by means of imagined utterances. It follows that utterances have various modes (*aḥwāl*) on account of which the modes of the notions corresponding to them in the soul vary so as to acquire qualifications (*aḥkām*) which would not have existed without the utterances. [*sequitur ut verba habeant diversas dispositiones per quas differant dispositiones intentionum que concomitantur esse in anima, ita quod fiant eis indicia que non haberentur nisi per verba*]. It is for this reason that the art of logic must be concerned in part with investigating the modes of utterances. . . . But there is no value in the doctrine of those who say that the subject of logic is to investigate utterances in so far as they indicate notions . . . but rather the matter should be understood in the way we described (*al-Madkhal*, 22: 13-23: 4. Latin fol. 3ʳA: 60–B: 19).

The modes mentioned here are of course those secondary properties which concepts acquire when they constitute definitions and arguments. They are thoughts (*ma'qūlāt*) of a second order, twice removed from the things of the material world to which outward speech belongs. Avicenna now tells us that reasoning is impossible without utterances, whether spoken or imagined. By itself this is not a new thing to say. But the consequence he draws from this statement is not the simple language-thought parallelism noted by the writers whose views he found inadequate. He clearly says that the conceptual modifications are *brought about* by modifications in the utterances. This means that the secondary concepts, the proper object of logic, not only are reflected in language but are generated by it. Is this so because logical concepts arise only in the context of a process, reasoning, which is dependent on language in a peculiar way? In any case, however one interprets his words, and I am not sure I quite understand them, he seems to be making a stronger claim for the role of utterance in logic than I have encountered in any writer before and up to his time. Avicenna is not just saying that utterances are important in the study of logic. Having already pointed out that logic was not a (psychological) inquiry concerned with mental entities as such, he is now telling us that it is an inquiry primarily concerned with language. The reason is, or seems to be, that the properties constituting the subject matter of logic would be inconceivable without the exercise of a particular function of language.

VI

Avicenna returns to the question of the relation of logic to philosophy and the related question of the subject matter of logic in other parts of the Logic of *Kitāb al-Shifā'*. In the section on *Categories*, for example, he again asserts his independence from the Peripatetics (including Fārābī and Ibn al-Ṭayyib) by emphatically excluding the doctrine of categories from the proper domain of logic. This agrees with his understanding of logic as concerned with second-order concepts. And in the section of *Syllogism* he devotes a chapter to showing how the function of logic as an instrument is to be understood. His interesting views in these and in other places are, however, too detailed and too complex to be dealt with adequately here.[25]

CORRECTIONS

V

p. 83, line 9: delete this line and replace it by the following:
ha-'areṣ ha-'edim ha-daḳḳim ha-'olim mi-mennah we-'omar she-'ammud ha-
(The correction was noted in *Isis*, 58 (1967), NO. 194, p. 560.)

IX

p. 222, line 12: *for* decreases, *read* decrease
p. 230, line 31: for *TQ:QO*, read *TQ:QE*
p. 245, line 28: *for A*-passage, *read PH*-passage

XIV

p. 117, n. 1, line 3: *read* is extant
p. 118, line 16: *read* the traditionists (*aṣḥāb al-ḥadīth*)
p. 118, n. 6, line 2: *read* and through mean distances
p. 121, line 18: *read* his arguments
p. 121, line 22: *read* failed to discover
p. 123, line 24: *read* Ṭūsī's researches
p. 124, line 33: read *yuḥādhī*

XV

p. 145, line 9: The Abū Ḥanīfa, here conjectured to be the well-known founder of the legal school named after him, is perhaps more likely Abū Ḥanīfa al-Dīnawarī, the ninth-century philologist who wrote on Indian reckoning, algebra, astronomical phenomena and plants, and an extant historical work, *al-Akhbār al-ṭiwāl*. See the article on him by B. Lewin, in *Encyclopaedia of Islam*, new edition, under 'al-Dīnawarī'.

XVII

Throughout this chapter, the final ζ in every quoted Greek word should be replaced by a ς.

INDEX OF NAMES AND TERMS